

Review on the inclusion of isospin breaking effects in lattice calculations

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Isospin symmetry is explicitly broken in the Standard Model by the non-zero differences of mass and electric charge between the up and down quarks. Both of these corrections are expected to have a comparable size of the order of one percent relatively to hadronic energies. Although these contributions are small, they play a crucial role in hadronic and nuclear physics. In this review we explain how to properly define QCD and QED on a finite and discrete space-time so that isospin corrections to hadronic observables can be computed *ab-initio*. We then consider the different approaches to compute lattice correlation functions of QCD and QED observables. Finally we summarise the actual lattice results concerning the isospin corrections to the light hadron spectrum.

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1. Motivations

In an isospin symmetric world, the up (u) and down (d) quarks are identical particles. It is known (*cf.* table 1) that in Nature isospin symmetry is explicitly broken by the non-zero mass and electric charge differences of the u and d quarks. However, the effects of this breaking are expected to be small relative to typical strong interaction energies such as hadron masses. Indeed, from [1] it is clear that the light quark mass difference $\delta m = m_u - m_d$ represents one percent or less of any typical QCD energy scale. Similarly, the typical relative size of the electromagnetic (EM) breaking of isospin symmetry is given by the fine structure constant $\alpha \simeq 0.007$. For those reasons we can reasonably state that, for observables with a non-vanishing isospin symmetric part, isospin symmetry is a good approximation of reality with an $O(1\%)$ relative error.

Nevertheless, it is interesting to notice that these little isospin breaking corrections are crucial to describe the structure of atomic matter in the Universe. Indeed, one particular effect of isospin symmetry breaking is the mass splitting between the proton (p) and the neutron (n). This mass difference is known experimentally with an impressive accuracy [1]:

$$\Delta M_N = M_p - M_n = -1.2933322(4) \text{ MeV} \quad (1.1)$$

The sign of this splitting makes the proton, and thus the hydrogen atom, a stable physical state. Also, the size of ΔM_N determines the phase space volume for the neutron β -decay $n \rightarrow p + e^- + \bar{\nu}_e$. At early times of the Universe ($t \sim 1 \text{ s}$ and $T \sim 1 \text{ MeV}$) and under standard assumptions¹, the existence of β -decay allows to infer that the ratio of the number of neutrons and protons is approximately equal to:

$$\frac{n_n}{n_p} \simeq \exp\left(\frac{\Delta M_N}{T}\right) \quad (1.2)$$

This ratio is one important initial condition of Big Bang Nucleosynthesis. Also, in our actual Universe, β -decay and its inverse process are known to be responsible for the generation of a large majority of the stable nuclides chart through nuclear transmutation. Even if the nucleon isospin mass splitting is a well known quantity, predicting it from first principles is still an open problem because of the complex non-perturbative interactions of quarks inside the nucleon. The proton carries an additional EM self-energy compared to the neutron, so just from QED one would expect to have $\Delta M_N > 0$. However, the fact that the experimental value of ΔM_N has the opposite sign suggests that the strong isospin breaking effects are competing against the EM effects with a larger magnitude. This would mean that an important part of the structure of nuclear matter as we know it relies on

	u	d
Mass (MeV) [1]	$2.3 \begin{pmatrix} +0.7 \\ -0.5 \end{pmatrix}$	$4.8 \begin{pmatrix} +0.7 \\ -0.3 \end{pmatrix}$
Charge	$\frac{2}{3}e$	$-\frac{1}{3}e$

Table 1: Physical properties of the up and down quarks.

¹The neutrino number density n_ν/n_γ is assumed to have the order of the baryon density number which is very small. This assumption is not valid anymore in some new physics scenarios but even in these hypothetical cases n_n/n_p depends strongly on ΔM_N .

a subtle cancellation between the small EM and strong breaking effects of isospin symmetry in the nucleon system. Therefore, it is fundamental to have a theoretical understanding of the nucleon isospin mass splitting.

