

An effective theory for QCD with an axial chemical potential

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We consider the low energy realization of QCD in terms of meson fields when an axial chemical potential is present; a situation that may be relevant in heavy ion collisions. We shall demonstrate that the presence of an axial charge constitutes an explicit source of parity breaking. The eigenstates of strong interactions do not have a definite parity and interactions that would otherwise be forbidden compete with the familiar ones. In this work, we first focus on scalars and pseudoscalars that are described by a generalized linear sigma model; and next, we give some hints on how the Vector Meson Dominance model describes the vector sector.

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1. Motivation of local parity breaking

It is well known that parity is a well established global symmetry of strong interactions. However, there are reasons to believe that it may be broken in a finite volume since the Vafa-Witten theorem [1] does not apply for $\mu \neq 0$. Accordingly, some time ago, it was proposed that the QCD vacuum can possess metastable domains leading to P violation.

Two different types of works have been of special interest in the study of the origin of this possible parity breaking effect. On the one hand, there is an important list of contributions that were devoted to the investigation of the existence of stable P - and CP -odd "pion" condensates (see [3]). On the other hand, local large topological fluctuations also had a relevant role in a hot environment [4]. This work deals with the last of these hypothesis.

It is conjectured that the presence of a non-trivial angular momentum (or magnetic field) in peripheral heavy ion collisions (HIC) leads to the so-called Chiral Magnetic Effect (CME) [2], where a separation of electrical charges takes place thus implying a clear signature for local parity breaking (LPB). However, in central collisions, fluctuations in the topological charge lead to an ephemeral phase with axial chemical potential $\mu_5 \neq 0$. In Fig. 1 we show where the LPB phase may exist within the known QCD phase diagram.

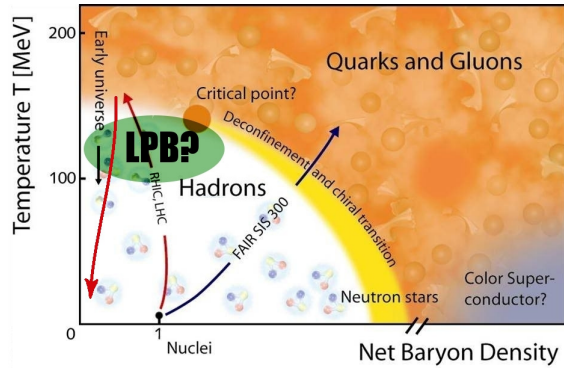


Figure 1: QCD phase diagram depending on the temperature and density with the hypothetical region where LPB may occur.

2. Axial baryon charge and axial chemical potential

The strong interaction shows a highly non-trivial vacuum that allows different topological sectors, separated by high energy barriers. These non-equivalent configurations may be connected due to large quantum fluctuations of the vacuum state in the presence of a hot medium via sphaleron transitions. A topological charge T_5 may arise in the fireball

$$T_5 = \frac{1}{8\pi^2} \int_{\text{vol.}} d^3x \varepsilon_{jkl} \text{Tr} \left(G^j \partial^k G^l - i \frac{2}{3} G^j G^k G^l \right)$$

and survive for a sizeable lifetime. During this period, one can control the jump in the topological charge $\langle \Delta T_5 \rangle$, a gauge invariant object, by introducing into the QCD Lagrangian a topological chemical potential μ_θ through $\Delta \mathcal{L}_{\text{top}} = \mu_\theta \Delta T_5$, where

$$\Delta T_5 = T_5(t_f) - T_5(0) = \frac{1}{8\pi^2} \int_0^{t_f} dt \int_{\text{vol.}} d^3x \text{Tr} \left(G^{\mu\nu} \tilde{G}_{\mu\nu} \right).$$

In the chiral limit, the integration of the local PCAC, broken by the gluon anomaly, predicts a conserved axial charge Q_5^q during $\tau_{\text{fireball}} \sim 5 \div 10$ fm connected to T_5 via

$$\frac{d}{dt} (Q_5^q - 2N_f T_5) \simeq 0, \quad Q_5^q = \int_{\text{vol.}} d^3x \bar{q} \gamma_0 \gamma_5 q = \langle N_L - N_R \rangle.$$

Thus, the presence of a non-trivial topological charge in the finite volume is directly related to the emergence of a quark axial charge, which has a characteristic oscillation time governed by the inverse quark masses. Therefore, for u, d quarks $1/m_q \sim 1/5 \text{ MeV}^{-1} \sim 40 \text{ fm}$, so the oscillation can be neglected and during the fireball lifetime we may consider the chiral charge to be approximately constant. However, for the s quark one has $1/m_s \sim 1/150 \text{ MeV}^{-1} \sim 1 \text{ fm} \ll \tau_{\text{fireball}}$ and even if a topological charge persists during fireball lifetime, the mean value of strange quark chiral charge is close to zero due to essential left-right oscillations.

