PROCEEDINGS OF SCIENCE

Nuclear Dynamics in the Superfluid Regime or Trampling Around the "Funny Hills" of the Nuclear Landscape

Luciano G Moretto[*]
University of California, Berkeley and Lawrence Berkeley National Laboratory
E-mail: lgmoretto@lbl.gov

The dynamics of collective motion on the nuclear potential energy landscape demands the knowledge of the inertia (and viscosity) tensors. Pairing affects both, either directly or indirectly. The effect of pairing in dynamical motion is evaluated semi-classically in the case of finite barrier penetration and saddle to scission descent. The dramatic result obtained in both cases speaks to the necessity of extensive work in this direction.

This paper is dedicated to Adriano, in fond remembrance

International Winter Meeting on Nuclear Physics,
21-25 January 2013
Bormio, Italy

[*]Speaker.

© Copyright owned by the author(s) under the terms of the Creative Commons Attribution-NonCommercial-ShareAlike Licence. http://pos.sissa.it/

PoS(Bormio 2013)071

1. Introduction

The extensive work on the collective nuclear potential landscape done by means of the microscopic-macroscopic models or by other more modern methods is in stark contrast to the minimal work done on the inertia (viscosity) tensors. Yet classical and quantum dynamics requires the knowledge of both the potential energy and the inertia tensor in order to define the true evolution of the system, or, semiclassically, its trajectory in collective space.

In fact, it is worth reflecting that of all the potential energy landscape, only the stationary points (i.e. - minima, maxima, saddle points) are invariant under a generic coordinate transformation. Even downhill may become uphill and vice versa. The only way to make use of the rest of the potential energy surface is to consider it in conjunction with the associated tensors mentioned above. Only then can the classical and semiclassical trajectories be determined through the extremization of the action integral.

The presence of pairing in nuclei also affects the dynamics. Normally the pairing energy of a nucleus is evaluated in its ground state and is taken as a static constant as opposed to it being evaluated along each point in a trajectory. It is shown in this presentation that the effect of a dynamical pairing energy greatly influences nuclear dynamics.

This work is done in the context of spontaneous fission, where a dynamical pairing energy can influence both the dynamics of tunneling under the fission barrier along with the evolution of the system beyond the barrier. Thus the rate of spontaneous fission is controlled by both the height of the fission barrier and the effective mass of the collective mode undergoing fission. An interesting application of this effect is to consider the relative rate of spontaneous fission of an actinide nucleus in its ground state versus a shape isomeric state.

2. The role of pairing

2.1 The potential energy

Pairing is typically included in the potential energy calculations. However the gap parameter Δ is calculated by requiring the expectation value of the pairing Hamiltonian H to be minimum

$$\frac{\partial \langle H(\Delta) \rangle}{\partial \Delta} = 0. \tag{2.1}$$

This is the gap equation and can be solved for Δ, which however, is a static value! Dynamically Δ can be very different.

2.2 The inertia

The inertia is also strongly pairing dependent. An estimate from the cranking model [1] gives the effective mass B of a collective motion as

$$B = \hbar^2 \left\langle \left(\frac{\partial \varepsilon_k}{\partial \alpha} \right)^2 \right\rangle \frac{g}{3\Delta^2} \simeq \frac{K}{\Delta^2}. \tag{2.2}$$

Here g is the doubly degenerate single particle level density and K is approximated to be constant. For this approximation the uniform model is used, where it is presumed that the single particle states are uniformly distributed. But, what value of Δ should one use?