Considering that isospin breaking effects in the hadron mass spectrum are generally measured quite precisely, it is also interesting to understand how one can use this information to deduce the masses of the individual u and d quark masses. For example, it is important to know if $m_u = 0$ could be a realistic solution to the strong CP problem. While recently (*cf.* the FLAG review [2]) considerable progress has been made in determining precisely the average up-down quark mass m_{ud} from first principles, such a computation is still missing for the individual masses. Because the kaon (K) is a pseudo-Goldstone boson of chiral symmetry breaking, the isospin mass splitting $\Delta M_K^2 = M_{K^+}^2 - M_{K^0}^2$ is very sensitive to δm . But in order to extract δm , one has to understand how to subtract the EM contribution to this splitting. One well known result in this direction is Dashen's theorem [3] which states that, in the SU(3) chiral limit, the EM Kaon splitting is equal to the EM pion (π) splitting:

$$\Delta_{\text{QED}} M_K^2 = \Delta_{\text{QED}} M_\pi^2 + \mathcal{O}(\alpha m_s) \quad (1.3)$$

This result is important because it is known that with good accuracy (*cf.* [2]), $\Delta_{\text{QED}} M_\pi^2 \simeq \Delta M_\pi^2$. The remaining question is: how large are the $\mathcal{O}(\alpha m_s)$ corrections in (1.3)? One way to quantify these corrections is to consider the dimensionless quantity ε defined in [2] as follows:

$$\varepsilon = \frac{\Delta_{\text{QED}} M_K^2 - \Delta_{\text{QED}} M_\pi^2}{\Delta M_\pi^2} \quad (1.4)$$

This number is constructed such that it vanishes in the SU(3) chiral limit. There were several attempts in the 1990s to compute these corrections analytically from effective theories, they are illustrated in figure 1. From these results, two conclusions can be made. First, there is a clear disagreement between the different determinations from effective theories which is an important motivation toward an *ab-initio* calculation of these corrections. Second, apart from this controversy it is interesting to notice that most of these results indicate rather large violations of Dashen's theorem. If this is the case, it means that corrections to Dashen's theorem have to be known and taken into account in order to compute the individual light quark masses.

The problems presented in this section, and more generally in any computation of isospin corrections to low-energy QCD observables, are difficult to solve because of the highly non-perturbative behavior of the strong interaction in this regime. It has been shown recently (*cf.* for example [12, 13]) that it is now possible to predict fundamental isospin symmetric QCD observables through lattice QCD simulations with a full control over the method's uncertainties. It is then reasonable to think that lattice simulations could be a reliable way to understand and compute isospin breaking effects. Moreover, besides the physical interest of these effects, actual lattice calculations are reaching a sub-percent precision on several standard observables and the assumption of isospin symmetry is becoming the dominant source of systematic uncertainty.

2. Lattice QCD and QED

In this section we summarise lattice QCD simulation techniques and explain how to include EM interactions in order to compute isospin breaking effects.

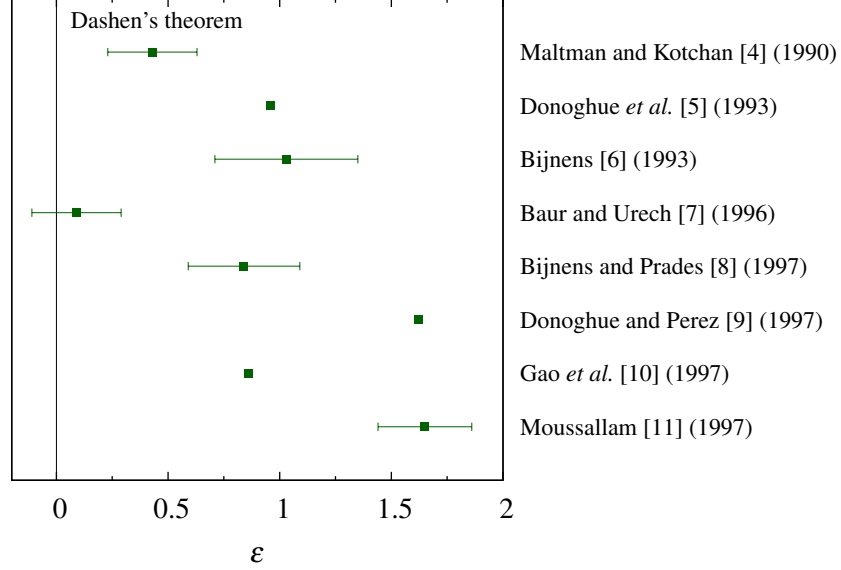


Figure 1: Summary of the analytical calculations of Dashen's theorem violation, ε , defined in (1.4). Results are presented in chronological order.

