Determination of the Δ resonance width from lattice QCD

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A method suitable for extracting resonance parameters of unstable baryons in lattice QCD is examined. The method is applied to the strong decay of the Δ to a pion-nucleon state, extracting the $\pi N \Delta$ coupling constant and Δ decay width.

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1. Introduction

The study of hadron resonances is of fundamental importance for nuclear and particle physics and one of the long-standing goals of lattice QCD. While methods for handling stable low-lying particles on the lattice are well developed, the study of resonances and decays is intrinsically more difficult within the Euclidean formulation of lattice QCD. The problem lies essentially in the fact, that scattering states cannot be realized for a theory formulated in Euclidean time.

Several ways to resolve the issue of relating data extracted from finite volume states in lattice QCD and properties of scattering states in infinite volume have been proposed. The method introduced by Lüscher relates the shifts in the energy spectrum of multi-hadron states due to their interaction in finite volume to the scattering phase shifts in infinite volume [1, 2]. In its full-fledged version this requires lattice simulations with a series of spatial lattice volumes with precise determinations of the spectrum of multi-hadron states. This method has been applied mainly to meson resonances [3, 4, 5] and also to the negative parity nucleon channel [6].

An alternative approach to study strong decays on the lattice has been proposed in Ref. [7]. While a true resonant behavior cannot appear for finite-volume states on a Euclidean lattice, initial and final states , $|\,i\,\rangle$, $|\,f\,\rangle$, with matching quantum numbers can mix, meaning that states realized on the lattice at asymptotically large times are a linear combination of states of the same quantum numbers. This mixing will be significant for states with equal energy. The so-called transfer matrix method provide a method to extract the overlap amplitude of such states and relate it to the leading-order continuum matrix element $\mathscr{M} \sim \langle f\,|H\,|\,i\rangle$. The matrix element is then related to the resonance parameters for the transition, namely the coupling and the width.

In this study we apply the transfer matrix method to baryonic resonances and specifically to the transition $\Delta \to \pi N$, which has been well-known studied experimentally. Thus, this work aims at benchmarking the method for an experimentally known width before applying it to other resonances where the width is either poorly determined or not yet measured. The methodology and some of the results were reported in Ref. [8].

2. Transfer matrix method

We consider the $\Delta \to \pi N$ transition depicted in Fig. 1, as described in an effective field theory approach, where an initial Δ state couples resonantly to the πN state. We associate this interac-

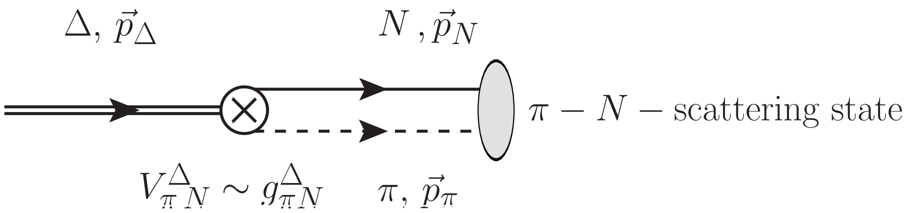

Figure 1: Schematic presentation of the Δ resonance and its decay channel, the pion-nucleon with coupling $g^{\Delta}_{\pi N}$.

tion with a vertex described by an effective coupling $g^{\Delta}_{\pi N}$. The transfer matrix in the subspace

$\{|\Delta\rangle, |\pi N\rangle\}$ spanned by the pion-nucleon and the Δ states, which without an interaction are orthogonal, can be written as

$$\mathrm{T} = \mathrm{e}^{-a\bar{E}} \begin{pmatrix} \mathrm{e}^{-a\delta/2} & ax & \cdots \\ ax & \mathrm{e}^{+a\delta/2} & \\ \vdots & & \ddots \end{pmatrix} \tag{2.1}$$

where we denote by $\bar{E} = (E_\Delta + E_{\pi N})/2$ the average energy of the Δ and πN system and by $\delta = E_{\pi N} - E_\Delta$ the energy difference between the pion-nucleon and the Δ state. The transition amplitude is denotes by $x = \langle \Delta \,|\, H \,|\, \pi N \rangle$.

