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Beyond the Standard Model B-parameters with improved staggered fermions in $N_f = 2+1$ QCD

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We calculate the kaon mixing B-parameters for operators arising generically in theories of physics beyond the standard model. We use HYP-smeared improved staggered fermions on the $N_f = 2+1$ MILC asqtad lattices. Operator matching is done perturbatively at one-loop order. Chiral extrapolations are done using "golden combinations" in which one-loop chiral logarithms are absent. For the combined sea-quark mass and continuum extrapolation, we use three lattice spacings: $a \approx 0.045,\ 0.06$ and 0.09 fm. Our results have a total error of 5-6%, which is dominated by the systematic error from matching and continuum extrapolation. For two of the BSM B-parameters, we agree with results obtained using domain-wall and twisted-mass dynamical fermions, but we disagree by $(4-5)\sigma$ for the other two.

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1. Introduction

Kaon mixing, and in particular the CP-violating component ε_K, can provide powerful constraints on theories of physics beyond the standard model (BSM). BSM theories lead to contributions to the mixing amplitude which add to that from the standard model (SM). Given that we now know the SM contributions to ε_K quite accurately, there is little room for BSM additions.

In order to determine the constraints on the parameters of BSM models, one needs to know the matrix elements of the local four-fermion $\Delta S = 2$ operators that arise when new, BSM heavy particles are integrated out. It turns out that, in addition to the operator appearing in the short-distance component of the SM contribution (which has a "left-left" Dirac structure and whose matrix element is parametrized by B_K), BSM theories lead to four other operators. Here we present our results for the matrix elements of all five operators, and compare with those of other recent calculations [1, 2]. More details of our results are given in Ref. [3].

2. Methodology and Results

We adopt the operator basis used in Ref. [4],

$$\begin{aligned}
Q_1 &= [\bar{s}^a \gamma_\mu (1-\gamma_5) d^a][\bar{s}^b \gamma_\mu (1-\gamma_5) d^b], \\
Q_2 &= [\bar{s}^a (1-\gamma_5) d^a][\bar{s}^b (1-\gamma_5) d^b], \\
Q_3 &= [\bar{s}^a \sigma_{\mu\nu} (1-\gamma_5) d^a][\bar{s}^b \sigma_{\mu\nu} (1-\gamma_5) d^b], \\
Q_4 &= [\bar{s}^a (1-\gamma_5) d^a][\bar{s}^b (1+\gamma_5) d^b], \\
Q_5 &= [\bar{s}^a \gamma_\mu (1-\gamma_5) d^a][\bar{s}^b \gamma_\mu (1+\gamma_5) d^b],
\end{aligned} \tag{2.1}$$

where a, b are color indices and $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2$. Q_1 is the SM operator while Q_{2-5} are the BSM operators. Matrix elements of the latter are parametrized as

$$\begin{aligned}
B_i(\mu) &= \frac{\langle \overline{K}_0 | Q_i(\mu) | K_0 \rangle}{N_i \langle \overline{K}_0 | \bar{s}\gamma_5 d(\mu) | 0 \rangle \langle 0 | \bar{s}\gamma_5 d(\mu) | K_0 \rangle} \\
(N_2,\, N_3,\, N_4,\, N_5) &= (5/3,\, 4,\, -2,\, 4/3)\,.
\end{aligned} \tag{2.2}$$

The basis (2.1) is that in which the the two-loop anomalous dimensions (which we use for renormalization group running) are known [4]. It differs from the "SUSY" basis of Ref. [5] (which has been used in previous lattice calculations of the BSM matrix elements), although the two bases are simply related, as discussed below.

Table 1 shows the lattices used in this work. They are generated using $N_f = 2+1$ flavors of asqtad staggered quarks [6]. For valence quarks, we use HYP-smeared staggered fermions [7].

The calculation of the BSM B-parameters follows closely the methodology used in our B_K calculation [8, 9]. The lattice operators are matched, at one-loop order, to continuum operators defined in the renormalization scheme of Ref. [4]. This scheme differs slightly from that used in the continuum-lattice matching calculation of Ref. [10], requiring an additional (continuum) matching factor which we have calculated. The matching on each lattice is done at a renormalization scale $\mu = 1/a$, with results subsequently run to a common final scale using the continuum two-loop anomalous dimensions.

