Composite flavor-singlet scalar in twelve-flavor QCD

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We report the calculation of the flavor-singlet scalar in the SU(3) gauge theory with the degenerate twelve fermions in the fundamental representation using a HISQ-type action at a fixed β. In order to reduce the large statistical error coming from the vacuum-subtracted disconnected correlator, we employ a noise reduction method and a large number of configurations. We observe that the flavor-singlet scalar is lighter than the pion in this theory from the calculations with the fermion bilinear and gluonic operators. This peculiar feature is considered to be due to the infrared conformality of this theory, and it is a promising signal for a walking technicolor, where a light composite Higgs boson is expected to emerge by approximate conformal dynamics.

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1. Introduction

Higgs boson was discovered at LHC experiment in the last year, and its mass has also been already measured, which is about 125 GeV. While the current experimental data insists that Higgs boson is consistent with the elementary particle in the Standard model, there still remains a possibility of a composite particle in a strongly coupled gauge theory. One of such attractive models is the walking technicolor [1], where a light composite flavor-singlet scalar is expected as a pseudo Nambu-Goldstone (NG) boson of the approximate conformal symmetry. Phenomenologically, a consistency of a composite Higgs boson with the current LHC data was discussed in Refs. [2, 3, 4]. Thus, the most urgent theoretical task is to check the existence of such a light flavor-singlet scalar state from first principle, lattice gauge calculation. (For reviews on the lattice studies in search for candidates for the walking technicolor, see [5, 6, 7] and references therein.)

In this study, we calculate the flavor-singlet scalar state (σ) in the SU(3) gauge theory with the fundamental twelve fermions ($N_f = 12$ QCD), which was studied by several groups. In our previous study [8] of this theory, it is found that m_f dependence of meson masses is consistent with the one expected in the conformal theory. We aim to investigate the properties of σ in this theory, especially whether σ is lighter or not, because σ in the walking technicolor is expected to have similar properties due to approximate conformal symmetry. Thus, this work is regarded as a pilot study for more attractive theories, such as the $N_f = 8$ QCD that was found to be a candidate of the walking technicolor in our work [9].

The calculation of σ state is one of the most challenging calculations in the lattice simulation due to huge statistical error of the vacuum-subtracted disconnected correlator. Using a noise reduction method and a large number of configurations for the disconnected correlator, we obtain the mass of σ (m_σ) with a reasonable error. From the results, we find that σ is lighter than pion π, corresponding to the NG boson in the chiral broken phase, in all the fermion masses. This peculiar feature is considered as a reflection of the conformal dynamics of this theory. While the light σ in the $N_f = 12$ QCD would not be regarded as a composite Higgs boson, it is a promising signal for a walking technicolor that has approximate conformal symmetry. All the results in this report have already been presented in Refs. [10, 11, 12].

2. Simulation details

We employ tree-level Symanzik gauge action and HISQ (highly improved staggered quark) [13] action without tadpole improvement and mass correction in the Naik term. The flavor symmetry breaking of this action is highly suppressed in QCD [14] and we observed that it is almost negligible in our $N_f = 12$ QCD simulations [8]. Using three degenerate staggered fermion species, we generate configurations by the standard HMC algorithm on three different lattice volumes ($V = L^3$), $L = 24, 30$ and 36, with fixed aspect ratio $T/L = 4/3$ at a single lattice spacing corresponding to $\beta \equiv 6/g^2 = 4.0$ for the four fermion masses, $m_f = 0.05, 0.06, 0.08$, and 0.10. All the simulation parameters and also the corresponding π mass (m_π) are tabulated in Table 1. We accumulate 8000–30000 trajectories depending on the parameters, and perform measurements every 2 trajectories. Such a large number of configurations allows us to obtain a reasonable signal of m_σ. The statistical error is estimated by the standard jackknife method with bin size of 200 trajectories.

We use the flavor-singlet scalar operator of the local staggered fermion bilinear

$$O_S(t) = \sum_{i=1}^{3} \sum_x \bar{\chi}_i(x,t)\chi_i(x,t), \tag{2.1}$$

where i denotes the index of the staggered fermion species. Using $O_s(t)$ we calculate the flavor-singlet scalar correlator, which is constructed by both the connected $C(t)$ and vacuum-subtracted disconnected $D(t)$ correlators as $\langle O_s(t)O_s^\dagger(0)\rangle \propto 3D(t) - C(t)$, where the factor in front of $D(t)$ comes from the number of species.