The coupling of the Δ to the πN state causes an energy shift of the energy eigenstates of the non-interacting theory. The energies in the the subspace $\{|\Delta\rangle, |\pi N\rangle\}$ are modified to

$$E_\pm = \bar{E} \pm \sqrt{\delta^2/4 + x^2}\,. \tag{2.2}$$

On the lattice we prepare the pion-nucleon state at an initial time t_i and the Δ state at a final time t_f and sum over all possibilities for one transition (leading-order) at each intermediate time. The resulting transition overlap amplitude is given by

$$\langle \Delta, t_f \,|\, \pi N, t_i \rangle = ax \frac{\sinh(\delta t/2)}{\sinh(a\delta/2)} \, \mathrm{e}^{-\bar{E}t} \xrightarrow{\delta t \ll 1} [axt] \, \mathrm{e}^{-\bar{E}t} + \ldots\,, \tag{2.3}$$

where $t \equiv t_f - f_i = an_{fi}$ and n_{if} is the number of transition steps. Higher order contributions, contributions from excited states and mixing with other states are denoted by the terms omitted in left-hand-side of Eq. (2.3).

For the method to be applicable the following conditions must hold:

1. the energy levels of the initial and final states must be sufficiently close
2. the transition amplitude needs to be sufficiently small so that only one transition occurs (leading-order contribution)

Since we need to have final states with non-zero spatial momentum on the lattice, with a sufficiently fine resolution in momentum space, a large enough spatial volume is preferable.

In our numerical study, we consider the $I_z = 3/2$ channel with an initial Δ^{++} and a final π^+-proton state. We illustrate how well the first condition on the energy levels is satisfied for our current study in Fig. 2, where we show the computed energy of the Δ^{++} and the sum of the energies of the π^+ and the proton $E_{\pi^+ p} = E_{\pi^+} + E_p$.

3. Lattice calculation

In the lattice calculation the unknown overlaps and exponential time dependence cancel by taking a suitable ratio of 3-point and 2-point functions

$$R_\mu(t, \vec{Q}, \vec{q}) = \frac{C_\mu^{\Delta \to \pi N}(t, \vec{Q}, \vec{q})}{\sqrt{C_\mu^{\Delta}(t, \vec{Q})\, C^{\pi N}(t, \vec{Q}, \vec{q})}}\,. \tag{3.1}$$

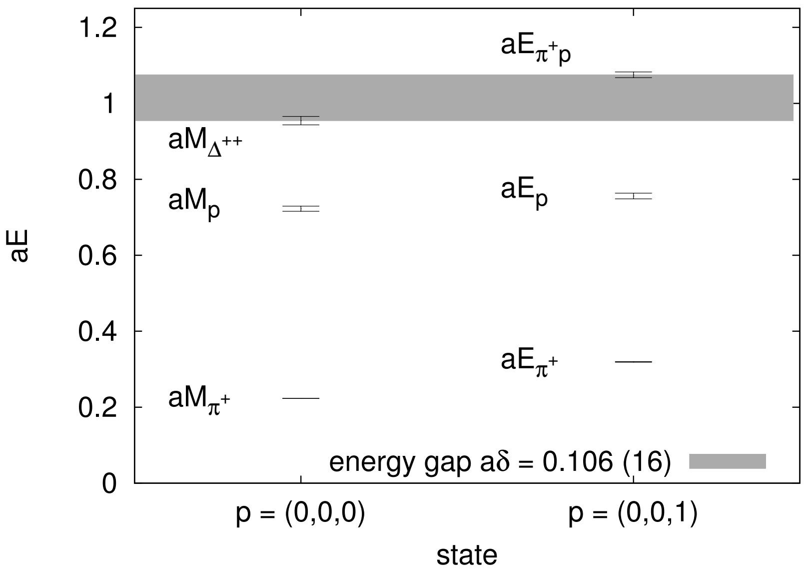

Figure 2: The relevant energy levels for the ensemble of domain wall valence quarks on a staggered sea for $m_\pi = 360\,\mathrm{MeV}$. We show the single particle masses M_{π^+}, M_p and $M_{\Delta^{++}}$ and the sum of the pion and proton energy $E_{\pi^+p} = E_{\pi^+} + E_p$.

As already pointed out, we consider the isospin $I_z = +3/2$ channel and use the standard interpolating fields for the Δ^{++}, the π^+ and the proton. The π^+p state is represented by the interpolating operator

$$J^\alpha_{\pi^+p}(t,\vec{q},\vec{x}) \,=\, \sum_{\vec{y}} J_{\pi^+}(t,\vec{y}+\vec{x})\,J^\alpha_p(t,\vec{x})\,\mathrm{e}^{-i\vec{q}\cdot\vec{y}}\,, \tag{3.2}$$

defined to have a relative momentum $\vec{q} \neq \vec{0}$ to generate overlap with the π^+p state with angular momentum $l = 1$. The dominant asymptotic contribution to the correlator will then come from the coupling $s_p \oplus l \to J_\Delta = 3/2$. Moreover, in this study we neglect the interaction between the pion and the proton in a finite box and calculate the pion-proton 2-point function, as a product of the pion and proton correlator, i.e. $C^{\pi^+p} \approx C^{\pi^+} \times C^p$.

