

Investigating a mixed action approach for η and η' mesons in $N_f = 2 + 1 + 1$ lattice QCD

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We present results for a test of a mixed action approach with Osterwalder-Seiler valence quarks on a Wilson twisted mass sea for the example η , η' mesons. Flavour singlet pseudoscalar mesons obtain significant contributions from disconnected diagrams and are, therefore, expected to be particularly sensitive to mixed regularisations. We employ different procedures for matching valence and sea quark actions and show that the results agree in the continuum limit.

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1. Introduction

Mixed action approaches, where valence and sea fermion actions are chosen differently, are used frequently in lattice QCD. They possess a number of important advantages compared to the so called *unitary* case, where valence and sea quark actions are identical. In particular, it is possible to use a valence action obeying more symmetries than the sea action in cases where the valence action cannot be used in the sea for theoretical reasons or because of too high computational costs. One can even go one step further and try to correct for small mismatches in bare parameters in the sea simulation by using a partially quenched mixed action approach.

Of course, a mixed action approach has also disadvantages, most prominently the breaking of unitarity, which might for instance drive certain correlators negative. Also, it is not clear a priori how big lattice artifacts one encounters in mixed formulations.

In this proceeding contribution we will present results on a mixed action approach with so called Osterwalder-Seiler [1] valence quarks on a $N_f = 2 + 1 + 1$ flavour Wilson twisted mass sea [2] and compare to unitary results [3, 4, 5]. This particular action combination has the advantage that the flavour symmetry breaking present in the sea formulation is avoided in the valence formulation. Moreover, the matching of valence and sea actions is particularly simple [1]. As physical example we study the η and η' system. The corresponding correlation functions obtain significant contributions from disconnected diagrams and are, therefore, uniquely sensitive to differences in between valence and sea formulations¹. We study the continuum limit with different matching conditions and find remarkably good agreement to the unitary case. First accounts of this work can be found in Ref. [8].

2. Lattice actions

The results we will present in this proceeding are obtained by evaluating gauge configuration provided by the European Twisted Mass Collaboration (ETMC) [9, 10, 11]. We use the ensembles specified in table 1 adopting the notation from Ref. [9]. More details can be found in this reference. All errors are computed using a blocked bootstrap procedure to account for autocorrelation.

The sea quark formulation is the Wilson twisted mass formulation with $N_f = 2 + 1 + 1$ dynamical quark flavours. The Dirac operator for u and d quarks reads [2]

$$D_\ell = D_W + m_0 + i\mu_\ell \gamma_5 \tau^3, \quad (2.1)$$

where D_W denotes the standard Wilson operator and μ_ℓ the bare light twisted mass parameter. For the heavy doublet of c and s quarks [12] the Dirac operator is given by

$$D_h = D_W + m_0 + i\mu_\sigma \gamma_5 \tau^1 + \mu_\delta \tau^3, \quad (2.2)$$

with τ^i representing the Pauli matrices in flavour space. We remark that the twisted term introduces flavour mixing between strange and charm quarks that needs to be taken into account in the analysis.

¹Note that this was also discussed in the context of the validity of the fourth root trick in staggered simulations, see Refs. [6, 7] and references therein.

The bare Wilson quark mass m_0 has been tuned to its critical value m_{crit} [13, 9] to achieve automatic order $\mathcal{O}(a)$ improvement [14], which is one of the main advantages of this formulation.

In the valence sector we employ the Osterwalder-Seiler (OS) action [1]. Formally, we introduce a twisted doublet both for valence strange and charm quarks [1, 15]. The Dirac operator for a single quark flavour $q \in \{s, c\}$

$$D_{q,q'} = D_W + m_{\text{crit}} \pm i\mu_q \gamma_5 \quad (2.3)$$

is formally identical to the one in the light sector Eq. 2.1. Flavour q (q') will come with $+\mu_q$ ($-\mu_q$). We denote the strange (charm) quark mass with μ_s (μ_c). In Ref. [1] it was shown that automatic $\mathcal{O}(a)$ -improvement stays valid and unitarity is restored in the continuum limit.

