

Dynamical quark mass generation

R. Frezzotti*

*Dipartimento di Fisica, Università di Roma Tor Vergata
and INFN, Sezione di Roma 2
Via della Ricerca Scientifica, 00133 Roma, Italy
E-mail: frezzotti@roma2.infn.it*

G.C. Rossi

*Dipartimento di Fisica, Università di Roma Tor Vergata
and INFN, Sezione di Roma 2
Via della Ricerca Scientifica, 00133 Roma, Italy
E-mail: rossig@roma2.infn.it*

Taking inspiration from lattice QCD results, we argue that a non-perturbative mass term for fermions can be generated as a consequence of the dynamical phenomenon of spontaneous chiral symmetry breaking, in turn triggered by the explicitly breaking of chiral symmetry induced by the critical Wilson term in the action. In a pure lattice QCD-like theory this mass term cannot be separated from the unavoidably associated linearly divergent contribution. However, if QCD with a Wilson term is enlarged to a theory where also a scalar field is present, coupled to a doublet of SU(2) fermions via a Yukawa interaction, then in the phase where the scalar field takes a non-vanishing (large) expectation value, a dynamically generated and “naturally” light fermion mass (numerically unrelated to the expectation value of the scalar field) is seen to emerge, at a critical value of the Yukawa coupling where the symmetry of the model is maximally enhanced.

*31st International Symposium on Lattice Field Theory - LATTICE 2013
July 29 - August 3, 2013
Mainz, Germany*

*Speaker.

1. Introduction and outlook

In this contribution we argue that in non-Abelian gauge theories with chiral symmetries broken at the UV cutoff by Wilson-like terms the dynamics of spontaneous chiral symmetry breaking (S χ SB) - triggered in the critical limit by the residual explicit chiral breaking - generates a dynamical mass for fermions. If one can solve, as we are going to show in a simple model including QCD, the ‘‘naturalness’’ problem [1] associated to the need of ‘‘fine tuning’’ the parameter controlling the recovery of chiral symmetry, this road may lead to a viable non-perturbative (NP) analog of the Higgs mechanism for mass generation [2]. In such a framework electroweak interactions can be naturally introduced. If a superstrong interaction at the TeV scale is also introduced, one can set up a model where mass hierarchy and the flavour properties of the Standard Model (recovered as the low energy theory) are understood and arise in a natural way.

2. Inspiration and numerical evidence from lattice QCD

As is well known, in lattice QCD (LQCD) with Wilson fermions [3] quark mass renormalization requires the subtraction of a linearly divergent counter-term, $m_{cr}\bar{q}q$ (q being the N_f -flavour quark field), arising because the Wilson term in the lattice Lagrangian explicitly breaks chiral symmetry. In general m_{cr} will have a formal small- a expansion of the kind

$$m_{cr} = \frac{c_0}{a} + c_1\Lambda_{QCD} + c_2a\Lambda_{QCD}^2 + O(a^2). \quad (2.1)$$

Eq. (2.1) suggests that, if we could set the mass parameter, m_0 , in the lattice fermion action just equal to the linearly divergent term in (2.1), then the $c_1\Lambda_{QCD}$ contribution (if non-zero) would play the role of a quark mass in the renormalized chiral Ward–Takahashi Identities (WTIs). To make use of this remark for NP mass generation one has to answer positively the following questions.

1) Are there numerical indications for the existence of a term like the second one in the r.h.s. of (2.1) in actual LQCD simulation data? 2) Do we understand its dynamical origin? 3) Is it possible to disentangle a (small) NP fermion mass from the much larger (perturbative) effect that comes along with it when chiral symmetry is broken at a high scale?