We use a hybrid action of staggered sea quarks and domain wall (DW) valence quarks. The gauge field configurations are taken from the MILC ensemble *2864f21b676m010m050* [9, 10] with pion mass $m_\pi \approx 360\,\mathrm{MeV}$, lattice spacing $a \approx 0.124\,\mathrm{fm}$ and spatial volume of $(3.4\,\mathrm{fm})^3$. We use 210 configurations with 4 measurements per configuration. The relative momentum in the pion-proton system is set to $\vec{q} = (2\pi/L)\,\vec{e}_k$, $k = \pm1,\,\pm2,\,\pm3$ and we combine data from both all six momentum directions as well as forward and backward propagation. In the left panel of Fig.3, we show the ratio R of Eq. (3.1), which results from averaging the six contributions R_j, $j = 1,2,3$ with $\vec{q} \propto \pm\vec{e}_j$. If $tx \ll 1$ then $R \propto t$. As can be seen, the data suggest the existence of a time interval $4 \lesssim t/a \lesssim 10$ where R has a linear dependence on the sink-source separation t. We remark, that the signal on the smallest timeslices is affected by excited states contamination, most notably by contributions from oscillatory contributions known to exist for domain wall fermions [12]. At large time separations,

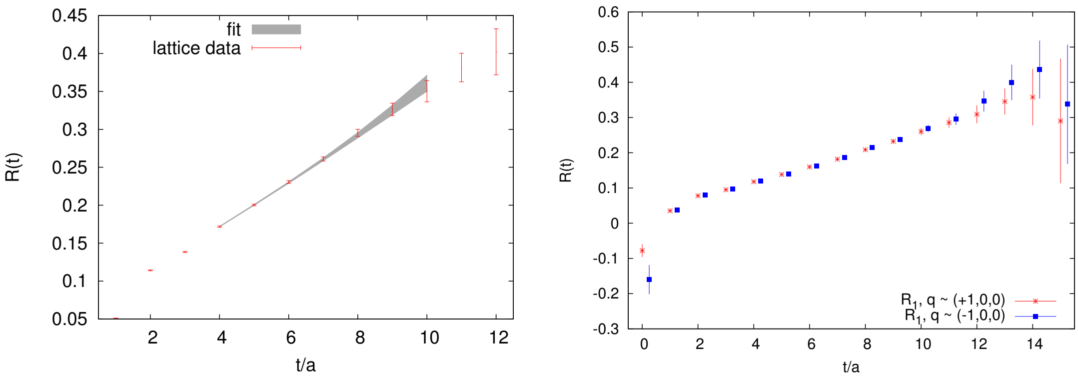

Figure 3: Ratio R averaged over over momentum directions and forward / backward propagation as measured in our numerical study.

	$t_{\min}/a$	$t_{\max}/a$	$A \cdot 10^2$	$aB \cdot 10^2$	$a^3C \cdot 10^5$	χ^2/dof
f_1	4	9	6.47 (49)	2.62 (15)	0.188 (68)	2.4/3
f_1	4	10	6.24 (47)	2.69 (14)	0.156 (79)	4.3/4
f_1	5	9	5.62 (103)	2.82 (26)	0.140 (104)	1.8/2
f_1	5	10	5.05 (84)	2.98 (21)	0.074 (122)	2.9/3
f_2	4	9	5.62 (25)	2.89 (06)		6.0/4
f_2	4	10	5.63 (25)	2.89 (06)		6.5/5
f_2	5	9	4.75 (51)	3.05 (10)		2.4/3
f_2	5	10	4.78 (52)	3.05 (11)		3.0/4
f_2	4	9	6.51 (53)	2.60 (16)	4.1 (22)	2.4/3
f_2	4	10	6.27 (52)	2.68 (16)	2.9 (21)	4.3/4
f_2	5	9	5.64 (128)	2.82 (33)	2.4 (32)	1.8/2
f_2	5	10	5.05 (117)	2.98 (30)	0.7 (28)	2.9/3

Table 1: Results for the fit parameters A, B and C for the fit functions f_1 and f_2 and several choices of the fit interval.

the evaluation of the ratio is essentially limited by the two-point function of the Δ, which enters to the power $-1/2$ and becomes statistically compatible with zero for $t \gtrsim T/4$.