For matching the strange quark mass we employ two procedures based on meson masses. In previous studies it was found that matching kaon masses is best in the sense that the residual lattice artifacts in the results computed in a mixed framework are small [16]. The corresponding interpolating operator in the OS framework reads

$$\mathcal{O}_{K^+} = \bar{\psi}_s i\gamma_5 \psi_d. \quad (2.4)$$

Note that we rely to the so called physical basis throughout this proceeding contribution [17]. For details on how to compute the kaon mass in the unitary case we refer to Ref. [10]. As a second matching observable we use the mass M_{η_s} of the so-called η_s meson – a pion made out of strange quarks which does not exist in nature. M_{η_s} can be obtained from the connected only correlation function of the OS interpolating operator

$$\mathcal{O}_{\eta_s} = \frac{1}{\sqrt{2}} (\bar{\psi}_s i\gamma_5 \psi_s + \bar{\psi}_{s'} i\gamma_5 \psi_{s'}). \quad (2.5)$$

In both cases we tune the value of $a\mu_s$ such that the kaon (η_s) masses agree within error in between the mixed and the unitary formulation. In order to compute the matching values for μ_s we performed inversions in a range of $a\mu_s$ values around a first guess obtained from $\mu_s = \mu_\sigma - Z_P/Z_S \mu_\delta$, computed M_{K^+} and M_{η_s} , and interpolated to the matching point where needed. The matching values for $a\mu_s$ for the two matching observables and all ensembles can be found in table 1.

The value of the charm quark mass turns out to be not very important for our investigation, because the charm does not contribute significantly to η and η' mesons. Therefore, we use the following relations for the bare twisted quark mass parameters to obtain

$$\mu_c = \mu_\sigma + Z_P/Z_S \mu_\delta. \quad (2.6)$$

The value for the ratio of renormalisation constants can be found in Ref. [18]. The actual values for μ_c can again be found in table 1.

3. Pseudoscalar flavour-singlet mesons

In order to extract η and η' states we compute the Euclidean correlation functions

$$\mathcal{C}(t)_{qq'} = \langle \mathcal{O}_q(t'+t) \mathcal{O}_{q'}(t') \rangle, \quad q, q' \in \{u, s, c\}, \quad (3.1)$$

ensemble	β	$a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	$a\mu_s^{K^+}$	$a\mu_s^{\eta_s}$	$a\mu_c$	L/a	N_{conf}
A60.24	1.90	0.0060	0.150	0.190	0.0232	0.0138	0.2768	24	1177
B55.32	1.95	0.0055	0.135	0.170	0.0186	0.0110	0.2514	32	1964
D45.32sc	2.10	0.0045	0.0937	0.1077	0.0149	0.0118	0.1720	32	885

Table 1: The ensembles used in this investigation. For the labeling we employ the notation of ref. [9]. Additionally, we give the number of the configurations N_{conf} which were used to compute correlators in light and strange quark sector, the mass of the OS valence strange quark μ_s for M_{K^+} and M_{η_s} matching and the OS valence charm quark mass μ_c .

with operators $\mathcal{O}_q = (\bar{q}i\gamma_5 q + \bar{q}'i\gamma_5 q')/\sqrt{2}$, with $q \in \{u, s, c\}$ and identifying $u' \equiv d$, again relying to the physical basis. The generalised eigenvalue problem is solved [19, 20, 21] for determining M_η and $M_{\eta'}$. Correlation functions are made of quark connected diagrams and disconnected quark loops. The quark connected pieces have been calculated via the so called ‘‘one-end-trick’’ [22] using stochastic timeslice sources and for the disconnected diagrams we resort to stochastic volume sources with complex Gaussian noise.

In the twisted mass formulation a very powerful variance reduction method is available for estimating the disconnected loop $(\bar{u}i\gamma_5 u + \bar{d}'i\gamma_5 d')/\sqrt{2}$, see Ref. [23]. In the OS case this variance reduction also applies to strange and charm disconnected loops [8]. Double counting of loops stemming from this approach needs to be taken into account by combinatorial factors.

Finally, we can define mixing angles ϕ_l, ϕ_s in the quark flavour basis using the pseudoscalar matrix elements, see Ref. [3]. From chiral perturbation theory combined with large N_C arguments $|\phi_l - \phi_s|/|\phi_l + \phi_s| \ll 1$ can be inferred according to Refs. [24, 25, 26, 27] which is confirmed by lattice QCD [4]. Therefore, we will consider only the average mixing angle

$$\tan^2 \phi \equiv \frac{A_{l,\eta'} A_{s,\eta}}{A_{l,\eta} A_{s,\eta'}}, \quad (3.2)$$

where the $A_{q,n} = \langle 0 | \bar{q}i\gamma_5 q | n \rangle$ are pseudoscalar matrix elements determined from the eigenvectors with $n \in \{\eta, \eta'\}$ and $q \in \{l, s\}$.

