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Scattering amplitude and pomeron loops in perturbative QCD at large N_c

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The amplitude for the collision of two hadrons with the lowest order pomeron loop is calculated. Numerical calculations show that the loop contribution to the amplitude begins to dominate the single pomeron exchange at rapidities $8-10$. Full dependence of the triple-pomeron vertex on intermediate conformal weights is taken into account.

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1. Introduction

In the framework of QCD with a large number of colours $N_c \gg 1$, strong interactions are mediated by the exchange of interacting BFKL pomerons, which split and fuse by triple pomeron vertices. This picture can be conveniently described by an effective non-local quantum field theory [1]. In terms of the relevant Feynman diagrams contributions standardly separate into tree diagrams and diagrams with pomeron loops, which corresponds to division into quasi-classical contribution and quantum corrections. The relative contribution of loops is characterized by the parameter $\alpha_s^4 N_c^2 \exp(\Delta_{BFKL} y)$, where y is the rapidity and Δ_{BFKL} is the intercept of the BFKL pomeron (minus unity) [2]. So although the loop contributions are damped by factor $1/N_c^2$ as compared with the tree contributions, which depend on $\alpha_s N_c$, the large exponential factor makes their relative order grow with energy. It is of vital importance then to estimate the role of pomeron loops at present energies to have some idea of the validity of the quasi-classical methods which are currently applied to the analysis of the high-energy scattering in the framework of QCD, such as the Balitski-Kovchegov equation [3, 4], its generalization to nucleus-nucleus collisions [5] and the Color Glass Condensate approach (see the review [6] and references therein).

There were quite a few attempts to estimate loop contribution, all suffering from crude approximations and uncertainties. In [7, 8] the pomeron loop was calculated under the assumption that the triple-pomeron vertex is independent of conformal weights. Later in [9] a step forward was taken to take into account the dependence of the loop on inner conformal weights. This allowed to study the relative magnitude of the loop for the BFKL propagator with a fixed (dominant) conformal weight. Still this result did not solve the real physical problem, the contribution of the loop to the scattering amplitude, which is obtained after integration over all conformal weights and requires knowledge of the triple pomeron vertex as a function of three conformal weights.

In this paper [10] we try to exploit explicit formulas for triple-pomeron vertex [11] to actually calculate the contribution of the pomeron loop to the hadronic scattering amplitude in full rigour. This contribution turns out to be quite large and becomes comparable to the tree result at rapidities of the order of 10.

2. Conformal invariant technique

The BFKL pomeron propagator satisfies the BFKL evolution equation:

$$(\frac{\partial}{\partial y} + H_{BFKL}) g_{y-\bar{y}}(r_1, r_2; \bar{r}_1, \bar{r}_2) = \delta(y-\bar{y}) \nabla_1^{-2} \nabla_2^{-2} \delta^{(2)}(r_1 - \bar{r}_1) \delta^{(2)}(r_2 - \bar{r}_2), \tag{2.1}$$

where y is the rapidity and r_i are transverse coordinates of the reggeized gluons, $r_{ij} = r_i - r_j$.

The solution of this equation was found in terms of expansion in the conformal basis [12]:

$$g_{y-y'}(r_1, r_2; \bar{r}_1, \bar{r}_2) = \sum_{\mu > 0} E_\mu(r_1, r_2) E_\mu^*(\bar{r}_1, \bar{r}_2) g_h(y - y'), \tag{2.2}$$

formed by functions E_μ. In complex notations they are

$$E_\mu(r_1, r_2) = \Big(\frac{r_{12}}{r_{10} r_{02}}\Big)^h \Big(\frac{r_{12}^*}{r_{10}^* r_{02}^*}\Big)^{\bar{h}}, \tag{2.3}$$

where $\mu = \{n, \nu, r_0\}$ denotes a set of conformal weights n, ν and two-dimensional center-of-mass coordinate r_0. Integer n and real ν enumerates functions of the basis. The conformal weight defined by n, ν is

$$h = \frac{1+n}{2} + i\nu; \quad \bar{h} = 1 - h^*. \tag{2.4}$$

For simplicity we shall also use h to denote a set $\{n, \nu\}$. Restriction $\mu > 0$ means using half of the whole set with $\nu > 0$, so that

$$\sum_{\mu>0} = \sum_{n=-\infty}^{\infty} \int_0^{\infty} d\nu \frac{1}{a_h} \int d^2 r_0, \quad a_h = \frac{\pi^4}{2} \frac{1}{\nu^2 + n^2/4}. \tag{2.5}$$