2.1 Lattice QCD

Let us consider a QCD observable O in Euclidean space-time. Once one integrates the quark fields, the expectation value of O is given by the following functional integral:

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}G_\mu O^{\text{Wick}}[G_\mu, (\not{D} + M)^{-1}] \det(\not{D} + M) e^{-S_{\text{YM}}[G_\mu]} \quad (2.1)$$

where \mathcal{Z} is the partition function, G_μ is the gluon field, O^{Wick} is the gluonic observable obtained from O once the quark fields have been Wick contracted, D_μ is the gauge covariant derivative, M is the quark mass matrix and S_{YM} is the pure gauge Yang-Mills action. A remarkable feature of this integral is that $\frac{1}{\mathcal{Z}} \det(\not{D} + M) e^{-S_{\text{YM}}[G_\mu]} \mathcal{D}G_\mu$ is a probability density. Therefore, if one is able to draw a large number of gluon field according to this law, the expectation value $\langle O \rangle$ can be evaluated numerically through a Monte-Carlo estimation. This idea is the essence of lattice QCD calculations.

To perform such Monte-Carlo calculations, it is of course needed to discretise QCD. To do this, we reduce space-time to a 4-dimensional periodic hypercubic lattice with a lattice spacing a . In order to discretise the gauge action while keeping exact gauge invariance, it is known that one needs to consider the gauge links $U_\mu = e^{iG_\mu}$ as fundamental variables of the theory. Writing an action for those link variables that converge to the Yang-Mills action in the continuum limit $a \rightarrow 0$ does not present any particular difficulties that require comment here. However, discretising the Dirac action is a non-trivial problem. Indeed, a naive discretisation of Dirac action leads to a theory where any fermionic flavour exist with a multiplicity of 16. This is the so-called *doubler problem*. This issue is deeply related to the difficulty of reconstructing chiral symmetry on a discrete space-time, a detailed description of this problem and its possible solutions can be found in [14].

2.2 QED in finite volume

In order to include QED in lattice simulations, one needs to understand how to formulate electrodynamics consistently in a finite volume. Compared to QCD, the main difficulty comes from the absence of a mass gap in the theory: physical QED states can have an energy arbitrary close to 0. This will have two important consequences. First, the propagation of gauge modes is not suppressed by any mass scale: the interaction is long-ranged and important finite volume (FV) effects are expected. Second, momentum quantisation in finite volume will alter the structure of infrared (IR) divergences in the theory.

We begin by discussing IR divergences. Let us consider a diagrammatic contribution \mathcal{D} to a correlation function featuring a photon loop (*e.g.* the 1-loop part of the electron EM self-energy). In infinite volume, \mathcal{D} will have the following form:

$$\mathcal{D} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} f(k, p_1, \dots, p_n) \quad (2.2)$$

where the p_j are the external momenta of the diagram. We assume that we are in the non-trivial situation where the degree of $f(k, p_1, \dots, p_n)$ in k for $k^2 \rightarrow 0$ is strictly less than 2. In this context, the integrand in (2.2) diverges for $k^2 \rightarrow 0$. Nevertheless, depending on the behaviour of f in the vicinity of $k = 0$ and the value of the external momenta, this divergence can be integrable and \mathcal{D} can be finite. It is well known in infinite volume QED that IR divergences do not affect physical observables. In finite volume, momenta are quantised and the integral in (2.2) becomes a sum over discrete momentum space which depends of the choice of boundary condition for the photon field. For periodic boundary conditions, $k = 0$ is part of momentum space² and the FV version of \mathcal{D} is infinite independently of the properties of f in the vicinity of $k = 0$. This shows that a naive formulation of FV QED can easily lead to a divergent theory. Therefore, it is needed to provide an appropriate IR regularisation of the FV theory. Here we present a possible solution which allows to conserve gauge invariance (*i.e.* a massless photon). We chose to use periodic boundary conditions and remove the zero mode of the gauge field from the path integral variables. This will subtract the infinite term in the momentum sum in the FV version of \mathcal{D} . Also, the gauge field is modified on a set of measure 0 so this alteration disappear in the infinite volume limit, where the momentum sum becomes an integral. This choice of regularisation is the one adopted by all the actual lattice QCD and QED projects. A more physical description of this regularisation can be found in [15].