Excited State Removal

To improve the η' (and η) mass determinations, we use a method first proposed in Ref. [28], successfully applied for the η_2 (the η' in $N_f = 2$ flavour QCD) in Ref. [23] and very recently to the $N_f = 2 + 1 + 1$ case in Ref. [4]. It grounds on the assumption that disconnected contributions are significant only for the η and η' state, but negligible for higher excited states. The method involves to subtract excited states from the connected correlators only. The subtracted connected and full disconnected are combined in \mathcal{C} , which is then used in the analysis. We refer to the discussion in Ref. [5, 4] for more details.

4. Results

In order to compare the mixed case with the unitary case we match the two actions as detailed in the previous sections using either the kaon or the η_s mass. Next we compute M_η^{OS} at this match-

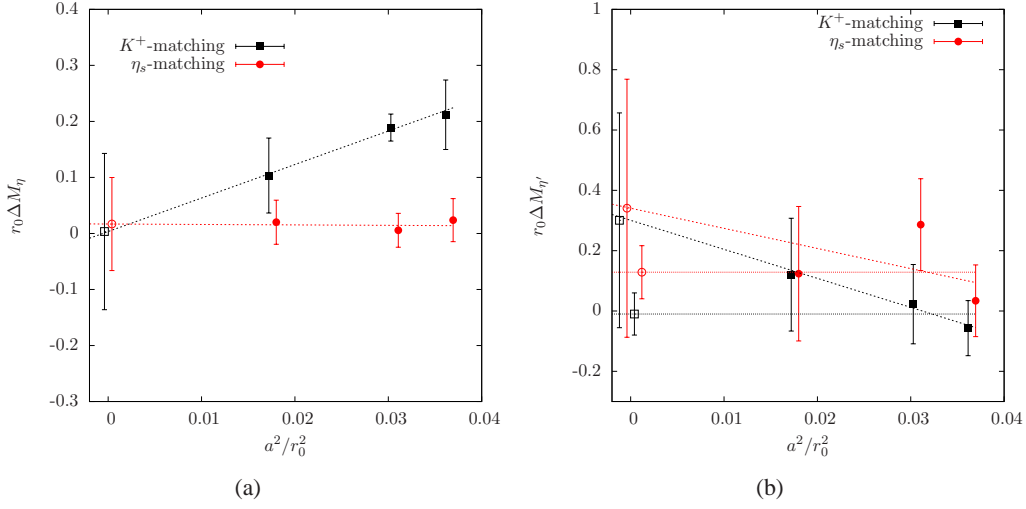


Figure 1: (a) We show ΔM_η for three ensembles (D45.32sc, B55.32, A60.24) as a function of the squared lattice spacing. Leftmost points correspond to linear continuum extrapolations indicated by the lines. (b) the same as (a) but for the η' state, for which we additionally show constant extrapolations.

ing points and the difference to the corresponding unitary masses ΔM_η . At fixed values of $r_0 M_\pi$ and $r_0 M_K$ – which is approximately the case for the ensembles D45, B55 and A60, $r_0 \Delta M_\eta$ should go to zero in the continuum limit with a rate of $\mathcal{O}(a^2)$. We do not expect the small differences in $r_0 M_K$ in between the different lattice spacing values to influence our results much, because unitary and OS data are affected likewise.

$r_0 \Delta M_\eta$ is shown in the left panel of figure 1 as a function of a^2/r_0^2 . For both matching observables we observe a linear dependence in a^2/r_0^2 . A corresponding continuum extrapolation in a^2/r_0^2 leads to the expected vanishing of this difference at $a = 0$. Kaon matching clearly exhibits larger differences, while η_s matching gives $r_0 \Delta M_\eta$ compatible with zero for each value of the lattice spacing separately. This indicates smaller a^2 artifacts for η_s matching because the unitary masses show constant scaling in a^2 [3]. $r_0 M_{\eta'}$ is shown in figure 1(b), which is in most cases compatible with zero even at finite lattice spacing. In the left panel of figure 2 we show $\Delta\phi$, again for both matching procedures, with the same conclusion.