To extract the overlap x we fit the time-dependence of R using two Ansätze, namely

$$f_1(t) = A + B\,a\,\frac{\sinh(\delta\,t/2)}{\sin(a\delta/2)}\,, \qquad f_2(t) = A + B\,t\,(+C\,t^3)\,. \tag{3.3}$$

f_1 is the form we expect from the transfer matrix analysis as given in Eq. (2.3), amended by an additive constant shift that incorporates excited state contributions. We expect the second Ansatz f_2 to be valid in the time window where R is approximated by a linear-dependence on time. The stability of the extracted slope B is checked by including a cubic term in t. The fit results for a number of choices of fit windows $[t_{\min}, t_{\max}]$ are compiled in Tab. 1.

4. Extraction of the coupling

To extract the effective coupling from the parameter B we connect to leading-order (LO) effective field theory (EFT) and find

$$B = \sum_{\sigma_3,\,\tau_3} \frac{\mathscr{M}(\vec{Q},\vec{q},\sigma_3,\tau_3)}{\sqrt{N_\Delta N_{\pi N}}}\, V\, \delta_{\vec{Q}\vec{Q}} \times \text{spin sum factor}\,. \tag{4.1}$$

The finite volume normalization of the states reads

$$N_\Delta = V\,\frac{E_\Delta}{m_\Delta}\,, \qquad N_{\pi N} = N_\pi \times N_N = 2V\,E_\pi \times V\,\frac{E_N}{m_N}\,. \tag{4.2}$$

The matrix element $\mathscr{M}$ is decomposed according to LO EFT as

$$\mathscr{M}(\vec{Q},\vec{q},\sigma_3,\tau_3) \,=\, \frac{g^{\Delta}_{\pi N}}{2m_N}\,\bar{u}^{\mu\,\alpha}_{\Delta}(\vec{Q},\sigma_3)\,q_\mu\,u^{\alpha}_{N}(\vec{Q}+\vec{q},\tau_3)\,. \tag{4.3}$$

As can be seen from Eq. (4.3) for $\vec{q} \propto \vec{e}_j$, the imaginary part of R_j is the only component that should have a non-zero signal. This serves as a consistency check of our results, which satisfy this expectation. Using the values of B given in Tab. 1 we extract the coupling

$$g^{\Delta}_{\pi N}\,(\mathrm{LAT}) = 27.0 \pm 0.6 \pm 1.5\,. \tag{4.4}$$

The second error is a systematic error estimated from the spread of the results due to the different fit intervals and fit functions. To compare this result on the coupling constant with the experimental value we relate the coupling to the width,

$$\Gamma \,=\, \frac{{g^{\Delta}_{\pi N}}^{2}}{48\pi}\,\frac{1}{m_N^2}\,\frac{E_N+m_N}{E_N+E_\pi}\,q^3\,. \tag{4.5}$$

Using the PDG value for the Δ width [13] we find $g^{\Delta}_{\pi N}\,(\mathrm{LO\ EFT}) = 29.4\,(4)$. In Ref. [11] a model-independent K-matrix analysis yielded the value $g^{\Delta}_{\pi N}\,(\mathrm{EXP}) = 28.6\,(3)$. We find reasonable agreement with both results. This, in turn, means that the width

$$\Gamma_\Delta\,(\mathrm{LAT}) \,=\, 99\,(12)\,\mathrm{MeV}\,, \tag{4.6}$$

obtained using our lattice result on the coupling constant is consistent with the experimental value $\Gamma_\Delta\,(\mathrm{EXP}) = 117\,(3)\,\mathrm{MeV}$.

5. Discussion and Outlook

We presented results on the Δ resonance for one ensemble of staggered sea quarks and domain wall valence quarks. Although we find good agreement with the experimental value one needs to investigate lattice artifacts as well as perform the computation with smaller pion mass before a final result can be given. However, this study has shown that the method yields robust results and therefore, one can apply it to study the decay of other baryons. As an outlook to future work we

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show preliminary results for the analysis of the decay $\Sigma^{*+} \to \pi^+\Lambda^0$ in the right panel of Fig. 3. Using only momentum directions $\vec{q} \propto \pm\vec{e}_1$ we obtain the preliminary results

$$\begin{aligned} 4 \le t/a \le 12 \quad aB &= 0.0208\,(06) \text{ with } \chi^2/\text{dof} = 1.1 \Rightarrow g = 21.5 \pm 0.7 \\ 6 \le t/a \le 12 \quad aB &= 0.0228\,(16) \text{ with } \chi^2/\text{dof} = 1.1 \Rightarrow g = 23.6 \pm 1.6\,, \end{aligned} \tag{5.1}$$

which can be compared to the LO EFT result extracted from the width [13],

$$g_{\pi\Lambda}^{\Sigma^{*+}}\,(\text{LO EFT}) \approx 20.0 \tag{5.2}$$

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