

## Direct test of time-reversal symmetry in the entangled neutral kaon system at a $\phi$ -factory

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A novel method to perform a direct T (time reversal) symmetry test in the neutral kaon system is presented, independent of any CP and/or CPT symmetry tests. This is based on the comparison of suitable transition probabilities, where the required interchange of *in*  $\leftrightarrow$  *out* states for a given process is obtained exploiting the Einstein-Podolsky-Rosen correlations of neutral kaon pairs produced at a  $\phi$ -factory.

The statistical significance of the test achievable with the KLOE-2 experiment at DAΦNE, the Frascati  $\phi$ -factory, is also briefly discussed, together with the possibility to perform a novel CPT test, similarly to the proposed T test.

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## 1. Introduction

In the context of a local quantum field theory with Lorentz invariance and Hermiticity, as in the Standard Model, the validity of the CPT theorem ensures an automatic theoretical connection between any source of CP violation and a corresponding T (time reversal) violation. Even though CPT invariance has been confirmed by all present experimental tests, particularly in the neutral kaon system with stringent limits to possible CPT violation effects [1, 3, 4, 5, 6], the theoretical connection between CP and T symmetries does not imply an experimental identity between them. In fact a direct evidence of T violation should result from an experiment clearly showing the violation, independently from the results of CP violation and without any connection with them.

In case of a transition process, due to the antiunitarity of the operator implementing the symmetry transformation, T invariance requires that the rate for the reaction  $i \rightarrow f$  equals that of the reaction  $f_T \rightarrow i_T$ , with *in* and *out* states exchanged and T *inverted* (for spinless particles this corresponds to the reaction  $f \rightarrow i$ ).

In the past the measurement of a non-zero value of the Kabir asymmetry [7], comparing the rates of the process  $K^0 \rightarrow \bar{K}^0$  and its T conjugated one  $\bar{K}^0 \rightarrow K^0$ , has been presented as a proof for T violation [8, 9]. However, this process has the feature that  $K^0 \rightarrow \bar{K}^0$  is a CPT even transition, so that T and CP transformations are identical in this case, and the corresponding observables are not independent. Therefore it is impossible to separate T violation from CP violation in the Kabir asymmetry.

Another subtle drawback is the one discussed in Ref. [10], where it is pointed out that the T-violation contribution in the Kabir asymmetry is constant in time and directly proportional to  $\Delta\Gamma$ , the width difference of the  $K_{S,L}$  mass eigenstates. Therefore T non-invariance would not be present in the limit  $\Delta\Gamma \rightarrow 0$ , as one might like in a direct genuine test. In the  $B^0 - \bar{B}^0$  system no asymmetry is expected to be found, since in this case  $\Delta\Gamma$  almost vanishes (within the SM).

In order to overcome these difficulties it has been suggested to exploit the Einstein-Podolsky-Rosen (EPR) [11] entanglement of neutral mesons produced at a  $\phi$ -factory (or B-factory) [12, 13, 14]. In fact in this case other transitions than  $K^0 \rightarrow \bar{K}^0$  or  $\bar{K}^0 \rightarrow K^0$  can be observed, independently from CP or CPT test results, thus implementing a *genuine* test of the T symmetry.

Recently, the first direct observation of T violation, in this sense, has been accomplished in the neutral B meson system [15]. In the present paper the methodology to perform a direct test of T symmetry in the neutral kaon system at a  $\phi$ -factory is described<sup>1</sup>, which presents some notable differences with respect to the B meson case [16]. The statistical significance of the test achievable with the KLOE-2 experiment at DAΦNE, the Frascati  $\phi$ -factory [17], is also briefly discussed, together with the possibility to perform a novel CPT test, similarly to the proposed T test (see detailed discussion in Appendix A of Ref. [16], and also Ref.[18]).

