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The form factors for $B \to \pi l \nu$ semileptonic decay from $2+1$ flavors of domain-wall fermions

Taichi Kawanai*
Theoretical Research Division, Nishina Center, RIKEN, Wako 351-0198, Japan
Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA
RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA
E-mail: taichi.kawanai@riken.jp

Ruth S. Van de Water
Theoretical Physics Department, Fermi National Accelerator Laboratory, Batavia, IL 60510, USA
E-mail: ruthv@fnal.gov

Oliver Witzel
Center for Computational Science, Boston University, 3 Cummington Mall, Boston, MA 02215, USA
E-mail: owitzel@bu.edu

We present a calculation of the $B \to \pi l \nu$ form factors with domain-wall light quarks and relativistic b-quarks on the lattice. We work with the $2+1$ flavor domain-wall fermion and Iwasaki gauge-field ensembles generated by the RBC and UKQCD Collaborations. The chiral-continuum extrapolation is performed using SU(2) hard-pion chiral perturbation theory. To extrapolate the lattice form factors to the full kinematic range, we use the model independent z-parameterization and impose the kinematic constraint $f_+(0) = f_0(0)$ at zero momentum transfer.

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*Speaker.

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1. Introduction

The precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$ provides a strong test of the standard model. The lattice calculation of the hadronic form factor f_+ plays an essential role in the determination of $|V_{ub}|$ from $B \to \pi l \nu$ exclusive semileptonic decay. When combined with the hadronic form factor $f_+(q^2)$, the value of $|V_{ub}|$ can be obtained by experimental measurements of the differential decay rate via

$$\frac{d\Gamma(B \to \pi l \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \left[(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2\right]^{3/2} |f_+(q^2)|^2, \tag{1.1}$$

where the momentum transfer $q^\mu \equiv p_B^\mu - p_\pi^\mu$ and we neglect the mass of the outgoing lepton. The form factors f_+ and f_0 parameterize the hadronic element of the $b \to u$ vector current $\mathcal{V}^\mu \equiv i\bar{u}\gamma^\mu b$:

$$\langle \pi | \mathcal{V}^\mu | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu. \tag{1.2}$$

Nonperturbative lattice-QCD provides a first-principles method for computing the $B \to \pi$ form factors with controlled uncertainties.

In these proceedings, we report on a lattice-QCD calculation of the $B \to \pi$ form factor with domain-wall light quarks using the $2+1$ flavor domain-wall fermion and Iwasaki gauge-field ensembles generated by the RBC and UKQCD Collaborations. There have been two $2+1$ flavor lattice calculations of $f_+(q^2)$ done by the HPQCD [1] and FNAL/MILC Collaborations [2]. Both groups use the MILC gauge configurations. Our calculation will provide an important independent check on existing calculations that use staggered light quarks.

2. Methodology

In practice, we calculate the form factors $f_\parallel$ and $f_\perp$, which are more convenient for lattice simulations:

$$\langle \pi | \mathcal{V}^\mu | B \rangle = \sqrt{2m_B}[v^\mu f_\parallel(E_\pi) + p_\perp^\mu f_\perp(E_\pi)], \tag{2.1}$$

where $v^\mu = p_B^\mu / m_B$ and $p_\perp^\mu = p_\pi^\mu - (p_\pi \cdot v) v^\mu$. In the B-meson rest frame, these form factors are directly proportional to the hadronic matrix elements of the temporal and spatial vector current:

$$f_\parallel = \frac{\langle \pi | \mathcal{V}^0 | B \rangle}{\sqrt{2m_B}}, \quad f_\perp = \frac{\langle \pi | \mathcal{V}^i | B \rangle}{\sqrt{2m_B}} \frac{1}{p_\pi^i}, \tag{2.2}$$

and the desired form factors f_+ and f_0 can be obtained by the following relations:

$$f_+(q^2) = \frac{1}{\sqrt{2m_B}}[f_\parallel(E_\pi) + (m_B - E_\pi) f_\perp(E_\pi)] \tag{2.3}$$

$$f_0(q^2) = \frac{\sqrt{2m_B}}{m_B^2 - m_\pi^2}\left[(m_B - E_\pi) f_\parallel(E_\pi) + (E_\pi^2 - m_\pi^2) f_\perp(E_\pi)\right]. \tag{2.4}$$