As a result, for u and d quarks, QCD with a background topological charge leads to the generation of an axial chemical potential μ_5 , conjugate to the quark axial charge Q_5^q , which can be related to the topological charge and chemical potential in the following way:

$$\langle \Delta T_5 \rangle \simeq \frac{1}{2N_f} \langle Q_5^q \rangle \iff \mu_5 \simeq \frac{1}{2N_f} \mu_\theta, \quad \Delta \mathcal{L}_{\text{top}} = \mu_\theta \Delta T_5 \iff \Delta \mathcal{L}_q = \mu_5 Q_5^q.$$

3. Effective scalar/pseudoscalar meson theory with μ_5

This section is devoted to show how the parity breaking effect is introduced in hadron physics and its main results. If we focus in the scalar meson sector, the spurion technique can be used by taking μ_5 as the time component of some external axial-vector field. In this work, we will consider μ_5 to be an isosinglet. Therefore the derivative operator is extended in the following way

$$D_v \implies D_v - i\{\mathbf{I}_q \mu_5 \delta_{0v}, \cdot\} = D_v - 2i\mathbf{I}_q \mu_5 \delta_{0v}.$$

Note the breaking of Lorentz symmetry when the time component of the external field is selected. In a medium where parity is broken, two new processes are likely to appear inside the fireball: the decays $\eta, \eta' \rightarrow \pi\pi$ that are strictly forbidden in QCD on parity grounds. And even more, they might reach thermal equilibrium. In order to investigate such an effect, we construct a generalized Σ model with the following effective Lagrangian:

$$\begin{aligned} \mathcal{L} = & \frac{1}{4} \text{Tr} (D_\mu H D^\mu H^\dagger) + \frac{b}{2} \text{Tr} [M(H + H^\dagger)] + \frac{M^2}{2} \text{Tr} (HH^\dagger) - \frac{\lambda_1}{2} \text{Tr} [(HH^\dagger)^2] - \frac{\lambda_2}{4} [\text{Tr} (HH^\dagger)]^2 \\ & + \frac{c}{2} (\det H + \det H^\dagger) + \frac{d_1}{2} \text{Tr} [M(HH^\dagger H + H^\dagger HH^\dagger)] + \frac{d_2}{2} \text{Tr} [M(H + H^\dagger)] \text{Tr} (HH^\dagger) \end{aligned} \quad (3.1)$$

where the main building blocks are

$$H = \xi \Sigma \xi, \quad \xi = \exp \left(i \frac{\Phi}{2f} \right), \quad \Phi = \lambda^a \phi^a, \quad \Sigma = \lambda^b \sigma^b.$$

The v.e.v. of the neutral scalars are defined as $v_i = \langle \Sigma_{ii} \rangle$ where $i = u, d, s$, and satisfy the following gap equations:

$$M^2 v_i - 2\lambda_1 v_i^3 - \lambda_2 \bar{v}^2 v_i + c \frac{v_u v_d v_s}{v_i} = 0.$$

For further purposes we need the non-strange meson sector (composed of pions, η_q , $\sigma(600)$, $\bar{a}_0(980)$) and η_s . The relation between η_q , η_s and the genuine degrees of freedom η and η' is given by a plain rotation of the fields. In summary, we have

$$\Phi = \begin{pmatrix} \eta_q + \pi^0 & \sqrt{2}\pi^+ & 0 \\ \sqrt{2}\pi^- & \eta_q - \pi^0 & 0 \\ 0 & 0 & \sqrt{2}\eta_s \end{pmatrix}, \quad \Sigma = \begin{pmatrix} v_u + \sigma + a_0^0 & \sqrt{2}a_0^+ & 0 \\ \sqrt{2}a_0^- & v_d + \sigma - a_0^0 & 0 \\ 0 & 0 & v_s \end{pmatrix}, \quad \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix}.$$