PoS(Bormio 2013)071

2.3 The action

The solution to this quandary is to use the action, where extremization with respect to Δ should give the correct and relevant value of Δ. In order to do so, we calculate the Δ dependence to second order in Δ starting with

$$V(\alpha,\Delta) \equiv \langle H\rangle = 2\sum v_k^2(\varepsilon_k-\lambda) - G\left[\sum u_k v_k\right]^2. \tag{2.3}$$

The second order term in the expansion is

$$\frac{1}{2}\frac{\partial^2\langle H\rangle}{\partial\Delta^2}\bigg|_{\Delta=\Delta_0} = \frac{1}{2}\Delta_0^2\sum E_k^{-3} - \frac{1}{4}\Delta_0^4 G\left[\sum E_k^{-3}\right]^2 \tag{2.4}$$

The uniform model can be applied to simplify this equation to

$$\frac{1}{2}\frac{\partial^2\langle H\rangle}{\partial\Delta^2}\bigg|_{\Delta=\Delta_0} = g\left(1-\frac{1}{2}gG\right) \approx g. \tag{2.5}$$

Hence, the potential energy expanded to second order in Δ is

$$V(\alpha,\Delta) = V_0(\alpha) + g(\Delta-\Delta_0)^2. \tag{2.6}$$

For the inertia we use the form

$$B = \frac{K}{\Delta^2} + \beta, \tag{2.7}$$

where the K/Δ^2 comes from the cranking model [1] and β is the irrotational limit of the inertia.

The action can now be written as

$$S = \int_a^b d\alpha\left[2\left(\frac{K}{\Delta^2}+\beta\right)\left\{V_0(\alpha)-E+g(\Delta-\Delta_0)^2\right\}\right]^{1/2}. \tag{2.8}$$

The gap parameter along the trajectory must be determined dynamically. Every point along the trajectory thus satisfies

$$\frac{d}{d\Delta}\left[\left(\frac{K}{\Delta^2}+\beta\right)\left\{V_0(\alpha)-E+g(\Delta-\Delta_0)^2\right\}\right] = 0. \tag{2.9}$$

This implies that the gap along each point of the trajectory is

$$\frac{\Delta}{\Delta_0} = 1 + \frac{V_0(\alpha)-E}{g\Delta_0^2}, \tag{2.10}$$

in the limiting case of $\beta = 0$.

3. The magic world of superfluidity

The use of the uniform model allows one to appreciate the surprising effects of dynamical pairing. The derived equations presented here are only valid for values of Δ sufficiently close to Δ_0. However, one can appreciate the main qualitative conclusion even from this very crude equation.

PoS(Bormio 2013)071

3.1 The "ordinary" world

Consider the effect of dynamical pairing on a collective motion with a positive kinetic energy [2]. As one tries to inject kinetic energy into the collective degree of freedom, the system reacts in such a way as to *decrease* the pairing correlation. This phenomenon, which is analogous to the Coriolis antipairing effect, depends only upon the sign of $\partial B/\partial\Delta$ and it is a very fundamental property of the classical equations of motion.

If one calls the actual kinetic energy $E_{kin} = E - V(\alpha,\Delta)$ and the expected kinetic energy $E^0_{kin} = E - V_0(\alpha,\Delta_0)$ one obtains for the ratio of the two quantities

$$\frac{E_{kin}}{E^0_{kin}} = 1 - \frac{E^0_{kin}}{g\Delta_0^2} = \frac{\Delta}{\Delta_0}. \tag{3.1}$$

This ratio, as one can see, decreases continuously with increasing E^0_{kin}. The actual kinetic energy E_{kin} first increases with E^0_{kin} and then decreases. The maximum occurs at:

$$\frac{dE_{kin}}{dE^0_{kin}} = 1 - 2\frac{E^0_{kin}}{g\Delta_0^2} = 0, \tag{3.2}$$

or at

$$E^0_{kin} = \frac{1}{2}g\Delta_0^2, \tag{3.3}$$

where E_{kin} assumes the value:

$$E_{kin} = \frac{1}{2}E^0_{kin}, \tag{3.4}$$

and

$$\Delta = \frac{1}{2}\Delta_0. \tag{3.5}$$

For larger values of E^0_{kin} the true kinetic energy decreases. Equation 3.1 is shown in figure 1. Again, one must be very cautious in using these results for values of Δ much different from Δ_0, both because of the quadratic expansion and because of the poor knowledge of the dependence of $B(\alpha,\Delta)$ upon Δ.