Passing from rapidity y to the complex angular momentum $j = 1 + \omega$, we have the pomeron propagator

$$g_\omega(r_1, r_2; \bar{r}_1, \bar{r}_2) = \int_{-\infty}^{\infty} dy e^{-\omega y} g_y(r_1, r_2; \bar{r}_1, \bar{r}_2). \tag{2.6}$$

As a function of ω, in the conformal basis, the propagator is

$$g_{\omega,h} = \frac{1}{l_{n\nu}} \frac{1}{\omega - \omega_h}, \tag{2.7}$$

where ω_h are the BFKL energy levels

$$\omega_h = 2\bar{\alpha}_s(\psi(1) - \mathrm{Re}\psi(h)) \tag{2.8}$$

and

$$l_h = \frac{4\pi^8}{a_{n+1,\nu} a_{n-1,\nu}}. \tag{2.9}$$

The form of the triple-pomeron vertex Γ can be extracted from the interaction part of the Lagrangian of the effective non-local field theory [5]. Passing to the conformal representation one presents:

$$\Gamma(r_1, r_2 | r_3, r_4; r_5, r_6) = \sum_{\mu_1,\mu_2,\mu_3>0} \Gamma_{\mu_1|\mu_2,\mu_3} E_{\mu_1}(r_1, r_2) E^*_{\mu_2}(r_3, r_4) E^*_{\mu_3}(r_5, r_6). \tag{2.10}$$

In the lowest order

$$\Gamma^{(0)}_{\mu_1|\mu_2,\mu_3} = R_{12}^{\alpha_{12}} R_{23}^{\alpha_{23}} R_{13}^{\alpha_{13}} \times (c.c.) \times \Omega(\bar{h}_1, h_2, h_3). \tag{2.11}$$

The conformal vertex Ω was introduced and studied by Korchemsky [11] and corresponds to planar diagrams contribution, which gives the dominant part in the limit of large number of colours.

In any pomeron Feynman diagram BFKL propagators and triple vertices are convoluted by integration over coordinates or momenta. One can perform this integration using expansions (2.2), (2.10) and completeness condition for the conformal basic functions. As a result this convolution is substituted by the summation over conformal weights and integration over intermediate center-of-mass coordinates.

With a help of this rule one can immediately write down the Schwinger-Dyson equation for a full conformal pomeron propagator $G_{\omega,h}$, which includes arbitrary number of loop insertions:

$$G_{\omega,h} = g_{\omega,h} - g_{\omega,h} l_h^2 \Sigma_{\omega,h} G_{\omega,h}, \tag{2.12}$$

where l_h^2 comes from the triple-pomeron vertex operator L acting on the incoming and outgoing pomeron propagators. Function $\Sigma_{\omega,h}$ is the pomeron self-mass in the conformal basis:

$$\Sigma_{\omega,h} = \frac{8\alpha_s^4 N_c^2}{\pi^2} \int \frac{d\omega_1}{2\pi i} \sum_{h_1 h_2} \Gamma^{(0)}_{h|h_1,h_2} G_{\omega,h_1} G_{\omega,h_2} \Gamma_{h_1,h_2|h}. \tag{2.13}$$

The formal solution of equation (2.12) is

$$G_{\omega,h} = \frac{1}{1/g_{\omega,h} + l_h^2 \Sigma_{\omega,h}}. \tag{2.14}$$

In the lowest order one should change $G_{\omega,h} \to g_{\omega_h}$ and $\Gamma_{h_1 h_2|h} \to \Gamma^{(0)}_{h_1 h_2|h}$ in (2.13).

3. Scattering amplitude

The amplitude for the scattering of two hadrons at rapidity y can be presented as

$$A(s,t) = is \int_{\sigma - i\infty}^{\sigma + i\infty} \frac{d\omega}{2\pi i} s^{\omega} f_{\omega}(q^2), \tag{3.1}$$

where function $f_\omega(q^2)$ is a t-channel partial wave for fixed transferred momentum.

In the approximation of a single bare pomeron exchange it is a convolution of the bare pomeron propagator g_ω with two impact factors describing interaction of the pomeron with external hadrons:

$$f_\omega(q^2) = \int d^2r d^2r' \; \Phi_1(r,q) g^q_\omega(r,r') \Phi_2^*(r',q), \tag{3.2}$$

where the pomeron propagator in the mixed representation is

$$(2\pi)^2 g^q_\omega(r,r') \delta^{(2)}(q-q') = \int d^2R d^2R' \; e^{iqR} e^{-iq'R'} g_\omega(r_1, r_2|r_3, r_4). \tag{3.3}$$

Here $g^q_\omega(r,r')$ describes propagation of the pomeron with momentum transfer q and can be interpreted as the amplitude for the scattering of two dipoles with sizes r and r'.

The expression $g^q_\omega(r,r')$ in the conformal basis can be easily found if one uses the corresponding conformal expansion for the bare propagator.