The other difficulty comes from the infinite range of the EM interaction: important FV effects are expected. These effects are generated by photons going around the finite, periodic space-time. Formally, these effects are given by the difference between the momentum integral and sum in contributions of the form (2.2). These effects have been worked out in partially quenched chiral perturbation theory coupled to photons (PQ χ PT+QED) in [16], in the approximation where the time dimension is infinite. In [17], an asymptotic expansion of these results is performed, and it is shown that the leading order FV effects are $O(L^{-1})$ where L is the length of spatial dimensions. However, it has been observed in [18, 19] that this PQ χ PT+QED model does not reproduce well the FV effects observed in the data. Nevertheless, a $O(L^{-1})$ volume dependence is observed in the

²With periodic boundary conditions, momentum is quantised in the form $k_\mu = \frac{2\pi}{L_\mu} n_\mu$ where L_μ is the size of the volume in direction μ and the n_μ are integers.

results of [20] using a wide range of volumes. This confirms the intuition of large FV QED effects. Indeed, the FV effects from QCD are known (*cf.* for example [21]) to decay exponentially with $M_\pi L$ and are quickly dominated by the power-like behaviour of the QED FV effects. To conclude, it seems crucial to use large volumes (above $\sim (6 \text{ fm})^3$) in lattice simulations in order to control completely the infinite volume limit.

2.3 Inclusion of QED in lattice QCD simulations

Up to now, there are essentially two ways of including the EM interactions in lattice calculations. First, similarly to the traditional lattice QCD methods, one can include QED stochastically by generating random EM fields in order to obtain a Monte-Carlo estimation of the path integral. Also, as it was done by the RM123 collaboration in [17], the correlation functions of QCD and QED can be expanded perturbatively in α . By doing this, the QED corrections can be calculated from expectation values of pure QCD operators.

First of all, one needs to formulate QED on a finite, periodic lattice. Up to now, the formulation commonly used is the so-called *non-compact lattice QED*. The gauge action of this theory is just a naive discretisation of Maxwell action:

$$S_{\text{Maxwell}}[A_\mu] = \frac{a^4}{4} \sum_{\mu, \nu, x} [\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)]^2 \quad (2.3)$$

where A_μ is the real EM potential and ∂_μ is the forward finite difference in direction μ :

$$\partial_\mu f(x) = \frac{1}{a} [f(x + a\hat{\mu}) - f(x)] \quad (2.4)$$

where $\hat{\mu}$ is the unitary vector in direction μ . Thanks to the abelianity of the U(1) group, this action remains gauge invariant even on a discrete space-time. Also, accordingly to the discussion in section 2.2, one must use a FV IR regularisation such as the subtraction of A_μ zero mode. Then this EM gauge field can be coupled to the lattice quarks in the following way: one first constructs the U(1) gauge links $U_\mu^{\text{QED}} = e^{ieQA_\mu}$ where Q is the charge matrix in flavour space. Then the links which are used in the discrete version of the gauge covariant Dirac operator are the $U_\mu = U_\mu^{\text{QED}} U_\mu^{\text{QCD}}$, where the U_μ^{QCD} are the lattice QCD link variables.

Let us now discuss how to stochastically simulate QCD and QED. One considerable advantage of pure gauge QED over QCD is that it is a free theory: there is no photon-photon interaction. In other words, the Maxwell action is quadratic and $e^{-S_{\text{Maxwell}}[A_\mu]}$ is just a Gaussian weight. So drawing pure gauge EM fields is a trivial matter of drawing Gaussian random numbers. There is no need to go through a Markov chain simulation as it is usually done in QCD. This simple Gaussian random drawing can then be easily included in any Markovian process used to simulate QCD (generally a variant of the Hybrid Monte-Carlo algorithm). However, the distribution of the QCD and QED fields in the path integral are not independent: they are mixed inside the fermionic determinant. It means that in principle one must generate simultaneously QCD and QED fields. As the generation of QCD configurations represents generally the large majority of the computational cost in lattice simulations, almost every calculations up to now used the *electro-quenched* approximation where one neglects the QED contribution to the determinant. This allows to reuse previously generated QCD configurations. Physically, this approximation is equivalent to assume that sea quarks are

neutral. We will discuss later the precision of this approximation. It is possible to make a full QCD and QED stochastic calculation reusing previously generated QCD configurations by computing *a posteriori* the needed correction to the fermionic determinant: this is the so-called *reweighting* method. This method has been used in [22, 23, 24]. Although interesting, the use of reweighting generally degrades the signal its computational cost increases very quickly with the lattice volume. This happens because of the non-local nature of the determinant. Therefore, if one considers the need to use large volumes to control EM FV effects, reweighting looks like a limited option to include QED sea effects.