In the above treatment one has assumed that Δ acts as a parameter which adjusts adiabatically without having any kinetic energy term associated with it. One may improve the picture by considering Δ to be a dynamical variable. In this case, $B(\alpha,\Delta)$ becomes a 2×2 matrix and the action becomes

$$S = \int_a^b d\alpha \left[2(E - V(\alpha,\Delta))\left\{B_{\alpha\alpha} + 2B_{\alpha\Delta}\Delta_\alpha + B_{\Delta\Delta}\Delta_\alpha^2\right\}\right]^{1/2}, \tag{3.6}$$

where Δ_α is the first derivative of Δ with respect to α. The Euler equation now becomes a differential equation. Because, at least in the uniform model,

$$B_{\alpha\alpha} \gg 2B_{\alpha\Delta}\Delta_\alpha \text{ and } B_{\alpha\alpha} \gg B_{\Delta\Delta}\Delta_\alpha^2, \tag{3.7}$$

the previously developed formulas still hold qualitatively. The overall qualitative result of this exercise is the following. A collective mode cannot bear much more kinetic energy than an amount of the order of the condensation energy $\frac{1}{2}g\Delta_0^2$, without dramatically decreasing the pairing correlation.

In the descent from saddle to scission, as the system starts increasing its velocity, pairing should quickly decrease to the point where quasiparticle excitations become very likely due to

PoS(Bormio 2013)071

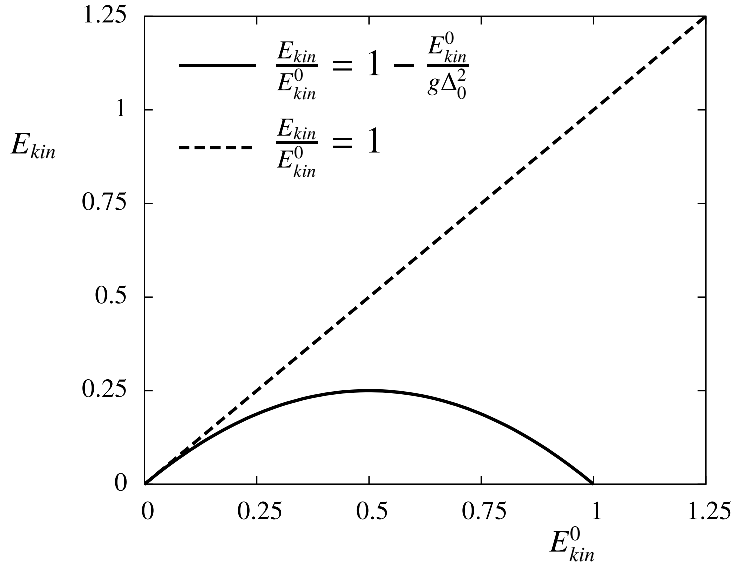

Figure 1: The solid line shows how loading energy into a collective mode does not necessarily increase the kinetic energy of the motion. The dashed line is if all the injected energy goes to kinetic energy. The energy is in units of $1/g\Delta_0^2$.

their decreased pairing gap energy. The creation of quasiparticles should further decrease pairing through "blocking", thus generating a catastrophic breakdown of the pairing correlation.

In conclusion it appears that:

1. A dynamical treatment of the pairing correlation must be used when dealing with nuclear collective motion;

2. The reaction of the pairing correlation to the collective velocity depends critically upon sign and magnitude of $\partial B/\partial\Delta$;

3. A negative $\partial B/\partial\Delta$ implies a decrease in pairing as the collective velocity increases, possibly leading to a catastrophic breakdown of the pairing correlation;

4. Any attempt to determine the degree of viscosity in the descent from saddle to scission should be preceded by a careful evaluation of $B_{\alpha\Delta}(\alpha,\Delta)$.