For $\mu_5 = 0$, we assume $v_u = v_d = v_s = v_0 \equiv f_\pi \approx 92$ MeV. The coupling constants (in units of MeV) are fitted to phenomenology assuming isospin symmetry via a χ^2 minimization with the subroutine MINUIT:

$$b = -3510100/m, \quad M^2 = 1255600, \quad c = 1252.2, \quad \lambda_1 = 67.007, \\ \lambda_2 = 9.3126, \quad d_1 = -1051.7/m, \quad d_2 = 523.21/m,$$

where $m \equiv m_q = (m_u + m_d)/2$ and $m/m_s \simeq 1/25$ is used.

Next, we present a simple case of mixing due to LPB in the isotriplet sector with π and a_0 . The kinetic and mixing terms in the Lagrangian are given by

$$\mathcal{L} = \frac{1}{2}(\partial a_0)^2 + \frac{1}{2}(\partial \pi)^2 - \frac{1}{2}m_1^2 a_0^2 - \frac{1}{2}m_2^2 \pi^2 - 4\mu_5 a_0 \pi,$$

where

$$m_1^2 = -2[M^2 - 2(3\lambda_1 + \lambda_2)v_q^2 - \lambda_2 v_s^2 - cv_s + 2(3d_1 + 2d_2)mv_q + 2d_2 m_s v_s + 2\mu_5^2], \\ m_2^2 = \frac{2m}{v_q} [b + (d_1 + 2d_2)v_q^2 + d_2 v_s^2].$$

After diagonalization in the momentum representation, the new (momentum-dependent) eigenstates are defined as $\tilde{\pi}$ and \tilde{a}_0 . Note that both states reduce to the known ones in the limit $\mu_5 = 0$, say $\tilde{\pi}(\mu_5 = 0) = \pi$ and equivalently for a_0 . In the left panel of Fig. 2 we see how the $\tilde{\pi}$ mass decreases as a function of μ_5 for a particle at rest and for $q = 100$ MeV while \tilde{a}_0 mass shows an enhancement, but μ_5 has to be understood as a perturbatively small parameter.

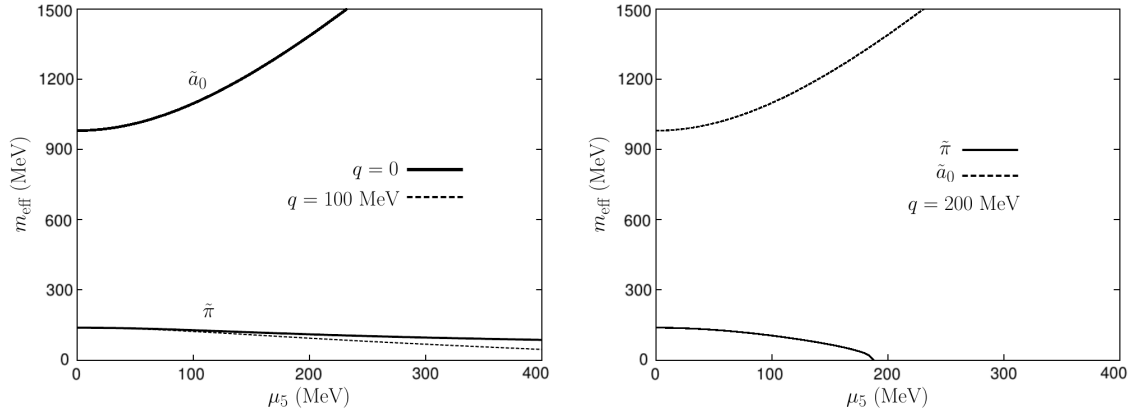


Figure 2: Effective mass dependence on μ_5 for $\tilde{\pi}$ and \tilde{a}_0 . Left panel: comparison of masses at rest and at low momentum $q = 100$ MeV. Right panel: same masses at $q = 200$ MeV, where the $\tilde{\pi}$ mass becomes tachyonic.