We employ the mostly nonperturbative method of Ref. [4] to match the lattice amplitude to the continuum matrix element:

$$\langle \pi | \mathcal{V}^\mu | B \rangle = Z_{V_\mu}^{bl} \langle \pi | V^\mu | B \rangle \ \text{ with } \ Z_{V_\mu}^{bl} = \rho_{V_\mu}^{bl} \sqrt{Z_V^{bb} Z_V^{ll}}. \tag{2.5}$$

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Table 1: Heavy-light current renormalization factors used in our analysis. The values of $\rho^{bl}_{V_\mu}$ are obtained in the chiral limit [3].

	$a \approx 0.11$ fm			$a \approx 0.086$ fm	
	$am_l = 0.005$	$am_l = 0.01$	$am_l = 0.004$	$am_l = 0.006$	$am_l = 0.008$
Z_V^{ll}	0.71732(14)	0.71783(15)	0.745053(54)	0.745222(45)	0.745328(48)
Z_V^{bb}	10.037(34)	10.042(37)	5.270(13)	5.237(12)	5.267(15)
$\rho^{bl}_{V_0}$	1.02658			1.01661	
$\rho^{bl}_{V_i}$	0.99723			0.99398	

Table 2: Lattice simulation parameters.

a [fm]	$L^3 \times T$	am_l	am_s	M_π [MeV]	# configs.	# time sources
≈ 0.11	$24^3 \times 64$	0.005	0.040	329	1636	1
≈ 0.11	$24^3 \times 64$	0.010	0.040	422	1419	1
≈ 0.086	$32^3 \times 64$	0.004	0.030	289	628	2
≈ 0.086	$32^3 \times 64$	0.006	0.030	345	889	2
≈ 0.086	$32^3 \times 64$	0.008	0.030	394	544	2

The flavor-conserving renormalization factors Z_V^{bb} and Z_V^{ll} are computed nonperturbatively on the lattice and the factor $\rho^{bl}_{V_\mu}$ is computed at one loop in mean-field improved lattice perturbation theory [5]. Most of the heavy-light current renormalization factor comes from Z_V^{bb} and Z_V^{ll}, such that $\rho^{bl}_{V_\mu}$ is expected to be close to unity [6]. In this study, the factor Z_V^{bb} is calculated using the charge-normalization condition $Z_V^{bb}\langle B_s|V^{bb,0}|B_s\rangle = 2m_B$ where $V^{bb,0}$ is the $b \to b$ lattice vector current. We use the values of Z_V^{ll} obtained by the RBC/UKQCD collaborations by exploiting the fact that $Z_A = Z_V$ for domain-wall fermions [7]. The renormalization factors used in this analysis are summarized in Table 1.

We improve the $b \to u$ vector current through $\mathcal{O}(\alpha_S a)$. At this order we need only compute one additional matrix element with a single-derivative operator. We calculate the improvement coefficient at 1-loop in mean-field improved lattice perturbation theory.

3. Computational setup

We use the $2+1$ flavor domain-wall fermion and Iwasaki gauge-field ensembles generated by the RBC and UKQCD Collaborations [7, 8]. The simulation parameters are summarized in Table 2. We use the domain-wall action for the light valence-quark propagators, and unitary pion masses. On the finer ensembles we compute two quark propagators on each configuration with their temporal source locations separated by $T/2$ to increase the statistics. Periodic boundary conditions are imposed in the time direction. Additional details of our computational setup are given in Ref. [9].