## 2. Observables at a $\phi$ -factory

The initial state of the neutral kaon pair produced in  $\phi \rightarrow K^0 \bar{K}^0$  decay can be rewritten in terms

<sup>1</sup>It is based on a work in collaboration with J. Bernabeu and P. Villanueva [16], developing an original idea of J. Bernabeu.

of any pair of orthogonal states  $|K_+\rangle$  and  $|K_-\rangle$ :

$$|i\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle\} = \frac{1}{\sqrt{2}}\{|K_+\rangle|K_-\rangle - |K_-\rangle|K_+\rangle\}. \quad (2.1)$$

Here one can consider the states  $|K_+\rangle$ ,  $|K_-\rangle$  defined as follows:  $|K_+\rangle$  is the state filtered by the decay into  $\pi\pi$  ( $\pi^+\pi^+$  or  $\pi^0\pi^0$ ), a pure CP = +1 state; analogously  $|K_-\rangle$  is the state filtered by the decay into  $3\pi^0$ , a pure CP = -1 state. Their orthogonal states correspond to the states which cannot decay into  $\pi\pi$  or  $3\pi^0$ , defined, respectively, as

$$\begin{aligned} |\tilde{K}_-\rangle &\propto [ |K_L\rangle - \eta_{\pi\pi}|K_S\rangle ] \\ |\tilde{K}_+\rangle &\propto [ |K_S\rangle - (\eta_{3\pi^0}^{-1})|K_L\rangle ], \end{aligned} \quad (2.2)$$

with  $\eta_{\pi\pi} = \langle\pi\pi|T|K_L\rangle/\langle\pi\pi|T|K_S\rangle$  and  $(\eta_{3\pi^0}^{-1}) = \langle 3\pi^0|T|K_S\rangle/\langle 3\pi^0|T|K_L\rangle$ . With these definitions of states, it can be shown that the condition of orthogonality  $\langle K_-|K_+\rangle = 0$ , (i.e.  $|K_+\rangle \equiv |\tilde{K}_+\rangle$  and  $|K_-\rangle \equiv |\tilde{K}_-\rangle$ ) corresponds to assume negligible direct CP violation (or CPT violation in decay) contributions (i.e.  $\varepsilon', \varepsilon'_{000} \ll \varepsilon$ ), assumption quite well satisfied for neutral kaons (see detailed discussion in Appendix A of Ref. [16]). The validity of the  $\Delta S = \Delta Q$  rule is also assumed, so that the two flavor orthogonal eigenstates  $|K^0\rangle$  and  $|\bar{K}^0\rangle$  are identified by the charge of the lepton in semileptonic decays, i.e. a  $|K^0\rangle$  can decay into  $\pi^-\ell^+\nu$  and not into  $\pi^+\ell^-\bar{\nu}$ , and vice-versa for a  $|\bar{K}^0\rangle$ .

Thus, exploiting the perfect anticorrelation of the states implied by Eq. (2.1), it is possible to have a ‘‘flavor-tag’’ or a ‘‘CP-tag’’, i.e. to infer the flavor ( $K^0$  or  $\bar{K}^0$ ) or the CP ( $K_+$  or  $K_-$ ) state of the still alive kaon by observing a specific flavor decay ( $\pi^+\ell^-\nu$  or  $\pi^-\ell^+\bar{\nu}$ ) or CP decay ( $\pi\pi$  or  $3\pi^0$ ) of the other (and first decaying) kaon in the pair.

In this way one can experimentally access other transitions than  $K^0 \rightarrow \bar{K}^0$  or  $\bar{K}^0 \rightarrow K^0$ . These new accessible processes can be divided into four categories of events, as summarized in Table 1.

Reference	T-conjug.	CP-conjug.	CPT-conjug.
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$

**Table 1:** Scheme of possible reference transitions and their associated T, CP or CPT conjugated processes accessible at a  $\phi$ -factory.

For instance, one can consider  $K^0 \rightarrow K_+$  as the reference process, where the initial state  $K^0$  is identified at time  $t_1$  with the flavor tag, and the final state  $K_+$  is identified at a subsequent time  $t_2 \geq t_1$  with a CP decay.

- I) The T transformed process is  $K_+ \rightarrow K^0$ ; any asymmetry in the rate between  $K^0 \rightarrow K_+$  and  $K_+ \rightarrow K^0$  would be a genuine T violating effect.
- II) The CP transformed process is  $\bar{K}^0 \rightarrow K_+$ ; any asymmetry in the rate between  $K^0 \rightarrow K_+$  and  $\bar{K}^0 \rightarrow K_+$  would be a genuine CP violating effect.

III) The CPT transformed process is  $K_+ \rightarrow \bar{K}^0$ ; any asymmetry in the rate between  $K^0 \rightarrow K_+$  and  $K_+ \rightarrow \bar{K}^0$  would be a genuine CPT violating effect.

One may check that the events used for the asymmetries in I), II), and III) are completely independent from each other. Moreover, T violating effects would depend on  $\Delta t = t_2 - t_1$  and would be present even in the limit  $\Delta\Gamma \rightarrow 0$ .