As discussed in the introduction, one can correct for small mismatches in the bare simulation parameters used for the sea action by going slightly partially quenched in the valence sector. In order to test also this approach, we have tuned the respective $a\mu_s$ values for the ensembles D45, B55 and A60 such that all ensembles yield the same values of $r_0 M_K = 1.341$ or $r_0 M_{\eta_s} = 1.571$.

At these matching points we again compute M_η . In the right panel of figure 2 we show $r_0 M_\eta$ as a function of a^2/r_0^2 for both matching conditions. We observe that, within statistical uncertainties, both matching procedures lead to the same continuum results.

A direct comparison to the unitary case is possible by also correcting the unitary values of M_η for the small mismatch in the bare strange quark mass values. This is possible by measuring dM_η^2/dM_K^2 and use it to shift all M_η values to the same value of $r_0 M_K = 1.341$ [3]. The result is again shown in the right panel of figure 2. The corresponding continuum extrapolation is linear in a^2/r_0^2 and also compatible to both OS continuum points within errors.

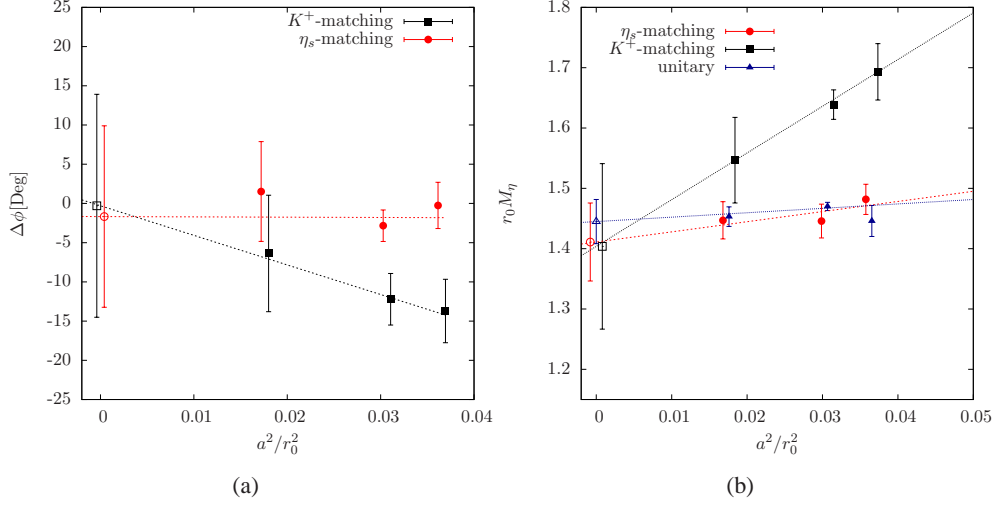


Figure 2: (a) We show $\Delta\phi$ for three ensembles (D45.32sc, B55.32, A60.24) as a function of the squared lattice spacing. Leftmost points correspond to linear continuum extrapolations indicated by the lines. (b) r_0M_η as a function of $(a/r_0)^2$ for the three ensembles. Different symbols correspond to two OS results with different matching conditions and to the unitary results. The leftmost points represent the corresponding continuum extrapolated values.

5. Conclusions and Outlook

We tested a mixed action approach with Osterwalder-Seiler valence quarks on a Wilson twisted mass sea for the η and η' system. We expect this system to be most sensitive to different valence and sea discretisation due to significant disconnected contributions in the correlation functions.

We employed two different conditions to match valence and sea actions, using the kaon mass and the η_s meson mass. For both matching procedures we observe agreement in the continuum limit, and in particular also with the unitary results. Moreover, we have also corrected for small mismatches in the bare strange quark mass value used in the gauge configuration generation, leading to a partially quenched mixed action approach. Also in this case we found within errors identical continuum extrapolated values for M_η and the mixing angle ϕ for both matching procedures.

By correcting also the unitary values of M_η we found agreement to the OS results. Therefore, we conclude that the mixed action approach can be used in the delicate case of the η and η' meson, at least to the precision that we could achieve here.

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