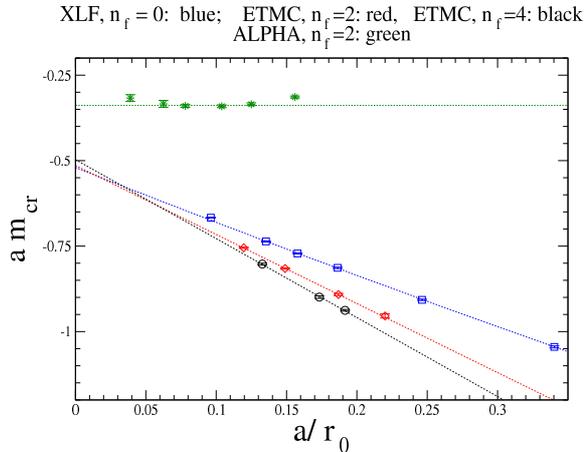


Figure 1: The critical value of am_0 in Wilson LQCD simulations as a function of a/r_0 .

To answer question 1), in Fig. 1 we present a compilation of LQCD data showing the behaviour of am_{cr} , which is the value of the am_0 Lagrangian parameter at which am_{PCAC} vanishes, as a function of a/r_0 (here r_0 denotes the Sommer scale). We show four sets of data taken from refs. [4], [5, 6], [7] and [8]. The three lower sets of points correspond to measurements of am_{cr} carried out at maximal twist using the Wilson twisted mass regularization of LQCD in the quenched ($N_f = 0$) approximation (blue squares [4]), with $N_f = 2$ dynamical flavours (red diamonds [5, 6]) and with $N_f = 4$ dynamical flavours (black circles [7]). The green stars correspond to Wilson clover-improved [10] data with $N_f = 2$ dynamical flavours [8] obtained with Schrödinger functional boundary conditions. For the sake of Fig. 1 we have taken $r_0 = 0.45$ fm [5, 6].

In the present notations the intercept of the fitted line through the data is the c_0 coefficient of (2.1), while its slope, $c_1\Lambda_{QCD} \times r_0$, is the quantity of interest. Indeed, the points of refs. [4], [5, 6], [7] all exhibit a nice linear behaviour (with a mild N_f dependence) in a wide a/r_0 window, which allows identifying a non-vanishing $c_1\Lambda_{QCD}$ slope taking values ¹ in the range 700 to 1000 MeV.

The Schrödinger functional data of ref. [8] are, instead, pretty flat implying that the c_1 coefficient is very small. As we shall argue in the next section, a non-zero slope is related to the $O(a)$ Wilson-like term in the Symanzik low energy effective Lagrangian (SLEL) [9]. The presence of the non-perturbatively tuned clover-term [10] in the lattice Lagrangian employed in ref. [8] effectively kills (the interesting NP effects originating from) the $d = 5$ SLEL operator [11].

3. The dynamical origin of the $c_1\Lambda_{QCD}$ term

The $c_1\Lambda_{QCD}$ term in (2.1) has its origin in a delicate interplay between $O(a)$ corrections to quark and gluon propagators and vertices ensuing from the spontaneous breaking of chiral symmetry, and the quadratic divergence of the loop integration in diagrams where one Wilson term vertex is inserted. Typical (lowest order in g_s^2) correlators where this occurs are depicted in Fig. 2, where the grey blob is a NP $O(a)$ correction to the gluon or (the helicity-preserving components of) the quark propagator and the gluon-quark-quark vertex, and aV_5 is the derivative vertex from the Wilson term. Such peculiar $O(a)$ corrections arise from NP contributions in the SLEL expansion

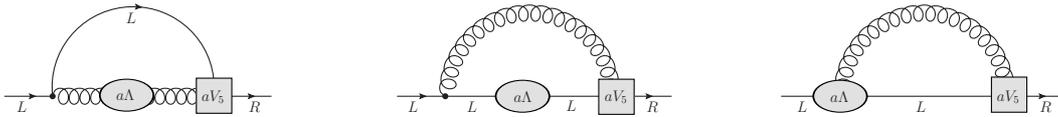


Figure 2: Typical lowest order “diagrams” giving rise to dynamically generated quark mass terms (L and R are quark-helicity labels). The grey blob represents the non-perturbative $a\Lambda_{QCD}\alpha_s$ effect in eqs. (3.2).