### 3. T symmetry test

For the direct T symmetry test one can define the following ratios of probabilities:

$$\begin{aligned} R_1(\Delta t) &= P [K^0(0) \rightarrow K_+(\Delta t)] / P [K_+(0) \rightarrow K^0(\Delta t)] \\ R_2(\Delta t) &= P [K^0(0) \rightarrow K_-(\Delta t)] / P [K_-(0) \rightarrow K^0(\Delta t)] \\ R_3(\Delta t) &= P [\bar{K}^0(0) \rightarrow K_+(\Delta t)] / P [K_+(0) \rightarrow \bar{K}^0(\Delta t)] \\ R_4(\Delta t) &= P [\bar{K}^0(0) \rightarrow K_-(\Delta t)] / P [K_-(0) \rightarrow \bar{K}^0(\Delta t)] . \end{aligned} \quad (3.1)$$

The measurement of any deviation from the prediction  $R_i(\Delta t) = 1$  imposed by T invariance is a signal of T violation.

At a  $\phi$ -factory the observable quantity is the double differential decay rate of the state  $|i\rangle$  into decay products  $f_1$  and  $f_2$  at proper times  $t_1$  and  $t_2$ , respectively [19]:

$$\begin{aligned} I(f_1, t_1; f_2, t_2) &= C_{12} \{ |\eta_1|^2 e^{-\Gamma_L t_1 - \Gamma_S t_2} + |\eta_2|^2 e^{-\Gamma_S t_1 - \Gamma_L t_2} \\ &\quad - 2|\eta_1||\eta_2| e^{-\frac{(\Gamma_S + \Gamma_L)}{2}(t_1 + t_2)} \cos[\Delta m(t_1 - t_2) + \phi_2 - \phi_1] \} \end{aligned} \quad (3.2)$$

with

$$\eta_i \equiv |\eta_i| e^{i\phi_i} = \frac{\langle f_i | T | K_L \rangle}{\langle f_i | T | K_S \rangle}, \quad (3.3)$$

$$C_{12} = \frac{|\mathcal{N}|^2}{2} |\langle f_1 | T | K_S \rangle \langle f_2 | T | K_S \rangle|^2,$$

and

$$|\mathcal{N}|^2 = [(1 + |\epsilon_S|^2)(1 + |\epsilon_L|^2)] / (1 - \epsilon_S \epsilon_L)^2 \simeq 1$$

a normalization factor.

Let us consider two generic orthogonal basis,  $\{K_X, \bar{K}_X\}$  and  $\{K_Y, \bar{K}_Y\}$  (in our case  $\{K^0, \bar{K}^0\}$  or  $\{K_+, K_-\}$ ), for which the decays  $K_X \rightarrow f_{\bar{X}}$  and  $\bar{K}_Y \rightarrow f_Y$  are forbidden while the decays  $\bar{K}_X \rightarrow f_{\bar{X}}$  and  $K_Y \rightarrow f_Y$  are allowed.

After integration on  $t_1$  at fixed time difference  $\Delta t = t_2 - t_1 > 0$ , the decay intensity (3.2) can be rewritten in a more suitable form, putting in evidence the probabilities we are aiming for. In particular it will be a function of the first decay product  $f_1 = f_{\bar{X}}$  (which takes place at time  $t_1$ , identifies a  $\bar{K}_X$  state, and tags a  $K_X$  state on the opposite side), and the second decay product  $f_2 = f_Y$  (which takes place at time  $t_2$  and identifies a  $K_Y$  state):

$$\begin{aligned} I(f_{\bar{X}}, f_Y; \Delta t) &= \int_0^\infty I(f_{\bar{X}}, t_1; f_Y; t_2) dt_1 \\ &= \frac{1}{\Gamma_S + \Gamma_L} |\langle K_X \bar{K}_X | i \rangle \langle f_{\bar{X}} | T | \bar{K}_X \rangle \langle K_Y | K_X(\Delta t) \rangle \langle f_Y | T | K_Y \rangle|^2 \\ &= C(f_{\bar{X}}, f_Y) \times P [K_X(0) \rightarrow K_Y(\Delta t)] , \end{aligned} \quad (3.4)$$

where the coefficient  $C(f_{\bar{X}}, f_Y)$ , depending only on the final states  $f_{\bar{X}}$  and  $f_Y$ , is given by:

$$C(f_{\bar{X}}, f_Y) = \frac{1}{2(\Gamma_S + \Gamma_L)} |\langle f_{\bar{X}} | T | \bar{K}_X \rangle \langle f_Y | T | K_Y \rangle|^2 . \quad (3.5)$$