As explained in the introduction of this section, a second option to include QED in lattice calculations is to do it in a perturbation theory fashion. If one considers, for example, the lattice 2-point function of some local spin 0 operator O in QCD and QED, it is easy to show that:

$$\langle O(x)O(0)^\dagger \rangle_{\text{QCD+QED}} = \langle O(x)O(0)^\dagger \rangle_{\text{QCD}} + e^2 \sum_{y,z,\mu,\nu} D_{\mu\nu}(y-z) \langle O(x) T[J_\mu(y)J_\nu(z)] O(0)^\dagger \rangle_{\text{QCD}} + \mathcal{O}(e^4) \quad (2.5)$$

where J_μ is the conserved EM current, $D_{\mu\nu}$ is the lattice free photon propagator and the indices on expectation values indicate in which theory the path integral is considered. Of course the definition of J_μ depends on the choice of lattice action for the quarks. With this expansion, the first order EM corrections to O propagator can be computed as a pure QCD 4-point function where one considers all possible insertions of two EM currents. This method is used and presented in detail in [17]. The authors of this paper also show that the $\mathcal{O}(\delta m)$ isospin corrections can be computed with the same kind of perturbative expansion. Also, in this work were neglected the quark disconnected diagrams featuring EM current self-contractions. This type of diagram is known to be difficult and expensive to compute in lattice QCD. This approximation is exactly equivalent to the electro-quenched approximation mentioned before. Technically, the computation of the 4-point function in (2.5) is much more complicated than the direct stochastic estimation, described before, of the QCD and QED 2-point function. Nevertheless, this perturbative expansion is a relevant method to obtain a specific diagrammatic contribution to a correlation function (*e.g.* the computation of hadronic light-by-light contribution to the muon anomalous magnetic moment).

Let us finish this section by discussing the precision of the electro-quenched approximation. In this context, the leading missing contributions are quark loops with one photon emission and an arbitrary number of gluon emissions. These contributions will always have the form $\text{tr}(QS)$ in flavour space where Q is the quark charge matrix and S is the full QCD quark propagator. In the three flavour theory (considering u, d and s flavours), Q is traceless. So if S is proportional to the unit matrix (*i.e.* if $m_u = m_d = m_s$), $\text{tr}(QS)$ vanishes. This means that the sea EM contributions are SU(3) flavour symmetry breaking effects. Also, it is easy to show that these contributions are suppressed by the number of colours $N_c = 3$. Building SU(3) flavour breaking ratios from the hadron spectrum (*i.e.* $(M_\Sigma - M_N)/M_N$) one can observe that SU(3) breaking effect are typically $\mathcal{O}(30\%)$ corrections. Dividing this number by N_c , one can then estimate that the electro-quenched approximation is correct up to $\mathcal{O}(10\%)$ corrections.

2.4 Summary of actual lattice QCD and QED calculations

In table 2 we summarise the parameters of lattice QCD and QED projects. In addition to this table, we should mention that most of these works are based on techniques that were developed for

collaboration references	RBC-UKQCD [32, 18]	MILC [33, 34, 35, 36]	BMWc [15, 19, 20, 37]	PACS-CS [24]	RM123 [38, 17]
fermion action	DW	Asqtad	2-HEX tISW	npSW	tmWil
N_f	2 + 1	2 + 1	2 + 1	1 + 1 + 1	2
QED type	qQED	qQED	qQED	QED	qpQED
$N_{\text{sim.}}$	7	7	39	1	13
$\min(M_\pi)$ (MeV)	250	233	120	135	270
a (fm)	0.11	0.06–0.12	0.05–0.12	0.09	0.05–0.10
N_a	1	3	5	1	4
L (fm)	1.8–2.7	2.4–3.6	1.9–6	2.9	1.6–2.6
$N_{\text{vol.}}$	2	5	17	1	6

Table 2: Summary of the main lattice QCD and QED calculations. The first line is the fermion action used, the abbreviations are from [2, p. 52]. The second line is the number of flavours used in the gauge configuration generation. The third line gives the method used to implement QED: “q” means that the electro-quenched approximation is used and “p” means that the perturbative method described in the previous section is used. The fourth line gives the number of simulation points used. The fifth line indicates the minimum pion mass reached. The sixth line is the range of lattice spacing used and the seventh line indicates their number. Similarly, the eighth and ninth lines are respectively the range and the number of lattice spatial extents used.

the first time in the pioneering work from A. Duncan *et al.* [25, 26]. Also there are some works from NPLQCD [27], UKQCD-QCDSF [28] and P.E. Shanahan *et al.* [29] which consider only strong isospin corrections in lattice QCD. Regarding analytical calculations with effective theories, beside the references cited in figure 1, there is a calculation of the EM mass splitting in the baryon octet in the review [30] and a determination of the EM mass splitting of the nucleon in [31].