3.2 The classically forbidden world

The quantum world is not limited to positive kinetic energies. Pairing also affects a system tunneling through a barrier [3]. Before penetrating the barrier, at the classical turning point given by the equation $V_0(\alpha) - E = 0$, the gap parameter is $\Delta = \Delta_0$: in other words the solution for Δ is the same as that given by the gap equation. As the system dives into the barrier, the least action principle tends to *decrease* the inertia by *increasing* Δ. The gap parameter is prevented from increasing indefinitely by the restoring force originated by the potential energy as in equation 2.6. Since in order to obtain equation 2.10 β has been set equal to zero, the gap parameter given by equation 2.10 is somewhat overestimated. Still, one can see that the effect is indeed very large. By using the following round numbers: $g = 7$ MeV^{-1} , $\Delta_0 = 1$ MeV, and $V_0(\alpha) - E = 7$ MeV,

PoS(Bormio 2013)071

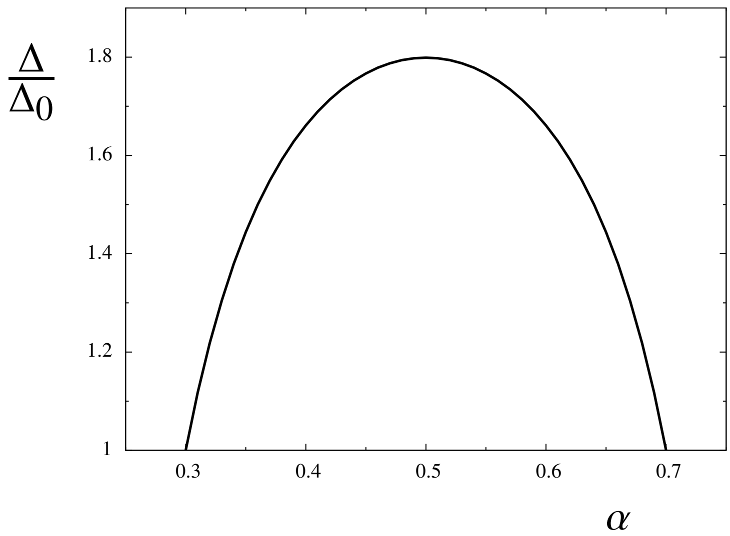

Figure 2: Enhancement of the gap parameter in the penetration of the fission barrier.

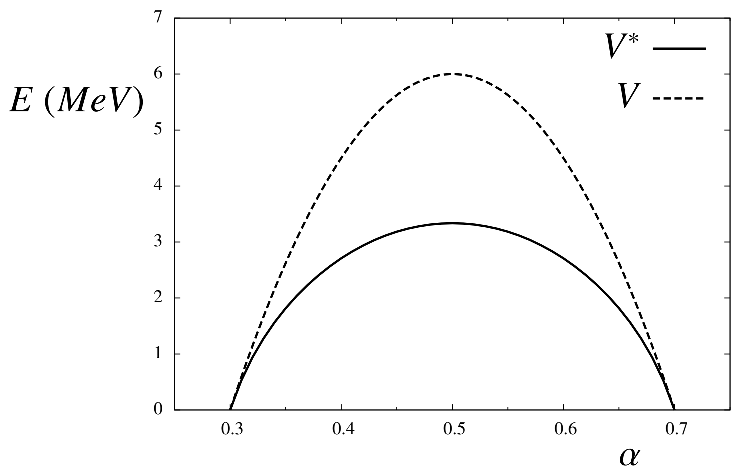

Figure 3: The effective fission barrier V^* is lower than the fission barrier V with no dynamical pairing.

one obtains $\Delta = 2\Delta_0$ for the enhancement under the top of the barrier. The effect of such a pairing increase can be incorporated into an effective potential V^*:

$$V^*(\alpha) - E = \frac{V_0(\alpha) - E}{1 + (V_0(\alpha) - E)/g\Delta_0^2}. \tag{3.8}$$

Again, by using the above mentioned parameters, it appears that, the deeper the system dives into the barrier, the more the effective barrier is reduced: in particular by using the numerical values of the parameters mentioned above, the height of the effective barrier is reduced by a factor of two with respect to the true barrier. Again, although equation 3.8 is overestimating the effect, it is clear that such a dramatic reduction of the barrier must have a substantial effect on the spontaneous fission half-lives.