In the right panel of Fig. 2, it can be shown that for high energies (or momenta) bigger than a certain threshold $k_\pi^c \equiv m_1 m_2 / (4\mu_5)$, the in-medium $\tilde{\pi}$ goes tachyonic. Nevertheless, energies are checked to be always positive so no vacuum instabilities arise. No appreciable differences occur for the \tilde{a}_0 mass.

We now turn to the isosinglet sector, where the mixing of η , σ and η' is studied. The kinetic and mixing terms in the Lagrangian are given by

$$\mathcal{L} = \frac{1}{2}[(\partial\sigma)^2 + (\partial\eta_q)^2 + (\partial\eta_s)^2] - \frac{1}{2}m_3^2\sigma^2 - \frac{1}{2}m_4^2\eta_q^2 - \frac{1}{2}m_5^2\eta_s^2 - 4\mu_5\sigma\eta_q - 2\sqrt{2}cv_q\eta_q\eta_s,$$

where

$$m_3^2 = -2(M^2 - 6(\lambda_1 + \lambda_2)v_q^2 - \lambda_2v_s^2 + cv_s + 6(d_1 + 2d_2)mv_q + 2d_2m_s v_s + 2\mu_5^2),$$

$$m_4^2 = \frac{2m}{v_q} [b + (d_1 + 2d_2)v_q^2 + d_2v_s^2] + 2cv_s, \quad m_5^2 = \frac{2m_s}{v_s} [b + 2d_2v_q^2 + (d_1 + d_2)v_s^2] + \frac{cv_q^2}{v_s}.$$

After diagonalization, and following the notation introduced before, the new eigenstates are defined as $\tilde{\sigma}$, $\tilde{\eta}$ and $\tilde{\eta}'$. In the left panel of Fig. 3 we find a similar behaviour as the one seen in the triplet sector for the particles at rest and at $q = 300$ MeV. Now $\tilde{\eta}$ decreases with respect to μ_5 while $\tilde{\sigma}$ and $\tilde{\eta}'$ increase. Boosting more these eigenstates, we find in the right panel of Fig. 3 that again for high enough energies (bigger than the threshold $k_{\tilde{\eta}}^c \equiv \frac{m_3}{4\mu_5 m_5} \sqrt{m_4^2 m_5^2 - 8c^2 v_q^2}$), the in-medium $\tilde{\eta}$ goes tachyonic.

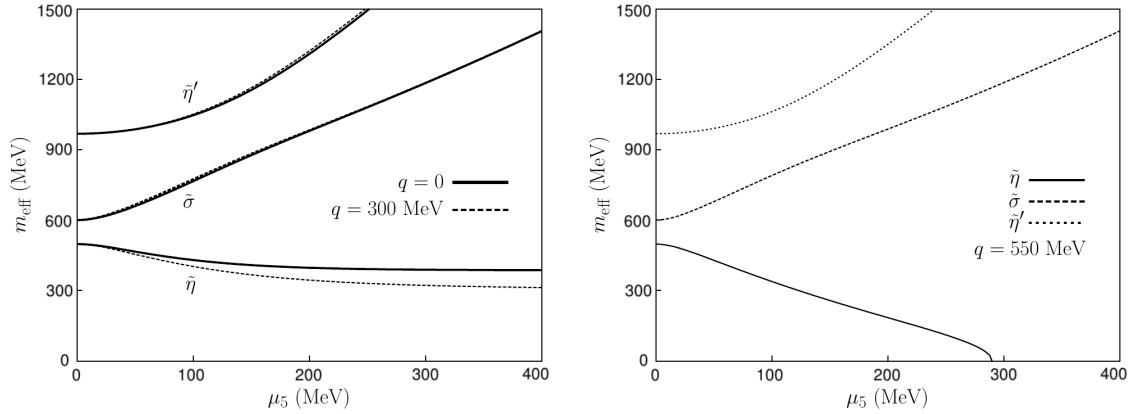


Figure 3: Effective mass dependence on μ_5 for $\tilde{\eta}$, $\tilde{\sigma}$ and $\tilde{\eta}'$. Left panel: comparison of masses at rest and at low momentum $q = 300$ MeV. Right panel: masses at $q = 550$ MeV, where the $\tilde{\eta}$ mass goes tachyonic.