As a consequence at a  $\phi$ -factory the experimentally observable quantities corresponding to the four ratios  $R_i(\Delta t)$  are two ratios of decay intensities (3.2):  $R_2^{\text{exp}}(\Delta t)$  and  $R_4^{\text{exp}}(\Delta t)$  (see Fig.1). They are defined and connected to the ratios  $R_i(\Delta t)$  as follows. For  $\Delta t > 0$  one has:

$$R_2^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)} = R_2(\Delta t) \times D , \quad R_4^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)} = R_4(\Delta t) \times D \quad (3.6)$$

while for  $\Delta t < 0$ :

$$R_2^{\text{exp}}(\Delta t) = \frac{D}{R_3(|\Delta t|)} , \quad R_4^{\text{exp}}(\Delta t) = \frac{D}{R_1(|\Delta t|)} . \quad (3.7)$$

Here the normalization constant  $D$ , assuming no CPT violation in semileptonic decays, is  $D = \{\text{BR}(K_L \rightarrow 3\pi^0) \cdot \Gamma_L\} / \{\text{BR}(K_S \rightarrow \pi\pi) \cdot \Gamma_S\}$ .

The KLOE-2 experiment at DAΦNE with an integrated luminosity of  $\mathcal{O}(10 \text{ fb}^{-1})$  [17] could make a statistically significant T test, measuring the ratios  $R_2^{\text{exp}}(\Delta t)$  and  $R_4^{\text{exp}}(\Delta t)$  integrated in the statistically most populated  $\Delta t$  region,  $0 \leq \Delta t \leq 300 \tau_S$  [16]. In this region the uncertainty on the knowledge of the  $(\eta_{3\pi^0}^{-1})$  parameter, which corresponds to the main uncertainty on the validity of the orthogonality condition  $\langle K_- | K_+ \rangle = 0$ , is negligible, as shown in Figs.1,2 (see detailed discussion in Ref.[16]). Moreover in this region  $R_2^{\text{exp}}(\Delta t)$  and  $R_4^{\text{exp}}(\Delta t)$  are expected to be constant (see Fig. 2), and a precise knowledge of the normalization  $D$  is needed in order to detect T violation. This also implies that T violation in this  $\Delta t$  region is constant, proportional to  $\Re\mathcal{E}$ , i.e. directly proportional to  $\Delta\Gamma$ . Therefore, in this  $\Delta t$  region, T violation would not be present in the limit  $\Delta\Gamma \rightarrow 0$ .

#### 4. CPT symmetry test

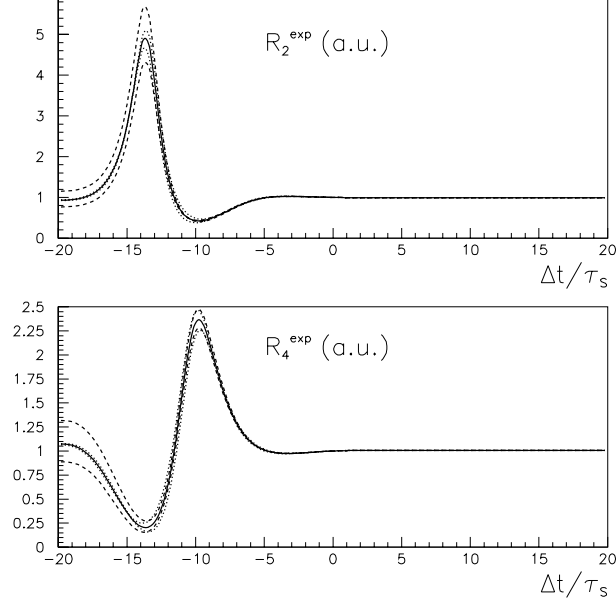
Similarly to the proposed T test, one can define the following ratios of probabilities for a CPT test:

$$\begin{aligned} R_{1,\text{CPT}}(\Delta t) &= P[K^0(0) \rightarrow K_+(\Delta t)] / P[K_+(0) \rightarrow \bar{K}^0(\Delta t)] \\ R_{2,\text{CPT}}(\Delta t) &= P[K^0(0) \rightarrow K_-(\Delta t)] / P[K_-(0) \rightarrow \bar{K}^0(\Delta t)] \\ R_{3,\text{CPT}}(\Delta t) &= P[\bar{K}^0(0) \rightarrow K_+(\Delta t)] / P[K_+(0) \rightarrow K^0(\Delta t)] \\ R_{4,\text{CPT}}(\Delta t) &= P[\bar{K}^0(0) \rightarrow K_-(\Delta t)] / P[K_-(0) \rightarrow K^0(\Delta t)] . \end{aligned} \quad (4.1)$$