PoS(Bormio 2013)071

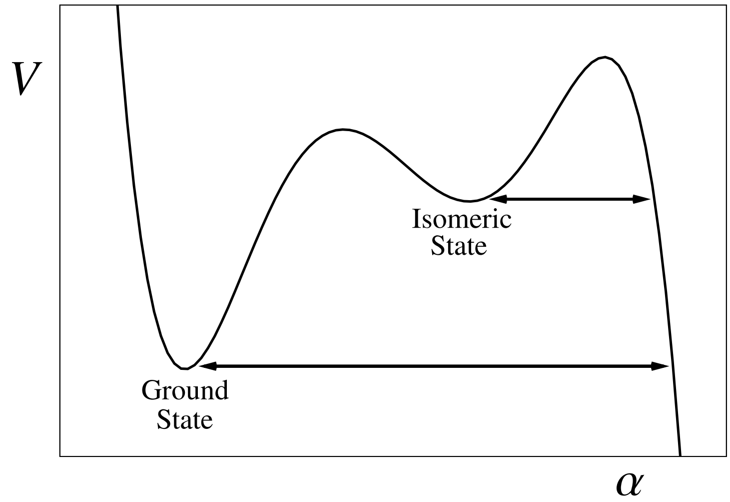

Figure 4: A schematic potential for fission barrier with an isomeric state. The arrows are to emphasize how the dynamical pairing effect is different tunneling from either.

In figure 2, a plot of Δ/Δ_0 is shown as a function of α. This calculation applied to the uniform model has been performed by substituting equation 2.7 and equation 2.3 into equation 2.8, and the following parameters have been used: $B = 1666\Delta_0^2/\Delta^2 + 225\hbar^2$ MeV^{-1}, $\Delta_0 = 0.775$ MeV, $g = 7$ MeV^{-1}, $V = 6 + \frac{1}{2} \times 300(\alpha - \alpha_0)^2$ MeV and $E = 0$. These quantities are expected to be realistic for an actinide nucleus. In figure 3, the effective potential energy is shown as a function of deformation. The overall features are still the same as those estimated by equations 2.10 and 3.8. A limitation of the above treatment is related to the fact that Δ should be considered as a true dynamical variable instead of a simple parameter. In other words, one should account for the kinetic energy associated with Δ as well as for the potential energy. The same arguments hold as for the classically allowed region and it is seen that equations 2.10 and 3.8 as well as the numerical results presented in figures 2 and 3 should be reasonably accurate.

An elegant verification of the above enhancement of barrier penetrability could be obtained by comparing the spontaneous fission half life for the ground state and the shape isomer in actinides. The much greater penetration enhancement in the ground state spontaneous fission should be readily observed.

4. Conclusion

Dynamics affects pairing dramatically. The gap parameter, typically calculated by minimizing the expectation value of the Hamiltonian, is relevant only to stationary systems.

When dynamical situations are considered, such as the saddle to scission descent, or the fission barrier penetration, the gap parameter must be determined by examination of the action.

The dynamical effects associated with dynamical pairing are major and need fuller attention.

Acknowledgments

This work was performed by Lawrence Berkeley National Laboratory and was supported by

PoS(Bormio 2013)071

the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the US Department of Energy under contract no DE-AC02-05CH11231.

References

[1] H.C. Pauli and T. Ledergerber, *The dynamics of fission in the subbarrier region of deformation*, in proceedings of *Physics and chemistry of fission 1973, Vol. 1*, International Atomic Energy Agency, Vienna 1974.

[2] L.G. Moretto, *Antipairing effects in the collective nuclear motion and their relevance to the saddle to scission descent in fission*, *Phys Lett* **57B** (1975) 317.

[3] L.G. Moretto and R.P. Babinet, *Large superfluidity enhancement in the penetration of the fission barrier*, *Phys Lett* **49B** (1974) 147.

PoS(Bormio 2013)071