of the relevant (gauge fixed) lattice correlators for $m_0 \rightarrow m_{cr}$, e.g.

$$\begin{aligned} \langle O(x, x', \dots) \rangle \Big|_L^L &= \langle O(x, x', \dots) \rangle \Big|_C^C - a \langle O(x, x', \dots) \int d^4z L_5(z) \rangle \Big|_C^C + O(a^2), \\ O(x, x', \dots) &\Leftrightarrow A_\mu^b(x) A_\nu^c(x'), \quad q_{L/R}(x) \bar{q}_{L/R}(x'), \quad q_{L/R}(x) \bar{q}_{L/R}(x') A_\mu^b(y), \end{aligned} \quad (3.1)$$

¹A word of caution is in order here. The quoted values of $d(am_{cr})/d(ar_0^{-1})$ are only indicative, as strictly speaking there isn't a mathematically rigorous way to identify an a/r_0 range where one can consider numerically negligible both the logarithmic a -behaviour of the gauge coupling upon renormalization (determining the behaviour of am_{cr} as $a \rightarrow 0$) and the higher order lattice artifacts that become dominant at large enough values of a .

where L_5 is the $d = 5$ SLEL operator, which breaks chirality. The label $|^C$ is to remind that the r.h.s. correlators are taken in continuum (renormalized) QCD. The key remark about these expansions is that the $O(a)$ correlators in (3.1) can be non-zero only due to the phenomenon of $S\chi$ SB. From these (amputated) correlators by using symmetry and dimensional arguments one reads off the NP contributions to quark and gluon propagators and vertices, namely (in continuum-like notations)

$$\begin{aligned}\Delta G_{\mu\nu}^{bc}(k)|^L &= -a\Lambda_{QCD}\alpha_s(\Lambda_{QCD})\delta^{bc}\frac{\delta_{\mu\nu}-k_\mu k_\nu/k^2}{k^2}f_{AA}\left(\frac{\Lambda_{QCD}^2}{k^2}\right), \\ \Delta S_{LL/RR}(k)|^L &= -a\Lambda_{QCD}\alpha_s(\Lambda_{QCD})\frac{ik_\mu(\gamma_\mu)_{LL/RR}}{k^2}f_{q\bar{q}}\left(\frac{\Lambda_{QCD}^2}{k^2}\right), \\ \Delta\Gamma_{Aq\bar{q}}^{b,\mu}(k,\ell)|^L &= a\Lambda_{QCD}\alpha_s(\Lambda_{QCD})ig_s\lambda^b\gamma_\mu f_{Aq\bar{q}}\left(\frac{\Lambda_{QCD}^2}{k^2},\frac{\Lambda_{QCD}^2}{\ell^2},\frac{\Lambda_{QCD}^2}{(k+\ell)^2}\right),\end{aligned}\quad (3.2)$$

where the factor $\alpha_s(\Lambda_{QCD})$ comes from the fact that the gluon emitted from the L_5 vertex has to be absorbed somewhere in the diagram². The scalar form factors f_{AA} , $f_{q\bar{q}}$ and $f_{Aq\bar{q}}$ are dimensionless functions depending on $\Lambda_{QCD}^2/(\text{momenta})^2$ ratios. From Symanzik's analysis of lattice artifacts, a -expansions like those in eqs. (3.1) are expected to be valid for small values of squared momenta compared to a^{-2} . Here we assume that the NP effects encoded in eqs. (3.1)–(3.2) persist up to large (i.e. comparable to a^{-1}) momenta, and *conjecture* the asymptotic behaviour

$$f_{AA}\left(\frac{\Lambda_{QCD}^2}{k^2}\right)\xrightarrow{k^2\rightarrow\infty}h_{AA}, \quad f_{q\bar{q}}\left(\frac{\Lambda_{QCD}^2}{k^2}\right)\xrightarrow{k^2\rightarrow\infty}h_{q\bar{q}}, \quad f_{Aq\bar{q}}\left(\frac{\Lambda_{QCD}^2}{k^2},\frac{\Lambda_{QCD}^2}{\ell^2},\frac{\Lambda_{QCD}^2}{(k+\ell)^2}\right)\xrightarrow{k^2,\ell^2,(k+\ell)^2\rightarrow\infty}h_{q\bar{q}},$$

where h_{AA} and $h_{q\bar{q}}$ are $O(1)$ constants and the last two limits are related by gauge invariance.