In the staggered fermion formulation, the operator O_s overlaps with σ state, but also with a flavor non-singlet pseudoscalar state ($\pi_{\overline{SC}}$) that is the staggered parity partner of σ. Therefore, in the large t region, the correlator behaves as

$$3D(t) - C(t) = A_\sigma(t) + (-1)^t A_{\pi_{\overline{SC}}}(t), \tag{2.2}$$

where $A_H(t) = A_H(e^{-m_H t} + e^{-m_H(T-t)})$, and the pseudoscalar state has a $(\gamma_5\gamma_4 \otimes \xi_5\xi_4)$ spin-taste structure, but is species-singlet.

Since $C(t)$ can be regarded as a flavor non-singlet scalar correlator, $C(t)$ has a contribution from the lightest non-singlet scalar state (a_0) and its staggered parity partner (π_{SC}). When t is large, we can therefore write

$$-C(t) = A_{a_0}(t) + (-1)^t A_{\pi_{SC}}(t), \tag{2.3}$$

where both a_0 and π_{SC} are species non-singlet and have the same taste structure as σ and $\pi_{\overline{SC}}$, respectively. The π_{SC} state is degenerate with the NG π and also with $\pi_{\overline{SC}}$, ($m_{\pi_{SC}} = m_\pi = m_{\pi_{\overline{SC}}}$) when the taste symmetry, thus the full flavor symmetry, is recovered. From Eqs. (2.2) and (2.3), the large t asymptotic form of $3D(t)$ is written as

$$3D(t) = A_\sigma(t) - A_{a_0}(t) + (-1)^t(A_{\pi_{SC}}(t) - A_{\pi_{\overline{SC}}}(t)). \tag{2.4}$$

The disconnected correlator $D(t)$, which is essential to obtain m_σ, is calculated by inverting the staggered Dirac operator at each spacetime point $(\vec{x},t)$. The computational cost of this inversion is mitigated by using a stochastic noise method. Moreover, its large fluctuations from the random noise in the method is dealt with by using a variance reduction method [15, 16, 17] already employed for the flavor-singlet pseudoscalar [15, 18] and chiral condensate [17] in usual QCD, and for the flavor-singlet scalar meson in $N_f = 12$ QCD [19] and in $N_f = 8$ QCD [20]. We employ 64 random sources spread in the spacetime and the color space for this reduction method.

3. Result

A typical result of $-C(t)$ and $3D(t)$ is shown in Fig. 1. In the large t region, $3D(t)$ behaves as a smooth function of t in contrast to $-C(t)$, which has a clear oscillating behavior. This means that the taste-symmetry breaking between $A_{\pi_{SC}}(t)$ and $A_{\pi_{\overline{SC}}}(t)$ in Eq. (2.4) is small, which can be expected from our previous work [8], where the taste-symmetry breaking in the meson masses were negligible.

$L^3 \times T$	m_f	N_{cfgs}	m_σ	m_π	m_σ/m_π
$24^3 \times 32$	0.05	11000	$0.237(13)\binom{02}{01}$	$0.3273(19)^*$	$0.73(4)\binom{1}{0}$
$24^3 \times 32$	0.06	14000	$0.279(17)\binom{07}{01}$	$0.3646(16)^*$	$0.77(5)\binom{2}{0}$
$24^3 \times 32$	0.08	15000	$0.359(21)\binom{01}{18}$	$0.4459(11)$	$0.81(5)\binom{0}{4}$
$24^3 \times 32$	0.10	9000	$0.453(42)\binom{37}{08}$	$0.5210(7)$	$0.87(8)\binom{7}{2}$
$30^3 \times 40$	0.05	10000	$0.275(13)\binom{21}{08}$	$0.3192(14)^*$	$0.86(4)\binom{7}{3}$
$30^3 \times 40$	0.06	15000	$0.329(15)\binom{47}{12}$	$0.3648(9)^*$	$0.90(4)\binom{13}{3}$
$30^3 \times 40$	0.08	15000	$0.382(21)\binom{03}{16}$	$0.4499(8)$	$0.85(5)\binom{1}{4}$
$30^3 \times 40$	0.10	4000	$0.431(51)\binom{06}{04}$	$0.5243(7)$	$0.82(10)\binom{1}{1}$
$36^3 \times 48$	0.05	5000	$0.283(23)\binom{01}{02}$	$0.3204(7)^*$	$0.88(7)\binom{0}{1}$
$36^3 \times 48$	0.06	6000	$0.305(22)\binom{25}{06}$	$0.3636(9)^*$	$0.84(6)\binom{7}{2}$