The measurement of any deviation from the prediction  $R_{i,\text{CPT}}(\Delta t) = 1$  imposed by CPT invariance is a signal of CPT violation.

At a  $\phi$ -factory the corresponding observable quantities are, for  $\Delta t > 0$ :

$$\begin{aligned} R_{2,\text{CPT}}^{\text{exp}}(\Delta t) &\equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)} = R_{2,\text{CPT}}(\Delta t) \times D_{\text{CPT}} \\ R_{4,\text{CPT}}^{\text{exp}}(\Delta t) &\equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)} = R_{4,\text{CPT}}(\Delta t) \times D_{\text{CPT}} \end{aligned} \quad (4.2)$$



**Figure 1:** The expected ratios  $R_2^{\text{exp}}(\Delta t)$  (top) and  $R_4^{\text{exp}}(\Delta t)$  (bottom) as a function of  $\Delta t$  (solid line); dashed lines correspond to  $\pm 10\%$  uncertainty on modulus and  $\pm 10^\circ$  on phase of  $(\eta_{3\pi^0}^{-1})$ ; the constant  $D$  has been fixed to one for simplicity.

while for  $\Delta t < 0$ :

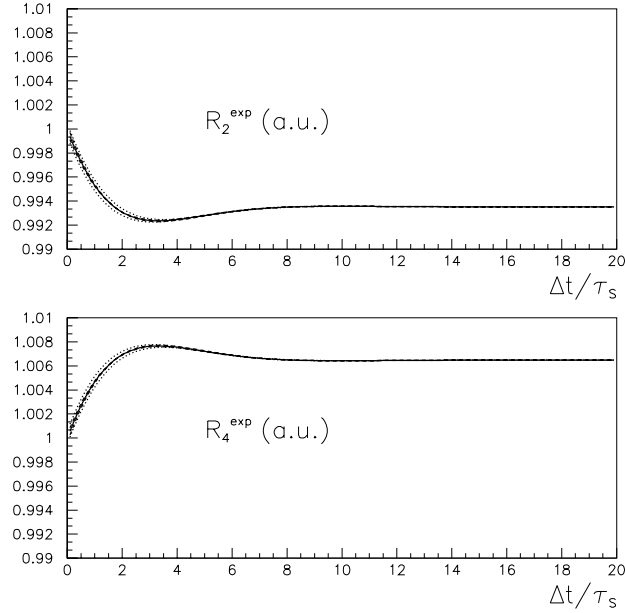
$$R_{2,\text{CPT}}^{\text{exp}}(\Delta t) = \frac{D_{\text{CPT}}}{R_{1,\text{CPT}}(|\Delta t|)}, \quad R_{4,\text{CPT}}^{\text{exp}}(\Delta t) = \frac{D_{\text{CPT}}}{R_{3,\text{CPT}}(|\Delta t|)}. \quad (4.3)$$

Here the normalization constant  $D_{\text{CPT}}$  is  $D_{\text{CPT}} = \{\text{BR}(\text{K}_L \rightarrow 3\pi^0) \cdot \Gamma_L\} / \{\text{BR}(\text{K}_S \rightarrow \pi\pi) \cdot \Gamma_S\}$  without any assumption on CPT violation in semileptonic decays.