The above relative $O(a\Lambda_{QCD}\alpha_s)$ corrections to propagators and vertices generate $O(\Lambda_{QCD}\alpha_s g_s^2)$ corrections to the quark self-energy, hence NP mass terms. Actually there are more relevant NP corrections besides those in eqs. (3.2) and fig. 2, i.e. corrections to the Wilson-term induced vertices and helicity-flip quark propagator components. Based on LQCD symmetries, to leading order in g_s^2 (and a) all the NP terms can be *effectively reproduced* in PT using *ad hoc modified Feynman rules*, namely those obtained adding to the LQCD Lagrangian (in continuum-like notations) the terms

$$\Delta\mathcal{L}|_{ad\,hoc} = a\Lambda_{QCD}\alpha_s\left\{h_{AA}\frac{1}{2}\text{tr}(FF) + h_{q\bar{q}}(\bar{q}\gamma_\mu D_\mu q) + h_{Wil}\left(-\frac{a}{2}r\right)(\bar{q}D^2q)\right\}. \quad (3.3)$$

To see how a dynamical mass gets generated, consider the loop momentum counting of, say, the “diagram” in the central panel of fig. 2 in the $a \rightarrow 0$ limit. One has factors $a\Lambda_{QCD}\alpha_s(\Lambda_{QCD})k_\mu/k^2$ and $1/k^2$ from the NP contribution to the quark propagator and the standard gluon propagator, respectively, and a factor ak_μ from the derivative coupling of the Wilson vertex. Including the extra g_s^2 power from the gluon loop, one gets schematically a fermion mass term of the order

$$a\Lambda_{QCD}g_s^2\alpha_s(\Lambda_{QCD})\int^{1/a}d^4k\frac{k_\mu}{k^2}\frac{1}{k^2}ak_\mu \sim g_s^2\alpha_s(\Lambda_{QCD})\Lambda_{QCD}. \quad (3.4)$$

Other “diagrams” give similar NP mass contributions, yielding in eq. (2.1) $c_1 = O(g_s^2\alpha_s(\Lambda_{QCD}))$.

²Since a soft quark line (where $S\chi$ SB occurs) exits the L_5 vertex, Λ_{QCD} is chosen as the scale of α_s . The scale at which the gauge coupling is evaluated will be a key feature to understand the fermion mass hierarchy problem [2].

4. Light mass fermions with natural fine tuning: a toy model

Separation of “large” (infinite in LQCD) from “small” (finite up to logs) contributions in formulae like (2.1) is only possible on the basis of some symmetry. Though absent in LQCD, this long sought for symmetry can be seen to exist in some enlarged theory where besides gauge interactions, an $SU(2)$ fermion doublet is coupled to a scalar field, Φ , via a Yukawa interaction and a Wilson-like term. To be concrete let us consider the renormalizable toy-model ($b^{-1} = UV$ cutoff)

$$\mathcal{L}_{\text{toy}}(Q, G, \Phi) = \mathcal{L}_{\text{kin}}(Q, G, \Phi) + \mathcal{V}(\Phi) + \mathcal{L}_{\text{Yuk}}(Q, \Phi) + \mathcal{L}_{\text{Wil}}(Q, G, \Phi), \quad (4.1)$$

$$\mathcal{L}_{\text{kin}}(Q, G, \Phi) = \frac{1}{4} F_{\mu\nu}^a G F_{\mu\nu}^a + \bar{Q}_L \gamma_\mu \mathcal{D}_\mu^G Q_L + \bar{Q}_R \gamma_\mu \mathcal{D}_\mu^G Q_R + \frac{1}{2} \text{tr} [\partial_\mu \Phi^\dagger \partial_\mu \Phi],$$