Table 1: Parameters of lattice simulations, m_σ, and m_π. N_{cfgs} is the number of saved gauge configurations. The second error of m_σ is a systematic error coming from the fit range. The values of m_π are from Ref. [8], but the ones with (*) have been updated. The error on m_σ/m_π comes only from m_σ.
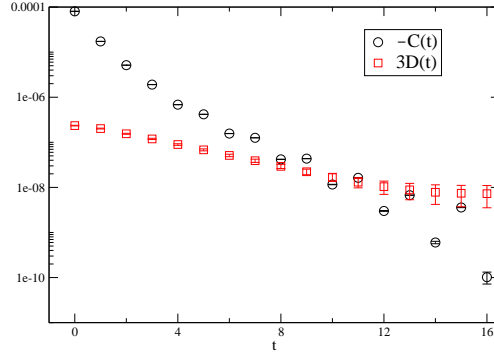

Figure 1: Connected $-C(t)$ and vacuum-subtracted disconnected $3D(t)$ correlators on $L = 24$ at $m_f = 0.06$.

The effective mass of each correlator is presented in Fig. 2. In order to minimize the parity partner contributions, such as $A_{\pi_{\overline{SC}}}(t)$ in Eq. (2.2), we adopt a projection, $C_+(t) = 2C(t) + C(t+1) + C(t-1)$, at even t. The effective mass of the full correlator $3D_+(t) - C_+(t)$ at large t is smaller than m_π, while the error is large. Since $m_\sigma < m_{a_0}$ as shown in the figure, we also employ $D(t)$ to extract m_σ. The effective mass plateau of $D(t)$ is consistent with the one of $3D_+(t) - C_+(t)$ in the large t region. Furthermore the plot clarifies the importance of using $D(t)$ to extract m_σ, because it performs better in identifying the lightest scalar state, even at small t region. This might be caused by a reasonable cancellation among contributions from excited scalar states and the a_0 state in $D(t)$.

We fit $3D(t)$ between $t = 4$ and $t = 8$, assuming a single light state propagating in this region, to obtain m_σ for all the parameters. A systematic error coming from the choice of the fitting range is estimated by the difference of central values obtained with several fit ranges. Figure 3 shows m_σ of all the parameters as a function of m_f, whose values are tabulated in Table. 1. Finite size effects are negligible on the largest two volumes at each m_f in our statistics. The difference of m_σ and m_π is more than one standard deviation when the statistic and systematic errors are combined in quadrature, except for $m_f = 0.06$ on $L = 30$, where there is a sizable systematic error. Recently, the light σ in the $N_f = 12$ QCD is also confirmed by the other group with the different lattice
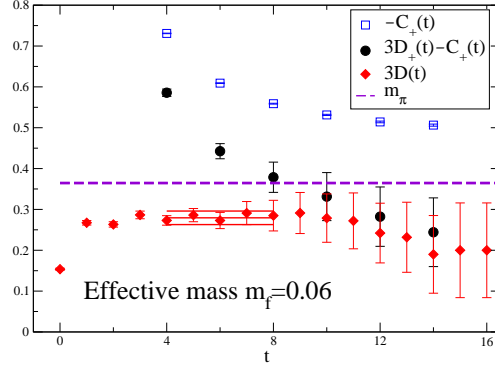
Figure 2: Effective masses for parity-projected connected correlator $-C_+(t)$, parity-projected full correlator $3D_+(t) - C_+(t)$, and vacuum-subtracted disconnected correlator $3D(t)$ on $L = 24$ at $m_f = 0.06$. Dashed and solid lines express m_π and fit result of m_σ from $3D(t)$ in $t = 4$–8 with the error band of one standard deviation, respectively.
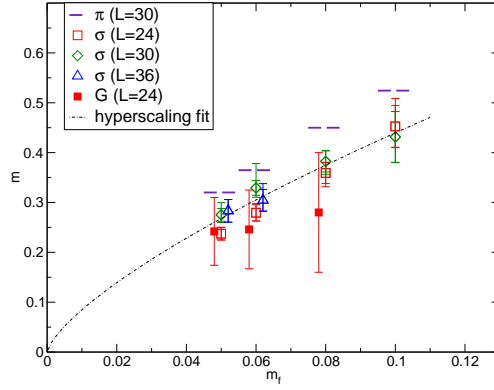