3. Dashen’s theorem and light quark masses

In order to compute corrections to Dashen’s theorem, one has to compute the QED contribution to the pion and kaon mass splittings. Let us write down the leading order isospin corrections to the squared mass splitting of a pseudoscalar P :

$$\Delta M_P^2 = M_{P^+}^2 - M_{P^0}^2 = \alpha A_P + \delta m B_P \quad (3.1)$$

where A_P and B_P are two functions of the isospin symmetric parameters of the theory. Then it is tempting to define:

$$\Delta_{\text{QED}} M_P^2 = \alpha A_P \quad \text{and} \quad \Delta_{\text{QCD}} M_P^2 = \delta m B_P \quad (3.2)$$

These definitions are ambiguous because α , m_u and m_d depend on each other through renormalisation. More explicitly, $\Delta_{\text{QED}} M_P^2$ also depends on δm and $\Delta_{\text{QCD}} M_P^2$ does on α . So in principle, one has to specify a scheme (and a scale) to separate QED and QCD isospin breaking effects. Let us estimate the typical size of the associated scheme ambiguity. If we assume that one determined the

QCD splitting of P for $\alpha = 0$, how does this result differ from a determination at the physical value of α ? Each individual u and d quark mass will differ by a relative $O(\alpha)$ contribution, yielding:

$$\Delta_{\text{QCD}}M_P^2(\alpha) = \Delta_{\text{QCD}}M_P^2(0) + O(\alpha\delta m, \alpha m_{ud}) \quad (3.3)$$

As we are only interested in leading order isospin corrections, we can ignore the $O(\alpha\delta m)$ contribution. At the physical quark masses, it is usual and reasonable to consider that $O(m_{ud}) = O(\delta m)$. Including this additional assumption in our power counting, we can consider that $O(\alpha m_{ud})$ corrections are negligible. A similar argument shows that the QED mass splittings ambiguity is also of order $O(\alpha\delta m, \alpha m_{ud})$. To conclude, we showed that the separation of QED and QCD contributions to isospin breaking effects can be obtained unambiguously, at leading order, at the physical quark masses and up to small $O(\alpha m_{ud})$ corrections.

Now that we have clarified how to separate QED and QCD isospin breaking effects, let us review the actual lattice results concerning the corrections to Dashen's theorem. These results are summarised in figure figure 2. If one compares these results to the phenomenological ones presented in figure 1, it is clear that the lattice results are really helping to give a consistent answer to the problem of Dashen's theorem corrections. In 2011, the last FLAG review [2] quoted $\varepsilon = 0.7(5)$ as a global result for Dashen's theorem corrections. Considering the results obtained since 2012, especially [17, 37], lattice calculations reduce significantly this uncertainty. So we can say that lattice QCD and QED are on the way to solve the long-lasting controversy around Dashen's theorem. Nevertheless, ε is computed from QED splittings and all the results up to now have been calculated in the electro-quenched approximation, which, as discussed in section 2.3, implies an $O(10\%)$ uncontrolled uncertainty on QED mass splittings.

In figure 3 we summarise the actual lattice calculation of the light quark mass ratio m_u/m_d . It is important to notice that, even with QED taken into account, this ratio is still independent of the renormalisation scheme up to higher order isospin corrections. If one compares the most recent lattice results and the PDG review average illustrated by the grey band in figure 3, it is clear that the inclusion of isospin breaking effect in lattice calculations greatly improves the determination of this ratio. We are now in a situation where we can state that $m_u \neq 0$ with a significance of the order of 10 standard deviations. This strongly excludes $m_u = 0$ as a possible explanation for the absence of CP violation in the strong interaction.

4. Octet baryon isospin mass splittings

Compared to meson masses, baryon masses are more difficult to compute precisely through lattice computation. Because of this, there are few works on the computation of the isospin breaking corrections to baryon masses. Most of the actual work is focused on the nucleon splitting. These results are summarised the results in figure 4. Since a number of projects only determine the QCD or QED contribution to this splitting, we choose to summarise results for each of these contributions rather than for the full splitting. Even if most of these results have still a small significance, they suggest that the nucleon splitting, whose value is crucial for nuclear matter stability, comes from a subtle cancelation between QED and QCD isospin breaking effects. Concerning the baryon octet, there are two pure QCD studies [28, 29]. Because QED was not taken into account in these two projects, they both had to use imprecise phenomenological inputs related to the light quark mass