$$\mathcal{L}_{\text{Yuk}}(Q, \Phi) = \eta (\bar{Q}_L \Phi Q_R + \bar{Q}_R \Phi^\dagger Q_L), \quad \mathcal{V}(\Phi) = \frac{\mu_0^2}{2} \text{tr} [\Phi^\dagger \Phi] + \frac{\gamma_0}{4} (\text{tr} [\Phi^\dagger \Phi])^2,$$

$$\mathcal{L}_{\text{Wil}}(Q, G, \Phi) = \frac{b^2}{2} (\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu^G \Phi \mathcal{D}_\mu^G Q_R + \bar{Q}_R \overleftarrow{\mathcal{D}}_\mu^G \Phi^\dagger \mathcal{D}_\mu^G Q_L).$$

Besides obvious symmetries, \mathcal{L}_{toy} is invariant under the (global) $\chi_L \times \chi_R$ transformations

$$\bullet \chi_L: \quad \tilde{\chi}_L \otimes (\Phi \rightarrow \Omega_L \Phi), \quad \text{with} \quad \tilde{\chi}_L: Q_L \rightarrow \Omega_L Q_L, \bar{Q}_L \rightarrow \bar{Q}_L \Omega_L^\dagger, \quad \Omega_L \in SU(2)_L \quad (4.2)$$

$$\bullet \chi_R: \quad \tilde{\chi}_R \otimes (\Phi \rightarrow \Phi \Omega_R^\dagger), \quad \text{with} \quad \tilde{\chi}_R: Q_R \rightarrow \Omega_R Q_R, \bar{Q}_R \rightarrow \bar{Q}_R \Omega_R^\dagger, \quad \Omega_R \in SU(2)_R \quad (4.3)$$

but not under the “chiral” transformations $\tilde{\chi}_L \times \tilde{\chi}_R$ acting only on fermions. However, much like it happens with the critical mass in LQCD when chiral symmetry is recovered [12], a critical value of the Yukawa coupling, η_{cr} , exists where the transformations $\tilde{\chi}_L \times \tilde{\chi}_R$ become up to $O(b^2)$ a symmetry of \mathcal{L}_{toy} . This is so because, ignoring possible NP effects, at η_{cr} the $\tilde{\chi}$ -breaking terms \mathcal{L}_{Wil} and \mathcal{L}_{Yuk} compensate each other to $O(b^0)$, and Φ decouples. Hence among Green functions involving quarks and gluons the same relations are implied by $\tilde{\chi}_L \times \tilde{\chi}_R$ and $\chi_L \times \chi_R$ invariances.

Moving beyond PT, always at η_{cr} , one has to consider the possibility of dynamical breaking of the (approximate) $\tilde{\chi}_L \times \tilde{\chi}_R$ symmetry ($S\tilde{\chi}SB$). We can have two very different scenarios. In both cases we assume that the scalar mass is much larger than Λ_s , the RGI scale of the theory.

If μ_0^2 is such that $\mathcal{V}(\Phi)$ has a single minimum with $\langle \Phi \rangle = 0$, \mathcal{L}_{Wil} and \mathcal{L}_{Yuk} (both linear in Φ) are expected to provide no seed for $S\tilde{\chi}SB$, so the $\tilde{\chi}_L \times \tilde{\chi}_R$ symmetry is thus realized à la Wigner [2].