For our purposes it will be sufficient co consider scattering at zero transferred momentum, i.e. forward scattering amplitude $f_\omega(0)$. The Fourier transformation of conformal functions (2.3) in the region of small momentum transfer was investigated in [12]. One can use this results to pass to the limit $q = 0$:

$$g^0_\omega(r,r') = \frac{1}{\pi^2} |rr'| \sum_n \int_0^\infty d\nu \left|\frac{r}{r'}\right|^{2i\nu} \left(\frac{r^* r'}{r r'^*}\right)^{n/2} g_{\omega,h}. \tag{3.4}$$

We shall be interested only in the leading contribution, which comes from $n = 0$. Therefore the forward scattering amplitude is

$$f_\omega(0) = \frac{1}{\pi^2} \int_0^\infty d\nu \; g_{\omega,\nu} \int d^2r \Phi(r,0) |r|^{1+2i\nu} \int d^2r' \Phi^*(r',0) |r'|^{1-2i\nu}. \tag{3.5}$$

It is natural to choose the impact factor in a Gaussian form

$$\Phi(r,0) = \frac{\lambda b}{\pi} e^{-br^2}, \tag{3.6}$$

where b is the inverse of the hadron radius squared $b = 1/R_N^2$. Putting this result into (3.5) we find

$$f_\omega(0) = \frac{1}{\pi^2}\int_0^\infty d\nu\, g_{\omega,\nu} \times \frac{\lambda^2}{b}\left(\nu^2+\frac{1}{4}\right)\frac{\pi}{\cosh(\pi\nu)}, \tag{3.7}$$

where $g_{\omega,\nu}$ is a conformal propagator (2.7) with $n = 0$.

To take into account the loop contribution one has to substitute the bare propagator $g_{\omega,\nu}$ by the full Green function. With a single loop insertion

$$G_{\omega,\nu} = \frac{1}{1/g_{\omega,\nu} + l_{0\nu}^2\Sigma_{\omega,\nu}} = \frac{1}{l_{0\nu}}\frac{1}{\omega-\omega_\nu} - \frac{\Sigma_{\omega,\nu}}{(\omega-\omega_\nu)^2}. \tag{3.8}$$

The first term in (3.8) comes from exchange of the bare pomeron. It is not difficult to show that in terms of rapidity y it gives

$$A_y^{(1)}(0) = \frac{1}{16\pi^2}\frac{\lambda^2}{b}\int_0^\infty d\nu\,\frac{1}{\left(\nu^2+\frac{1}{4}\right)}\frac{\pi}{\cosh(\pi\nu)}e^{\omega_\nu y}. \tag{3.9}$$

The second term in (3.8) corresponds to the lowest order loop contribution. As a function of ω

$$f_\omega^{(2)}(0) = -\frac{1}{16\pi^2}\frac{\lambda^2}{b}\int_0^\infty d\nu\, 16\left(\nu^2+\frac{1}{4}\right)\frac{\Sigma_{\omega,\nu}}{(\omega-\omega_\nu)^2}\frac{\pi}{\cosh(\pi\nu)} \tag{3.10}$$

with the explicit form of $\Sigma_{\omega,\nu}$ given in [9]:

$$\Sigma_{\omega,\nu} = \frac{\alpha_s^4 N_c^2}{8\pi^{10}}\int_0^\infty d\nu_1 d\nu_2 \frac{\nu_1^2}{\left(\nu_1^2+\frac{1}{4}\right)^2}\frac{\nu_2^2}{\left(\nu_2^2+\frac{1}{4}\right)^2}\frac{\Omega^2(1/2+i\nu,1/2+i\nu_1,1/2+i\nu_2)}{\omega-\omega(0,\nu_1)-\omega(0,\nu_2)} \tag{3.11}$$

and the conformal vertex Ω given by [11].

For numerical calculations it is convenient to pass to rapidity. Performing integration over ω we find

$$A_y^{(2)}(0) = -\frac{1}{16\pi^2}\frac{\lambda^2}{b}\int_0^\infty d\nu\, 16\left(\nu^2+\frac{1}{4}\right)\frac{\pi}{\cosh(\pi\nu)}$$

$$\times\frac{\alpha_s^4 N_c^2}{8\pi^{10}}\int_0^\infty d\nu_1 d\nu_2 \frac{\nu_1^2}{\left(\nu_1^2+\frac{1}{4}\right)^2}\frac{\nu_2^2}{\left(\nu_2^2+\frac{1}{4}\right)^2}\Omega^2(1/2+i\nu,1/2+i\nu_1,1/2+i\nu_2)$$

$$\times\left(\frac{e^{\omega_\nu y}y}{\omega_\nu-\omega_{\nu_1}-\omega_{\nu_2}} - \frac{e^{\omega_{\nu_1}y}}{\left(\omega_\nu-\omega_{\nu_1}-\omega_{\nu_2}\right)^2} + \frac{e^{(\omega_{\nu_1}+\omega_{\nu_2})y}}{\left(\omega_\nu-\omega_{\nu_1}-\omega_{\nu_2}\right)^2}\right). \tag{3.12}$$