Table 1: MILC lattices used in this calculation. Here "ens" is the number of gauge configurations, "meas" is the number of measurements per configuration, and ID identifies the ensemble.

a (fm)	am_ℓ / am_s	size	ens × meas	ID
0.09	0.0062 / 0.031	$28^3 \times 96$	995×9	F1
0.09	0.0093 / 0.031	$28^3 \times 96$	949×9	F2
0.09	0.0031 / 0.031	$40^3 \times 96$	959×9	F3
0.09	0.0124 / 0.031	$28^3 \times 96$	1995×9	F4
0.09	0.00465 / 0.031	$32^3 \times 96$	651×9	F5
0.06	0.0036 / 0.018	$48^3 \times 144$	749×9	S1
0.06	0.0072 / 0.018	$48^3 \times 144$	593×9	S2
0.06	0.0025 / 0.018	$56^3 \times 144$	799×9	S3
0.06	0.0054 / 0.018	$48^3 \times 144$	582×9	S4
0.045	0.0028 / 0.014	$64^3 \times 192$	747×1	U1

We use ten different valence quark masses, $am_{x,y} = am_s \times \frac{n}{10}$ (with $n = 1-10$), where m_x and m_y are the valence d and s quark masses, respectively, while m_s (given in Table 1) lies close to the physical strange quark mass. Thus our valence pion masses run down almost to 200 MeV. Results are extrapolated to physical light-quark masses using SU(2) staggered chiral perturbation theory (SChPT), taking the lightest four quark masses for m_x and the heaviest three for m_y. Thus we remain in the regime, $m_x \ll m_y \sim m_s$, where we expect SU(2) ChPT to be applicable.

As an example of the quality of our data, we show, in Fig. 1, the plateaus that we find for B_2 on the three lattice spacings. This is for our most kaon-like choice of valence quark masses. We use U(1) noise sources to create kaons with a fixed separation, and place the lattice operators between them. From plots such as these, as well as those for kaon correlators, we determine how far from the sources we must work to avoid excited-state contamination. We then fit to a constant.

Generalizing a proposal of Ref. [11], Ref. [12] suggested that chiral extrapolations of BSM B-parameters would be simpler if one uses the following "golden combinations":

$$G_{23} = \frac{B_2}{B_3}, \quad G_{45} = \frac{B_4}{B_5}, \quad G_{24} = B_2 \cdot B_4, \quad G_{21} = \frac{B_2}{B_K}. \tag{2.3}$$

This is because the leading chiral logarithms, which appear at next-to-leading order (NLO) in ChPT, cancel in these quantities (if one uses SU(2) ChPT). This observation is particularly important for staggered fermions, since chiral logarithms introduce taste-breaking effects which normally have to be corrected for. Hence, we perform the chiral and continuum extrapolations using the golden combinations, as well as B_K, and obtain our final results by inverting Eq. (2.3). We obtain consistent results by directly extrapolating the B-parameters themselves using the SU(2) SChPT forms of Ref. [12], following the analysis method previously followed for B_K [8, 9].

To extrapolate in m_x, we fit the golden combinations to

$$G_i(\text{X-fit}) = c_1 + c_2 X + c_3 X^2 + c_4 X^2 \ln^2 X + c_5 X^2 \ln X + c_6 X^3 \,, \tag{2.4}$$

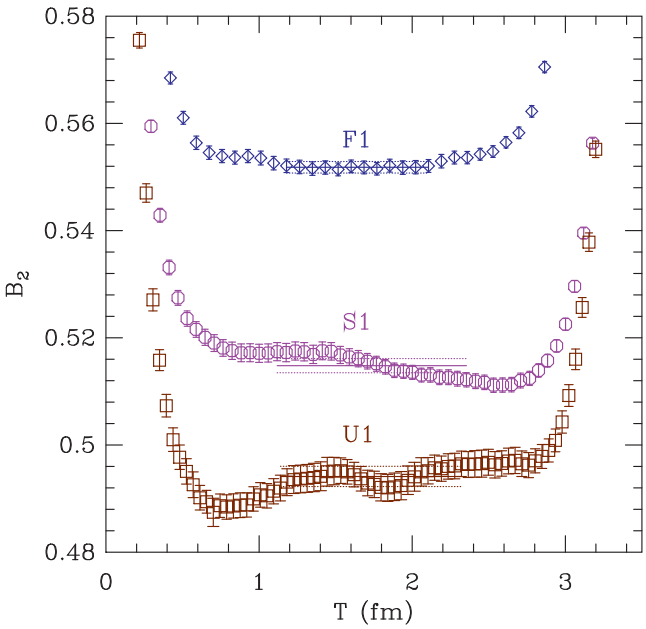


Figure 1: $B_2(\mu = 1/a)$ as a function of T, the distance between the left-hand kaon source and the operator. (Blue) diamonds, (purple) octagons and (brown) squares are from the F1, S1, and U1 ensembles, respectively, with $(m_x,\, m_y) = (m_s/10,\, m_s)$.