Figure 3: Mass of flavor-singlet scalar m_σ in all the parameters and mass obtained from gluonic calculation at $L = 24$ are presented. Inner error bar denotes statistical error and outer denotes combined error of statistical and systematic added in quadrature. Short dashed and dash-dot lines represent m_π at each m_f and hyperscaling fit result explained in text, respectively.

action [7]. We also obtain the mass from the 0^{++} glueball correlators [10, 12] with the variational analysis at $L = 24$ as plotted in the figure. The result is smaller than m_π by more than one standard deviation, and is consistent with m_σ obtained from the fermionic correlator at each parameter, while the statistical error is large.

For a check of consistency with the hyperscaling of m_π observed in the previous work [8], we fit m_σ on the largest volume data at each m_f using the hyperscaling form

$$m_\sigma = C m_f^{1/(1+\gamma)}, \tag{3.1}$$

with the fixed $\gamma = 0.414$ estimated from the finite-size hyperscaling analysis of m_π [8]. This fit gives a reasonable value of $\chi^2/\text{dof} = 0.12$, and the fit result is shown in Fig. 3. We also estimate the ratio m_σ/m_π at each parameter and present it in Fig. 4 and Table 1. All the ratios are smaller than unity by more than one standard deviation including the systematic error, except the one at $m_f = 0.06$ on $L = 30$, as previously explained. Since the ratio behaves as a constant for m_f, this
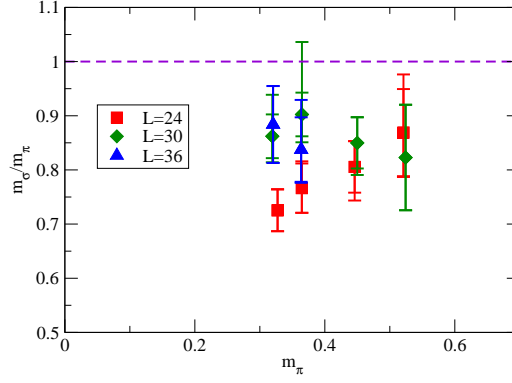
Figure 4: Ratio of m_σ/m_π in all the parameters. Inner error bar denotes statistical error and outer denotes combined error of statistical and systematic added in quadrature.

result is also shows that m_f dependence of m_σ is consistent with the one of m_π. A constant fit with the largest volume data at each m_f gives 0.86(3). These results are consistent with the theory being in the infrared conformal phase.

4. Summary

We have calculated the mass of the flavor-singlet scalar state in $N_f = 12$ QCD with a HISQ-type action at $\beta = 4.0$. We have obtained reasonable signals of m_σ from the vacuum-subtracted disconnected correlator, which is essential to calculate σ state. Clear signals in our simulations are possible thanks to the following salient features: 1. Small taste-symmetry breaking, 2. Efficient noise reduction methods, 3. Large configuration ensembles, and 4. Slow damping of $D(t)$ because of small m_σ.

From the results, we have confirmed that the obtained m_σ is smaller than m_π at all m_f we simulated. This feature is also observed from the gluonic measurements. The hyperscaling behavior of m_σ is confirmed from the analysis with a hyperscaling fit using the fixed value of γ, which was estimated in our previous study [8], and also from the ratio of m_σ/m_π. These results are consistent with that the $N_f = 12$ QCD has the infrared conformality. Calculations with lighter fermion and at different lattice spacing are important future works to understand more detailed property of σ in this theory.

We regard the light σ observed in the $N_f = 12$ QCD as a reflection of the dilatonic nature of the conformal dynamics. Thus, it is a promising signal for a walking theory [1], which has an approximate infrared conformality. It is the most pressing future direction to look at more viable candidates for walking technicolor models. For example, it will be interesting to investigate the scalar spectrum of the $N_f = 8$ QCD theory, which was shown to be a good candidate for the walking technicolor model [9]. We have already started this calculation and presented preliminary results in this conference [20].

Acknowledgments

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