The total forward scattering amplitude with the lowest order loop correction is a sum of (3.9) and (3.12):

$$A_y(0) = A_y^{(1)}(0) + A_y^{(2)}(0). \tag{3.13}$$

Numerical calculations of (3.9) and (3.12) will be presented in the next section.

4. Numerical studies

We have set up a program which calculates the bare pomeron exchange amplitude (3.9) and single-loop contribution (3.12). By far the most difficult part is the computation of the triple-pomeron vertex Ω. It requires complicated numerical procedures and is extremely time-consuming. The vertex is a complex function which depends on three conformal variables ν. We have restricted this variables to lie in the interval $0 < \nu < 3.0$ and introduced a grid dividing this interval into N points. We found the vertex on the grid, which requires calculation at N^3 points. This strongly limits numbers N admissible for given calculation resources. In our case we used $N = 30$ compatible with reasonable computation time. The value of the vertex in between the grid points was found by interpolation.

The integrals in (3.9) and (3.12) were calculated by the Newton-Cotes integration formulas. The limits in ν were taken as before $0 < \nu < 3.0$. The number of sample points was chosen to provide relative error of the order of 10^{-3}. The same integration strategy was used for the vertex integrals [11].

We have performed calculations for the standard value of the QCD coupling constant $\alpha_s = 0.2$ and $N_c = 3$. In Fig. 1 the dash-dotted line presents the bare pomeron exchange contribution (3.9). The behavior of the amplitude is determined by the initial pole of the conformal BFKL propagator (2.7). As expected, the curve grows with rapidity roughly as $e^{\Delta y}$, where $\Delta \approx 0.48$. The behavior of the single-loop contribution (3.12) is shown in Fig. 1 by the solid line. It roughly grows twice faster, as $\sim e^{2\Delta y}$, again as expected. Note that the results in Fig. 1 are normalized by factor $16\pi^2 b/\lambda^2$.

For small rapidities the loop term is suppressed by the smallness of the QCD coupling constant. However, its faster growth with rapidity compensates this very early. As a result we conclude that the loop contribution becomes visible already at rapidities $3-8$ and starts to dominate at $y \sim 8-10$.

5. Conclusions

We have studied a single loop contribution to the scattering amplitude of two colliding hadrons. We have found expression for the amplitude in a framework of conformal invariant technique with more or less general form of the impact factors. The triple-pomeron vertex with full dependence on the intermediate conformal weights was calculated and used.

Numerical analysis shows that smallness of the QCD coupling constant is compensated by rapid growth of the single-loop amplitude with rapidity. We found that the loop contribution manifests itself at relatively small rapidities $y \sim 3-8$ and dominates the bare pomeron exchange amplitude already at $y \sim 8-10$.

Thus loops have to be taken into account already at present energies. Higher order loops calculation is required for larger values of rapidity with summation of all loops in the limit $y \to \infty$. This nontrivial problem is left for future investigations together with inclusion of other type of scattering hadron and running coupling.

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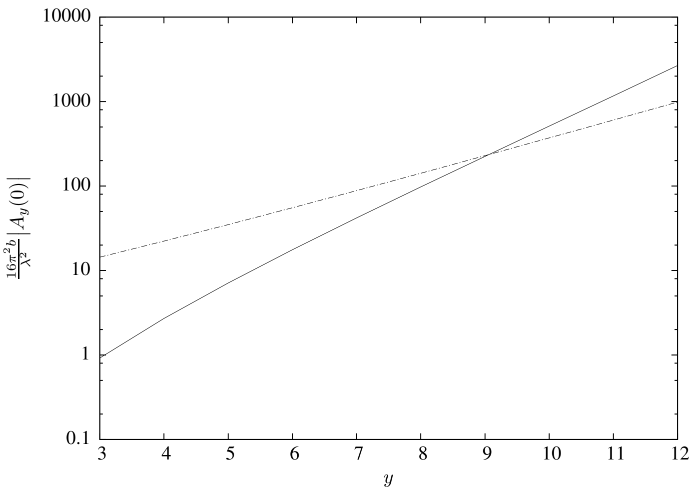

Figure 1: The bare pomeron exchange (3.9) (dash-dotted curve) and single-loop (3.12) (solid curve) contributions to the forward scattering amplitude as functions of rapidity.

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