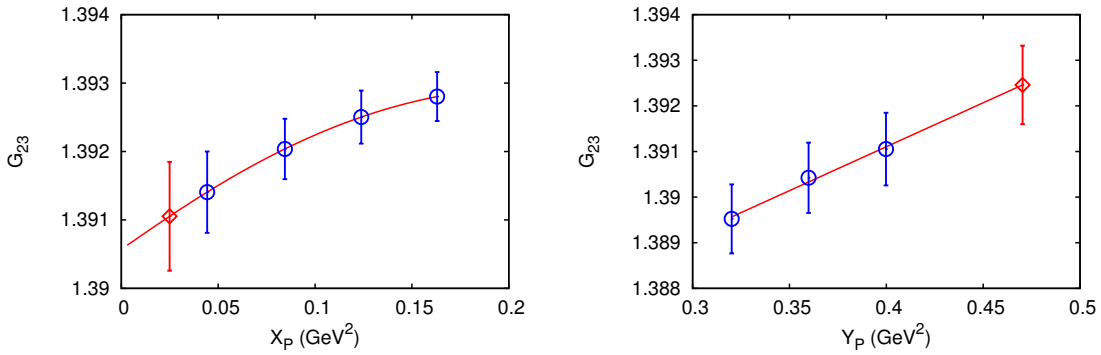
where $X \equiv X_P/\Lambda_\chi^2$ with $X_P = M^2_{\pi:x\bar{x}}$, and $\Lambda_\chi = 1\,\text{GeV}$, and generic NNLO continuum chiral logarithms are included. We call this the X-fit. To fit our four data points to this form, we constrain the coefficients c_{4-6} with Bayesian priors: $c_i = 0 \pm 1$. Systematic errors in X-fits are estimated by doubling the widths of these priors, and by comparing to the results of the eigenmode shift method [13]. The X-fits for B_K do involve NLO chiral logarithms, and we follow the same procedure as in Refs. [8, 9].

The extrapolation in the valence strange mass m_y is done using a linear function of $Y_P = M^2_{\pi:y\bar{y}}$, with a quadratic fit used to estimate a systematic error. We call this the Y-fit. In Fig. 2, we show X- and Y-fits for G_{23}, illustrating the mild dependence on the valence quark masses.

After these valence chiral extrapolations, we have, for each ensemble, results for the the G_i (and B_K) at scale $\mu = 1/a$. We next evolve these to a common scale, either 2 GeV or 3 GeV, using the two-loop anomalous dimensions of Ref. [4]. These results can then be extrapolated to the continuum and to physical sea-quark masses (with the dependence on the latter being analytic at NLO). We do a simultaneous extrapolation using

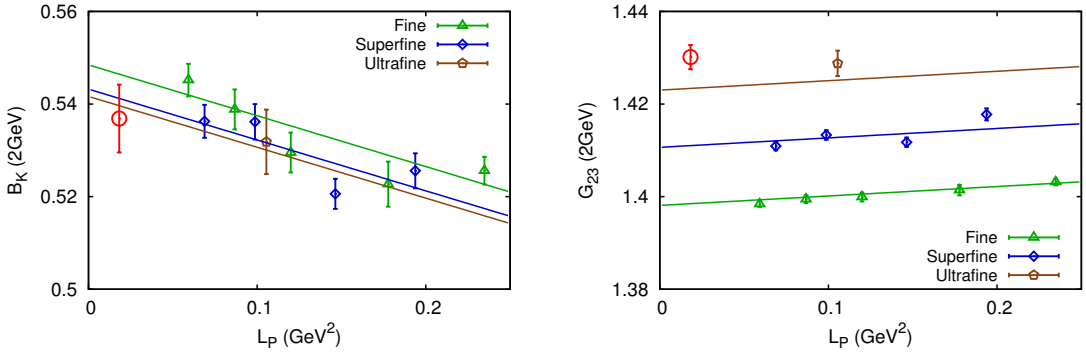
$$f_1 = d_1 + d_2(a\Lambda_Q)^2 + d_3 L_P/\Lambda_\chi^2 + d_4 S_P/\Lambda_\chi^2\,, \tag{2.5}$$

where L_P and S_P are squared masses of taste-ξ_5 pions composed of two light sea quarks and two strange sea quarks, respectively. Discretization errors are scaled with $\Lambda_Q = 300\,\text{MeV}$, so that for a typical discretization error one would have $d_2 \sim \mathcal{O}(1)$. However, we find that $d_2 \sim 2-7$ for the golden combinations, indicating enhanced lattice artifacts. The artifacts in B_K are much smaller. These results are illustrated in Fig. 3. The extrapolation fits have χ^2/dof in the range $1.6-2.7$.



(a) X-fit (b) Y-fit

Figure 2: (a) X-fit and (b) Y-fit of $G_{23}(\mu = 1/a)$ on F1 ensemble. In the X-fit, m_y is fixed at $am_y = 0.03$. The red diamond represents the extrapolated physical point in both plots.