The KLOE-2 experiment could make a statistically significant CPT test, measuring the ratios  $R_{2,\text{CPT}}^{\text{exp}}(\Delta t)$  and  $R_{4,\text{CPT}}^{\text{exp}}(\Delta t)$  integrated in the statistically most populated  $\Delta t$  region,  $0 \leq \Delta t \leq 300 \tau_S$  [16, 18]. In this region  $R_{2,\text{CPT}}^{\text{exp}}(\Delta t)$  and  $R_{4,\text{CPT}}^{\text{exp}}(\Delta t)$  are expected to be constant, and in particular one has:

$$R_{2,\text{CPT}}^{\text{exp}}(\Delta t \gg \tau_S) = (1 - 4\Re\delta) \times D_{\text{CPT}} \quad (4.4)$$

$$R_{4,\text{CPT}}^{\text{exp}}(\Delta t \gg \tau_S) = (1 + 4\Re\delta) \times D_{\text{CPT}}, \quad (4.5)$$



**Figure 2:** Same as in Fig. 1 but with zoomed plots in the region  $\Delta t > 0$ .

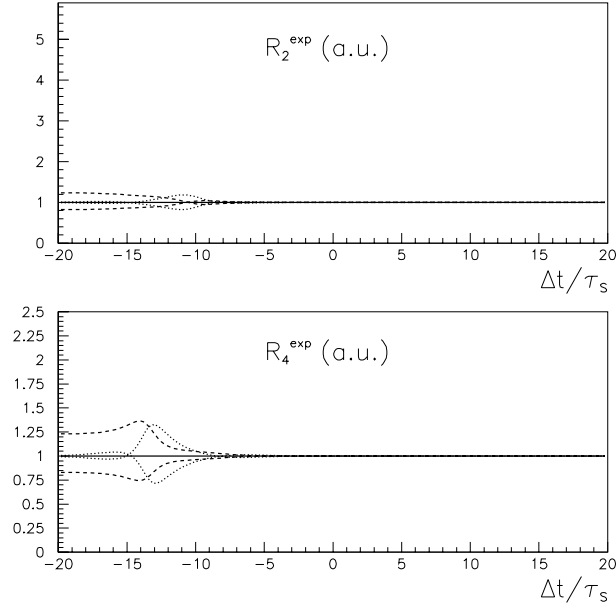
with  $\delta$  the usual CPT violation parameter in the effective Hamiltonian of the neutral kaon system. In this case CPT violation is proportional to  $\Re\delta$ , which do not vanish in the limit  $\Delta\Gamma \rightarrow 0$ , escaping the previous controversy for the T test. Moreover the impact of the uncertainty on the knowledge of  $(\eta_{3\pi^0}^{-1})$  is negligible in this  $\Delta t$  region (see Figs.3, 4), and would not appreciably affect the significance of the CPT test.

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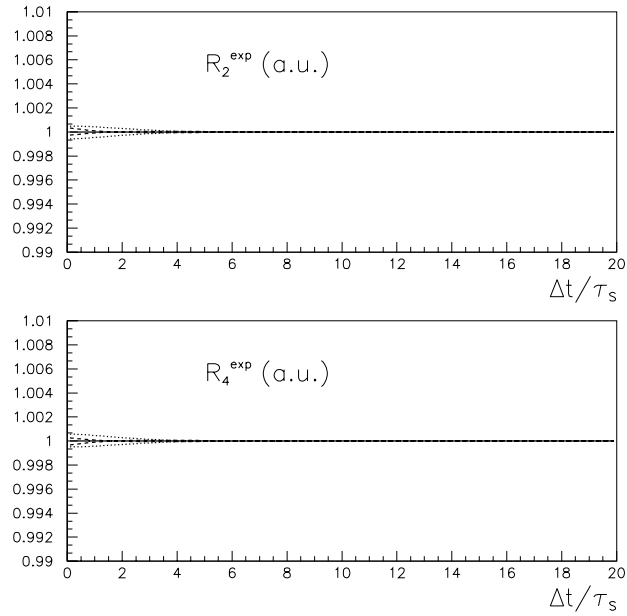
- [1] See “CPT invariance tests in neutral kaon decay“ and “Tests of conservations laws“ reviews in [2].



**Figure 3:** The expected ratios  $R_{2,\text{CPT}}^{\text{exp}}(\Delta t)$  (top) and  $R_{4,\text{CPT}}^{\text{exp}}(\Delta t)$  (bottom) (with the CPT violation parameter  $\delta$  fixed to zero) as a function of  $\Delta t$  (solid line); dashed lines correspond to  $\pm 10\%$  uncertainty on modulus and  $\pm 10^\circ$  on phase of  $(\eta_{3\pi^0}^{-1})$ ; the constant  $D_{\text{CPT}}$  has been fixed to one for simplicity.

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**Figure 4:** Same as in Fig. 3 but with zoomed plots in the region  $\Delta t > 0$ .

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