For the bottom quark, we use the relativistic heavy quark (RHQ) action [10] to control heavy-quark discretization errors introduced by the large bottom quark mass [11]. For the parameters of the RHQ action, we need calibration of three parameters (the bare-quark mass $m_0 a$, the clover

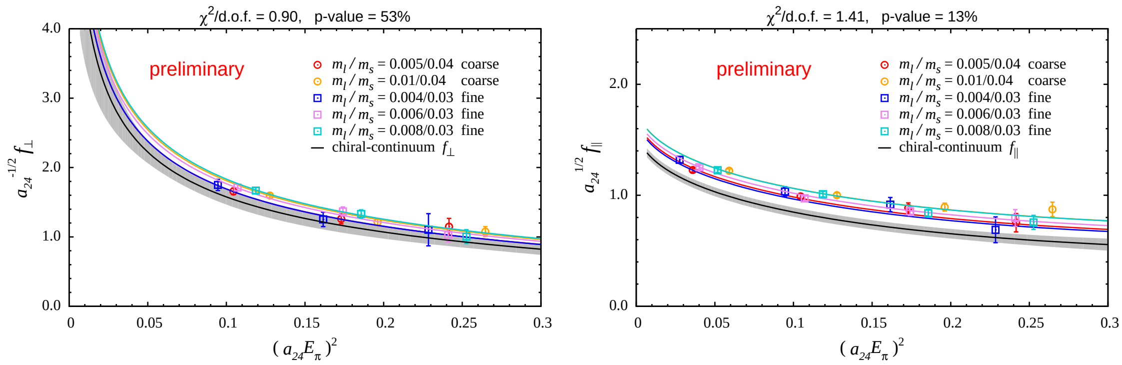

Figure 1: The $B \to \pi$ form factors $f_\perp$ (left) and $f_\parallel$ (right), shown in units of the lattice spacing on the 24^3 ensembles. The black curves with gray error bands show the chiral-continuum extrapolated $f_\parallel$ and $f_\perp$ with statistical errors.

coefficient c_P, and the anisotropy parameter ξ) [10, 12]. Here we employ values determined non-perturbatively in Ref. [13].

4. Analysis

4.1 Lattice form factors $f_\parallel^{\text{lat}}$ and $f_\perp^{\text{lat}}$

The lattice form factors $f_\parallel^{\text{lat}}$ and $f_\perp^{\text{lat}}$ are obtained from the following ratios of correlation functions at large source-sink separation:

$$R_{3,\mu}^{B\to\pi}(E_\pi, t, t_{\text{snk}}) = \frac{C_{3,\mu}^{B\to\pi}(E_\pi, t, t_{\text{snk}})}{\sqrt{C_2^\pi(E_\pi, t) C_2^B(t_{\text{snk}} - t)}} \sqrt{\frac{2E_\pi}{e^{-E_\pi t} e^{-m_B(t_{\text{snk}} - t)}}}. \tag{4.1}$$

After multiplying the results for $R_{3,0}^{B\to\pi}$ and $R_{3,i}^{B\to\pi}$ by Z_V^{bl}:

$$f_\perp(E_\pi) = Z_{V_i}^{bl} \lim_{t, t_{\text{snk}} \to \infty} \frac{1}{p_i} R_{3,i}^{B\to\pi}(E_\pi, t, t_{\text{snk}}) \tag{4.2}$$

$$f_\parallel(E_\pi) = Z_{V_0}^{bl} \lim_{t, t_{\text{snk}} \to \infty} R_{3,0}^{B\to\pi}(E_\pi, t, t_{\text{snk}}), \tag{4.3}$$

we obtain the renormalized form factors $f_\parallel$ and $f_\perp$ as a function of pion energy on each ensemble, shown in Fig. 1. We use the form factor data through momentum $\vec{p} = 2\pi(1,1,1)/L$ in this analysis.