The cubic couplings from our Lagrangian in Eq. (3.1) can be extracted and used to calculate the widths $\tilde{\eta}, \tilde{\sigma}, \tilde{\eta}' \rightarrow \tilde{\pi}\tilde{\pi}$. The important pieces are the following ones:

$$\mathcal{L}_{\sigma aa} = 2[(3d_1 + 2d_2)m - 2(3\lambda_1 + \lambda_2)v_q]\sigma\tilde{a}_0^2, \quad \mathcal{L}_{\sigma a\pi} = -\frac{4\mu_5}{v_q}\sigma\tilde{a}_0\tilde{\pi},$$

$$\mathcal{L}_{\sigma\pi\pi} = \frac{1}{v_q^2} [(\partial\tilde{\pi})^2 v_q - (b + 3(d_1 + 2d_2)v_q^2 + d_2v_s^2)m\tilde{\pi}^2] \sigma, \quad \mathcal{L}_{\eta aa} = -\frac{2\mu_5}{v_q}\eta_q\tilde{a}_0^2,$$

$$\mathcal{L}_{\eta a\pi} = \frac{2}{v_q^2}\tilde{a}_0[\partial\eta_q\partial\tilde{\pi}v_q - (b + (3d_1 + 2d_2)v_q^2 + d_2v_s^2)m\eta_q\tilde{\pi}], \quad \mathcal{L}_{\eta\pi\pi} = 0.$$

After the diagonalization just explained in both the isosinglet and triplet sectors, one replaces the initial degrees of freedom $\{\eta_q, \eta_s, \sigma\}$ and $\{\pi, a_0\}$ to the new ones $\{\tilde{\eta}, \tilde{\sigma}, \tilde{\eta}'\}$ and $\{\tilde{\pi}, \tilde{a}_0\}$. Hence, the widths may be numerically computed.

First of all, in the left panel of Fig. 4 we present these results at the rest frame of the decaying particle. Down to $\mu_5 = 50$ MeV, $\tilde{\eta}$ exhibits a smooth behaviour with an approximate average width of around 60 MeV, what corresponds to a mean free path of 3 fm, which is smaller than the typical fireball size of $L_{\text{fireball}} \sim 5 \div 10$ fm. As a result, a regeneration of such particles into (distorted) pions may take place in a HIC, as it is well known to happen with the ρ^0 meson, and they could possibly thermalize within the pion gas. On the other hand, down to $\mu_5 \sim 100$ MeV, $\tilde{\sigma}$ width decreases and it becomes stable. The bumps seem to reflect the tachyonic nature of the decaying $\tilde{\pi}$ but they are not trivial to analyse. Finally $\tilde{\eta}'$ width grows up to the GeV scale, which shows that the effective theory is not applicable.

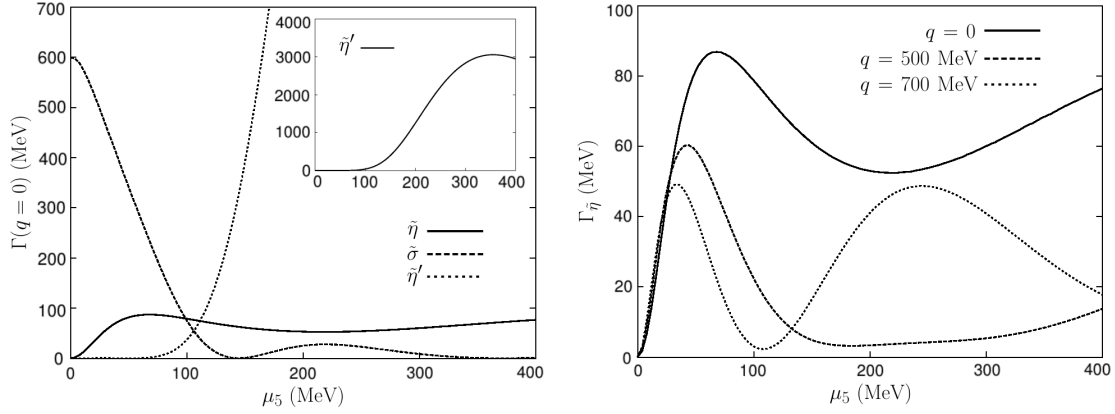


Figure 4: Left panel: $\tilde{\eta}$, $\tilde{\sigma}$ and $\tilde{\eta}'$ widths at rest depending on μ_5 . Right panel: $\tilde{\eta}$ width depending on μ_5 for different values of the incoming 3-momentum: $q = 0, 500, 700$ MeV.