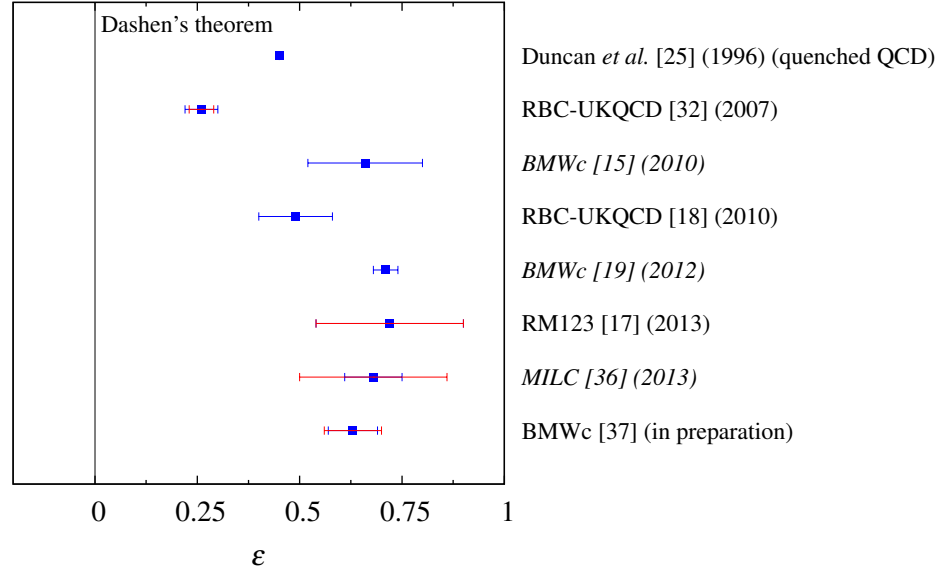


Figure 2: Summary of lattice calculations of Dashen's theorem violation, ϵ , defined in (1.4). Results are presented in chronological order. A blue error bar represents the statistical error and a red one the systematic error. The RBC-UKQCD paper [32] gives two different results corresponding to two cuts in their dataset. We decided to interpret this interval as a systematic error. Most of these papers do not quote explicitly ϵ as defined in (1.4) and the FLAG review [2]. When missing, we computed the value of ϵ only using information from the relevant paper, the PDG review [1] and the FLAG review [2]. The results in italic come from conference proceedings

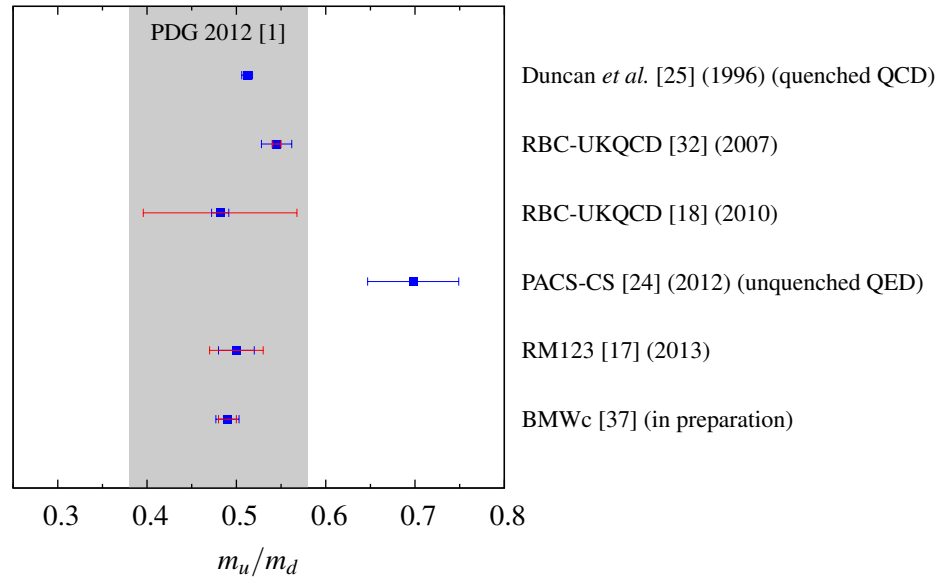


Figure 3: Summary of lattice calculations of the up and down quark mass ratio. Graphical conventions are the same than in figure 2.

ratio. Beside the early work [26], which was performed in quenched QCD, the recent results from BMWc [20] constitute the only complete determination of QCD and QED isospin corrections to the baryon octet. Their results are summarised in figure 5.