4.2 Chiral-continuum extrapolation

In order to extrapolate simultaneously the form factors to the physical light-quark mass and the continuum, we employ NLO $SU(2)$ hard-pion chiral perturbation theory [14] suitably modified to incorporate leading discretization errors from the domain-wall and Iwasaki actions. Thus the fit functions depend on the pion mass m_{ll}, pion-energy E_π and squared lattice spacing a^2:

$$f_\perp(m_{ll}, E_\pi, a^2) = \frac{c_\perp^{(1)}}{E_\pi + \Delta} \left(1 + \delta f_\perp + c_\perp^{(2)} m_{ll}^2 + c_\perp^{(4)} E_\pi + c_\perp^{(5)} E_\pi^2 + c_\perp^{(6)} a^2\right) \tag{4.4}$$

$$f_\parallel(m_{ll}, E_\pi, a^2) = c_\parallel^{(1)} \left(1 + \delta f_\parallel + c_\parallel^{(2)} m_{ll}^2 + c_\parallel^{(4)} E_\pi + c_\parallel^{(5)} E_\pi^2 + c_\parallel^{(6)} a^2\right), \tag{4.5}$$

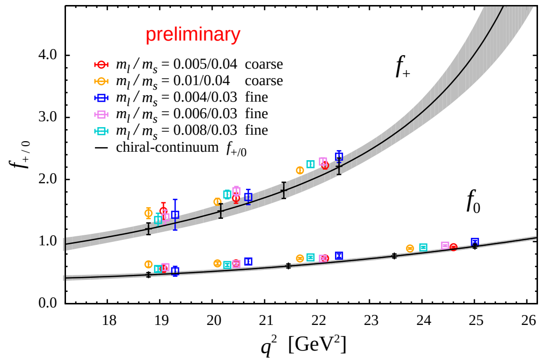

Figure 2: The $B \to \pi$ form factors f_+ and f_0 The black curves with gray error bands show the chiral-continuum extrapolated form factors with statistical errors with the four evenly-spaced synthetic data points used in the q^2 extrapolation overlaid. Errors shown are statistical only.

where the quantity Δ is the mass difference $m_{B^*} - m_B$, fixed to the experimental value [15], and ensures the proper location of the pole at the B^* mass in the physical form factor f_+. The function δf contains logarithmic functions of the pion mass:

$$(4\pi f_\pi)^2 \delta f_{\parallel/\perp} = -\frac{9g^2}{4} I_1(m_{ll}) - I_1(m_{ll}) + \frac{1}{4} I_1(m_{ll}) + \frac{1}{4}(m_{ll}^2 - m_{ll}^2)\frac{\partial I_1(m_{ll})}{\partial m_{ll}^2}, \tag{4.6}$$

where $I_1(m_{ll}) = m_{ll}^2 \log(m_{ll}^2/\Lambda^2)$ and $f_\pi = 130.4$ MeV [15]. For the parameter g, we use $g_{B^*B\pi} = 0.569$ calculated as a part of this RHQ project [16]. Fig. 1 shows the resulting chiral-continuum extrapolation of $f_\perp$ and $f_\parallel$.

4.3 Extrapolation in q^2 to zero recoil

We must extrapolate the lattice data to lower q^2 (larger E_π^2) to reach the kinematic region where experimental measurements are most precise. Using chiral-continuum extrapolated lattice data in the range of simulated pion energies, we first generate four synthetic data points of the form factors f_+ and f_0 used in the q^2 extrapolation to the full kinematic range, as shown in Fig. 2.

In this study, we employ the model-independent z-expansion fit to extrapolate to low momentum transfer [17–20]. As a first step, we consider mapping the variable q^2 on to a new variable z defined as

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \tag{4.7}$$

where $t_\pm = (m_B \pm m_\pi)^2$. This transformation maps the semileptonic region $0 < q^2 < t_-$ onto small values of z between $0.34 < z < 0.22$ when we choose $t_0 = 0.65 t_+$. The $B \to \pi$ form factors are analytic in the semileptonic region except at the location of the B^* pole, so the form factors f_+ and f_0 can be expressed as convergent power series:

$$f(q^2) = \frac{1}{P(q^2)\phi(q^2, t_0)} \sum_{k=0}^{\infty} a^{(k)}(t_0) z(q^2, t_0)^k, \tag{4.8}$$

where the function $P(q^2)$ is the Blaschke factor that contains subthreshold poles, and the outer function $\phi(q^2, t_0)$ is an arbitrary analytic function. Unitarity constrains the sum of the squares of