Physics is drastically different if μ_0^2 is such that a double well potential develops. In this case it is convenient to expand the scalar field around its vacuum expectation value (vev) writing

$$\Phi(x) = (v + \sigma(x)) \mathbf{1}_{2 \times 2} + i \vec{\pi}(x) \vec{\tau}, \quad (4.4)$$

with $\vec{\pi}$ a triplet of massless pseudoscalar Goldstone bosons and σ a scalar of mass $m_\sigma = O(v) \gg \Lambda_s$. As (ignoring Φ fluctuations) the $d = 6$ term \mathcal{L}_{Wil} looks much like the $d = 5$ Wilson term in LQCD, we expect the $\tilde{\chi}$ -breaking terms in \mathcal{L}_{Wil} (and \mathcal{L}_{Yuk}) to now trigger dynamical $S\tilde{\chi}SB$ [2]. In particular NP terms coming from $S\tilde{\chi}SB$ effects are expected to give rise to modifications of (gluon and fermion) propagators and fermion-antifermion-gluon vertices, as well as to peculiar gluon-gluon-scalar, fermion-antifermion-scalar, fermion-antifermion-gluon-scalar vertices, etc. Consider, for instance, the small- b^2 expansions (terms odd in b are excluded by \mathcal{L}_{toy} symmetries)

$$\langle O(x, x', \dots) \rangle \Big|_R^F = \langle O(x, x', \dots) \rangle \Big|_F^F - b^2 \langle O(x, x', \dots) \int d^4 z [L_6^\chi + L_6^\chi](z) \rangle \Big|_F^F + O(b^4), \quad (4.5)$$

$$O(x, x', \dots) \Leftrightarrow A_\mu^b(x) A_\nu^c(x') [\Phi^\dagger \Phi](y), \quad Q_{L/R}(x) \bar{Q}_{L/R}(x') [\Phi^\dagger \Phi](y), \quad Q_{L/R}(x) \bar{Q}_{L/R}(x') [\Phi^\dagger \Phi](y) A_\mu^b(y'),$$

where the label $|^R$ ($|^F$) means that vev's are taken in the UV-regulated (formal) \mathcal{L}_{toy} model and L_6^χ (L_6^χ) is the $d = 6$ $\tilde{\chi}$ -breaking (conserving) SLEL operator. Focusing on the $\mathcal{O}(b^2)$ terms from L_6^χ in the r.h.s. and looking at the contributions with just one σ -propagator, we read off the NP corrections to the gluon-gluon-scalar, $Q_{L/R}-\bar{Q}_{L/R}$ -scalar and $Q_{L/R}-\bar{Q}_{L/R}$ -gluon-scalar vertices, getting

$$\begin{aligned}\Delta\Gamma_{AA\Phi}^{bc\mu\nu}(k,\ell)|^R &= b^2\Lambda_s\alpha_s(\Lambda_s)\frac{\delta^{bc}}{2}\{[k(k+\ell)\delta_{\mu\nu}-k_\mu(k+\ell)_\nu]+[\mu\rightarrow\nu]\}F_{AA\Phi}\left(\frac{\Lambda_s^2}{k^2},\frac{\Lambda_s^2}{\ell^2},\frac{\Lambda_s^2}{(k+\ell)^2}\right), \\ \Delta\Gamma_{Q\bar{Q}\Phi}(k,\ell)|^R &= b^2\Lambda_s\alpha_s(\Lambda_s)\frac{i}{2}\gamma_\mu(2k+\ell)_\mu F_{Q\bar{Q}\Phi}\left(\frac{\Lambda_s^2}{k^2},\frac{\Lambda_s^2}{\ell^2},\frac{\Lambda_s^2}{(k+\ell)^2}\right), \\ \Delta\Gamma_{Q\bar{Q}\Phi}^{b,\mu}(k,\ell,\ell')|^R &= b^2\Lambda_s\alpha_s(\Lambda_s)ig_s\lambda^b\gamma_\mu F_{Q\bar{Q}\Phi}\left(\frac{\Lambda_s^2}{\text{mom}^2}\right), \quad \text{mom} \in \{k,\ell,\ell',\dots,\ell'+\ell,k+\ell\}.\end{aligned}\quad (4.6)$$