Secondly, we present the same result compared with a boosted decaying $\tilde{\eta}$. In the right panel of Fig. 4 one may observe that strong dependences on the 3-momentum appear but they have nothing to do with Lorentz time dilatation. It is remarkable that beyond $q = 500$ MeV an extra bump arises at $\mu_5 \approx 240$ MeV and grows rather fast. As to $\tilde{\sigma}$ and $\tilde{\eta}'$, no big differences occur when they are boosted.

4. Vector Meson Dominance approach to LPB

At this point, one may now wonder if such a mixing of states of different parities occurs in the vector/axial-vector sector. Many models are dealing with vector particles phenomenologically but if we assume that the vector mesons appear as part of a covariant derivative, no mixing term can be generated by operators of dimension 4 if μ_5 is an isosinglet.

Hence, in this work, vector mesons are introduced and treated in the conventional way using the Vector Meson Dominance model with no mixing of states with different parities. The only LPB effect will be the Chern-Simons term

$$\Delta\mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma} \text{Tr} [\zeta_\mu V_\nu V_\rho \sigma],$$

where $\zeta_\mu \propto \mu_5 \delta_{\mu 0}$. After implementing this extension, it may be found that vector mesons exhibit the following dispersion relation:

$$m_{V,\varepsilon}^2 - m_V^2 \propto \varepsilon \mu_5 |\vec{k}|, \quad (4.1)$$

where $\varepsilon = 0, \pm 1$ is the meson polarization and the breaking of Lorentz symmetry is explicit again. According to this result, massive vector mesons split into three polarizations with masses $m_{V,-}^2 < m_{V,L}^2 < m_{V,+}^2$, which is a clear signature of parity breaking. This effect may be investigated in the HIC phenomenology and in this work, we will focus in the $\rho^0 \rightarrow e^+e^-$ decay channel.

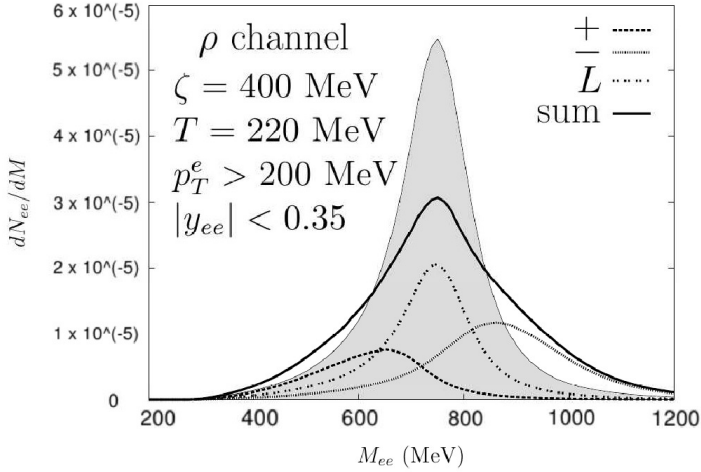


Figure 5: QCD phase diagram depending on the temperature and density with the hypothetical region where LPB may occur.

In Fig. 5 we present a plot of the number of lepton pairs depending on their invariant mass. Therein we compare the QCD result preserving parity with our calculation for $\mu_5 = 290$ MeV. The separation of the different polarization contributions is displayed together with their sum in order to show the distorted shape of the parity breaking result. Moreover, it is remarkable the polarization asymmetry aside the peak and the fact that it could be experimentally measured.

5. Conclusions

As we have seen, LPB is not forbidden by any physical principle in QCD at finite temperature and density. Therefore, we argued how topological fluctuations may be the origin of a parity breaking effect which is transmitted to hadronic physics via an axial chemical potential μ_5 . This phenomenon leads to unexpected modifications of the in-medium properties of both scalar and vector mesons. In the scalar sector, new eigenstates arise from the mixture of the known ones with interesting features such as the fact that $\tilde{\pi}$ and $\tilde{\eta}$ become tachyonic for big 3-momentum. Also $\tilde{\eta}$ acquires a non-negligible decay width meaning a significant indication for thermal equilibrium in the HIC fireball. Finally, in the vector meson sector, we found a distorted ρ spectral function in comparison with the QCD expectation.

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