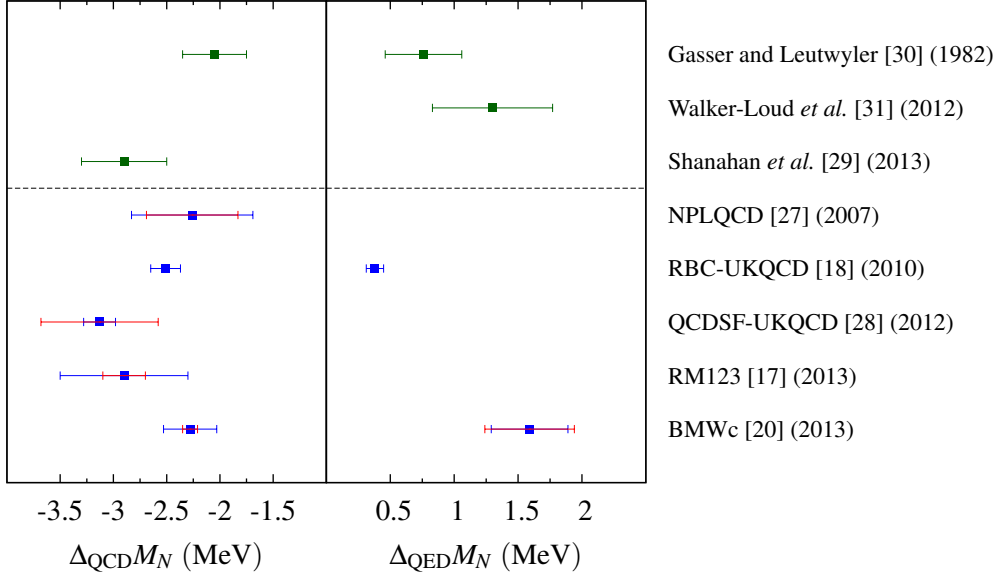


Figure 4: Summary of calculations of the QCD and QED contributions to the nucleon mass splitting $\Delta M_N = M_p - M_n$. As in previous plots, green points are phenomenological results and blue/red points are lattice results.

5. Conclusion and perspectives

The first lattice studies of isospin breaking effects presented in this review show that lattice simulations are encouragingly entering the era where the computation of such small effects is possible.

It is clear that it is now time to go beyond the electro-quenched approximation. This will allow to produce fully controlled high precision predictions for corrections to Dashen's theorem and for individual up and down quark masses. This is also needed in order to compute the nucleon splitting with a high level of significance, which would constitute an *ab-initio* proof of nuclear matter stability.

Also one can think about computing more sophisticated isospin corrections such as, for example, corrections to decay constants and hadronic matrix elements. This would require theoretical work to define properly these quantities in QCD and QED (*cf.* the interesting discussion in [39]). Once this is achieved, lattice calculations will be able to fully simulate the Standard Model in the low energy regime.

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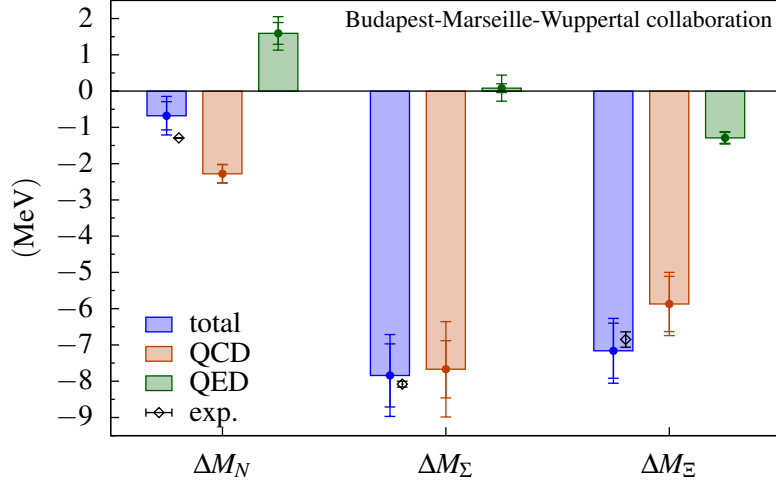


Figure 5: Summary of BMWc results [20] for the isospin mass splittings of the octet baryons. Also shown are the individual QCD and QED contributions to these splittings. The bands indicate the size of the splittings and contributions. On the points, the error bars are the statistical and total uncertainties (statistical and systematic combined in quadrature). For comparison, the experimental values for the total splittings are also displayed as black points.

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