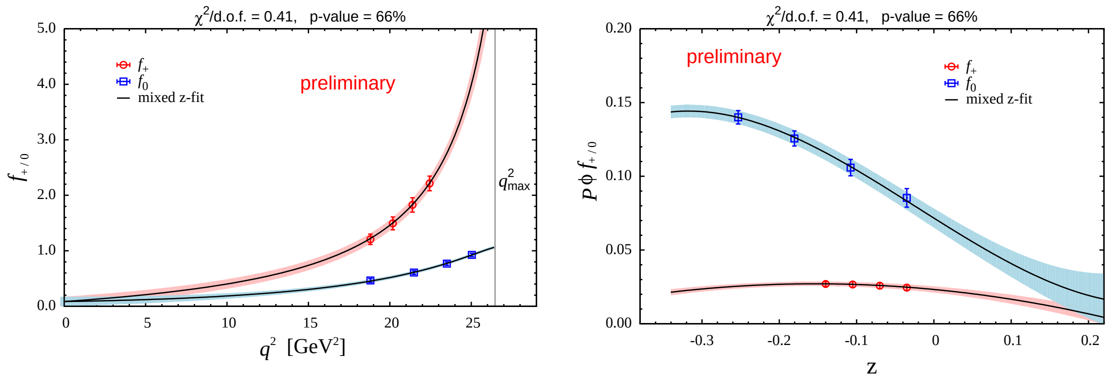

Figure 3: Extrapolation in q^2 of the factors f_+ and f_0 using the model-independent BGL z-parametrization, and imposing the kinematic constraint $f_+(q^2=0) = f_0(q^2=0)$. The left plot shows $f_{+/0}$ vs. q^2, while the right plot shows $P\phi f_{+/0}$ vs. z. The black curves with error bands show the parametrization of the form factors over the full kinematic range. Errors are statistical only.

the coefficients because the decay process of $B \to \pi l \nu$ semileptonic decay is related to the scattering process $l\nu \to B\pi$ by crossing symmetry. When the outer function is chosen as in Ref. [18], the sum of the squares of the coefficients is bounded by unity: $\sum_{k=0}^{N} (a^{(k)})^2 \leq 1$ for any N. Therefore only a small number of terms is needed to accurately describe the shape of the form factors over the full kinematic range.

Fig. 3 shows fits of the form factors f_+ and f_0 using the z-parametrization in Boyd, Grinstein, and Lebed (BGL) [18]. In the fits, we includes terms up to z^2 for f_+ and z^3 for f_0, and the kinematic constraint $f_+(q^2=0) = f_0(q^2=0)$ is imposed. The resulting slope and curvature for the $B \to \pi l \nu$ vector form factor f_+ are

$$a_+^{(1)}/a_+^{(0)} = -2.15 \pm 0.63 \tag{4.9}$$
$$a_+^{(2)}/a_+^{(0)} = -7.02 \pm 1.20, \tag{4.10}$$

where the errors are from statistics only. Our preliminary values of the slope and curvature are consistent with the independent lattice determination from the Fermilab Lattice and MILC Collaborations [2].

5. Outlook

We are currently estimating the systematic uncertainties in the form factors f_+ and f_0. We expect the dominant source of error to be from the chiral-continuum extrapolation, and that our total error will be competitive with that of Ref. [2].

Once we have a complete error budget, we will extrapolate our results from the simulated pion energies to the full q^2 range using the z-parametrization. Currently we are using the z-parametrization of Boyd, Grinstein, and Lebed, but we will also consider the alternative parametrization of Bourrely, Caprini, and Lellouch [20]. We will perform the z-fit to the lattice data alone to provide a model-independent parametrization of our result valid over the full kinematic range. We

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will also perform a simultaneous z-fit of our data and experimental measurements of the $B \to \pi$ differential branching fraction to obtain $|V_{ub}|$.

Our results will provide an important independent check on existing calculations, all of which use staggered light quarks.

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