(a) B_K (b) G_{23}

Figure 3: Examples of simultaneous sea-quark mass and continuum extrapolations. Red circles denote extrapolated results.

To estimate the systematic uncertainty in the continuum extrapolation, we compare the results of fitting to Eq. (2.5) with those obtained using the following fitting function, which includes higher order terms in a^2 and α_s [which here means $\alpha_s^{\overline{\rm MS}}(1/a)$]:

$$f_2 = f_1 + d_5(a\Lambda_Q)^2\alpha_s + d_6\alpha_s^2 + d_7(a\Lambda_Q)^4\,. \tag{2.6}$$

The difference, $\Delta f_{12} = |f_1 - f_2|$, is about 5%, and is comparable with our estimate of the systematic error coming from the one-loop matching, which we estimate by $\alpha_s^2(\mathrm{U1}) = 4.4\%$. Since Eq. (2.6) includes terms of α_s^2, it also captures the systematic error from using one-loop perturbative matching. Hence, we quote the larger of Δf_{12} and $\alpha_s^2(\mathrm{U1})$ in our final error budget for the matching/continuum extrapolation error.

In Table 2, we show our final results for the B-parameters evaluated at $\mu = 2\,\mathrm{GeV}$ and $3\,\mathrm{GeV}$. Total errors are $5-6\%$, and are dominated the systematic errors, as can be seen from the error budget given in Table 3. Further details concerning error estimates are explained in Ref. [3]. Overall, we see that the matching/continuum extrapolation dominates.

Table 2: Results for the B_i at $\mu = 2\,\text{GeV}$ and $3\,\text{GeV}$.

	$\mu = 2\,\text{GeV}$	$\mu = 3\,\text{GeV}$
B_K	0.537 (7)(24)	0.519 (7)(23)
B_2	0.620 (4)(31)	0.549 (3)(28)
B_3	0.433 (3)(19)	0.390 (2)(17)
B_4	1.081 (6)(48)	1.033 (6)(46)
B_5	0.853 (6)(49)	0.855 (6)(43)

Table 3: Error budget (in percent) for the $B_i(2\text{ GeV})$.

source of error	B_K	B_2	B_3	B_4	B_5
statistics	1.37	0.64	0.63	0.60	0.66
matching, cont-extrap.	4.40	4.95	4.40	4.40	5.69
X-fit (F1)	0.10	0.10	0.10	0.12	0.12
Y-fit (F1)	0.62	0.12	0.19	0.22	0.16
finite volume	0.50	0.50	0.50	0.50	0.50
$r_1 = 0.3117(22)$ fm	0.34	0.18	0.17	0.05	0.02
$f_\pi = 132$ vs. $124\,\text{MeV}$ (F1)	0.46	0.46	0.46	0.46	0.46

3. Comparisons and Outlook

As noted above, there are two previous calculations of the BSM B-parameters using dynamical quarks: one using $N_f = 2+1$ flavors of domain-wall fermions at a single lattice spacing [1], the other using $N_f = 2$ flavors of twisted-mass fermions at three lattice spacings [2]. The results from these two calculations are consistent. We pick those of Ref. [1] for a detailed comparison with our results, since then both calculations use the same number of dynamical flavors.

We begin by noting that the results for B_K in all three calculations are consistent (within the small errors). For the BSM B-parameters, Ref. [1] finds, at $\mu = 3\,\text{GeV}$, that $B_{2-5}^{\text{SUSY}} = 0.43(5)$, $0.75(9)$, $0.69(7)$ and $0.47(6)$, respectively. These are calculated in the SUSY basis, in which $B_3^{\text{SUSY}} = (5B_2 - 3B_3)/2$, while the other three B-parameters are the same. Our results convert (at $\mu = 3\,\text{GeV}$) to $B_3^{\text{SUSY}} = 0.79(3)$. Using this, and Table 2, we see that, while B_2 and B_3 are consistent, B_4 and B_5 are not. Our results are 1.5 and 1.8 times larger, respectively, a difference of $4-5\sigma$.

At the present time, we do not know the origin of this difference. We have ruled out the simplest possibilities, such as using incorrect continuum anomalous dimensions, by various cross checks with results in the literature. It thus seems that systematic errors in one or both calculations are being underestimated, the most likely culprit being the estimate of the truncation error in matching. This enters at the two-loop level in both calculations, since both are ultimately connected to a

continuum scheme by one-loop matching.

We think it is important to resolve this disagreement, not only because the matrix elements are wanted for phenomenology, but also because it may impact the reliability of other calculations. One way to proceed is for all calculations to use a common (S)MOM scheme and non-perturbative renormalization and running. This avoids the matching to the continuum. We are working in this direction. Another idea is for the other calculations to do the chiral extrapolations using the golden combinations described above.

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