$F_{AA\Phi}$, $F_{Q\bar{Q}\Phi}$ and $F_{Q\bar{Q}\Phi}$ are dimensionless functions depending on Λ_s^2/mom^2 ratios. From standard Symanzik arguments, small- b expansions like those in (4.5) are expected to be valid for momenta much smaller than the UV-cutoff b^{-1} . Like in Wilson LQCD, we assume that the NP effects encoded in (4.5)–(4.6) persist up to $\text{mom}^2 = \mathcal{O}(b^{-2})$, and *conjecture* the asymptotic behaviour

$$F_{AA\Phi}\left(\frac{\Lambda_s^2}{\text{mom}^2}\right) \xrightarrow{\text{mom}^2 \rightarrow \infty} H_{AA}, \quad F_{Q\bar{Q}\Phi}\left(\frac{\Lambda_s^2}{\text{mom}^2}\right) \xrightarrow{\text{mom}^2 \rightarrow \infty} H_{Q\bar{Q}}, \quad F_{Q\bar{Q}\Phi}\left(\frac{\Lambda_s^2}{\text{mom}^2}\right) \xrightarrow{\text{mom}^2 \rightarrow \infty} H_{Q\bar{Q}},$$

where H_{AA} and $H_{Q\bar{Q}}$ are $\mathcal{O}(1)$ constants and the last two limits are related by gauge invariance.

Further NP corrections analogous to the ones of eqs. (4.6) actually occur for \mathcal{L}_{Wil} -induced vertices. Based on the symmetries of the model (4.1), to leading order in g_s^2 (and b^2) these NP terms can be *effectively reproduced* in PT by using *ad hoc modified Feynman rules*, namely those that one infers after adding to the \mathcal{L}_{toy} Lagrangian the terms

$$\Delta\mathcal{L}|_{ad\,hoc} = \frac{b^2}{2}\Lambda_s\alpha_s\left\{\text{tr}[\Phi^\dagger U + \text{h.c.}]\left[\frac{H_{AA}}{2}\text{tr}(FF) + H_{Q\bar{Q}}(\bar{Q}\not{D}Q)\right] + H_{\text{Wil}}[\bar{Q}_L\overleftarrow{D}U\not{D}Q_R + \text{h.c.}]\right\} \quad (4.7)$$

with $U = \Phi[\Phi^\dagger\Phi]^{-1/2}$. The dimensionless field U , which transforms like Φ under $\chi_L \times \chi_R$, must necessarily arise in NP corrections owing to the exact $\chi_L \times \chi_R$ symmetry of the theory.

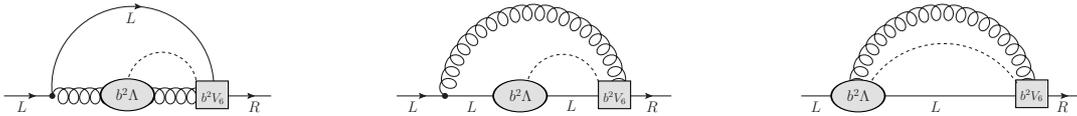


Figure 3: Typical lowest order “diagrams” giving rise to dynamically generated quark mass terms (L and R are fermion-helicity labels). The grey blob represents the non-perturbative $b^2\Lambda_s\alpha_s(\Lambda_s)$ effect in eqs. (4.6).

Formally using PT with modified Feynman rules, one checks that diagrams like those in fig. 3 (the dotted line represents the propagation of a σ/π particle) yield a fermion mass $\mathcal{O}(g_s^2\alpha_s(\Lambda_s)\Lambda_s)$. Indeed, for the counting of loop momenta of, say, the central diagram of fig. 3 in the $b \rightarrow 0$ limit, one finds a double integral with factors $1/k^2$ and $1/(\ell^2 + m_{\sigma/\pi}^2)$ from the standard gluon and σ/π propagators, the factors $\gamma_\mu k_\mu/k^2$ and $\gamma_\nu(k+\ell)_\nu/(k+\ell)^2$ for the quark propagators, a factor $b^2(k+\ell)_\lambda$ from the \mathcal{L}_{Wil} derivative coupling and a factor $b^2(2k+\ell)_\rho\gamma_\rho\Lambda_s$ from the NP vertex $\Delta\Gamma_{Q\bar{Q}\Phi}(k,\ell)|^R$. Thus the overall b^4 power is compensated by the two-loop integral quartic divergency.

From the NP results we got on the fermion self-energy diagrams and the exact $\chi_L \times \chi_R$ invariance of the theory (see eq. (4.7)), we expect the generating functional of 1PI Green functions to display a NP term of the form ³

$$C_1 \Lambda_s [\bar{Q}_L U Q_R + \bar{Q}_R U^\dagger Q_L], \quad C_1 = O(g_s^2 (b^{-1}) \alpha_s(\Lambda_s)). \quad (4.8)$$

Besides a fermion mass, this term in the Nambu-Goldstone phase of the model at $\eta = \eta_{cr}$, gives rise to NP Φ -to-fermions and (via fermion loops) Φ -to-gluons couplings, which also stem from the dynamical breaking of the $\tilde{\chi}_L \times \tilde{\chi}_R$ invariance. Nevertheless, the fine tuning $\eta \rightarrow \eta_{cr}$, that is crucial to get a fermion mass $\ll v$, is “natural” because, besides yielding the restoration of the $\tilde{\chi}_L \times \tilde{\chi}_R$ invariance in the Wigner phase, it leads in the Nambu-Goldstone phase to the maximal enhancement of this symmetry that is compatible with its dynamical SSB and related NP effects.

5. Conclusions

We have discussed the possibility that $O(g_s^2 \alpha_s(\Lambda_s) \Lambda_s)$ fermion masses are dynamically generated from an interplay between vanishingly small chirally breaking effects left-over in the “critical” theory and power divergencies of loop integrals with the insertion of Wilson-like vertices. We have also shown that it is possible to solve the “fine tuning” problem associated with the need of separating “large” from “small” mass contributions, in a toy-model where an SU(2) doublet of strongly interacting fermions is coupled to a scalar via Yukawa and Wilson-like ($\tilde{\chi}$ -breaking) terms.

Acknowledgments

We wish to thank M. Testa, G. Veneziano and P. Weisz for numerous, illuminating discussions.

References

- [1] G. 't Hooft, “*Naturalness, Chiral Symmetry and Spontaneous Chiral Symmetry Breaking*”, in “Recent Developments in Gauge Theories” (Plenum Press, 1980) - ISBN 978-0-306-40479-5.
- [2] R. Frezzotti and G.C. Rossi, in preparation.
- [3] K.G. Wilson, Phys. Rev. D **10** (1974) 2445.
- [4] K. Jansen *et al.* [XLF Collaboration], JHEP **0509** (2005) 071.
- [5] R. Baron *et al.* [ETM Collaboration], JHEP **1008** (2010) 097.
- [6] B. Blossier *et al.* [ETM Collaboration], Phys. Rev. D **82** (2010) 114513.
- [7] R. Baron, *et al.* [ETM Collaboration], JHEP **1006** (2010) 111 and PoS LATTICE **2010** (2010) 123.
- [8] M. Della Morte *et al.* [ALPHA Collaboration], Nucl. Phys. B **713** (2005) 378.
- [9] K. Symanzik, Nucl. Phys. B **226** (1983) 187 and 205.
- [10] B. Sheikholeslami and R. Wohlert, Nucl. Phys. B **259** (1985) 572.
- [11] M. Lüscher, S. Sint, R. Sommer and P. Weisz, Nucl. Phys. B **478** (1996) 365
- [12] M. Bochicchio, L. Maiani, G. Martinelli, G.C. Rossi, M. Testa, Nucl. Phys. **B262** (1985) 331.

³An analysis of the WTIs of the $\tilde{\chi}_L \times \tilde{\chi}_R$ transformations shows that the term (4.8) is RG-invariant [2]. This fixes the dependence of C_1 on the UV cutoff b^{-1} , and means that $C_1 \Lambda_s$ represents the fermion mass value at the UV cutoff.