Model independent determination of $f_0(500)$ and $f_0(980)$ scalar meson parameters

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We propose a theoretical approach based on a model independent pion scalar form factor analysis, which allows to determine masses and widths of $f_0(500)$ and $f_0(980)$ mesons. The procedure leads to $m_{f_0(500)} = (360 \pm 33)\,\mathrm{MeV}$, $\Gamma_{f_0(500)} = (587 \pm 85)\,\mathrm{MeV}$, $m_{f_0(980)} = (957 \pm 72)\,\mathrm{MeV}$ and $\Gamma_{f_0(980)} = (164 \pm 142)\,\mathrm{MeV}$.

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1. Introduction

Scalar mesons $f_0(500)$ and $f_0(980)$ were long time subject to a controversy, appearing, disappearing and reappearing in the PDG tables under different names: ε, σ, η_{0+}, $f_0(400-1200)$, $f_0(600)$, etc. Presently [1] the existence of the two particles seems to be well experimentally established with, however, important uncertainties $m_{f_0(500)} = (400-550)\,\text{MeV}$, $\Gamma_{f_0(500)} = (400-700)\,\text{MeV}$, $m_{f_0(980)} = (990\pm 20)\,\text{MeV}$ and $\Gamma_{f_0(980)} = (40-100)\,\text{MeV}$. A model independent analysis we propose might give a handle on what values are to be expected from the theory side.

2. Pion scalar form factor $\Gamma_\pi(t)$ analysis

Pion scalar form factor $\Gamma_\pi(t)$ is defined through the relation

$$< \pi^i(p_2)|\hat{m}(\bar{u}u+\bar{d}d)|\pi^j(p_1) >= \delta^{ij}\Gamma_\pi(t), \quad t=(p_2-p_1)^2, \quad \hat{m}=(m_u+m_d)/2. \tag{2.1}$$

It is an analytic function in the whole complex t-plane besides the cut on the positive real axis above $t=4m_\pi^2$, it obeys the so-called reality condition $\Gamma_\pi(t^*)=\Gamma_\pi^*(t)$ and its asymptotic behavior is known to be $\Gamma_\pi(t)_{|t|\to\infty}\sim 1/t$. We choose to normalize the form factor to unity $\Gamma_\pi(0)=1$. In the elastic region $4m_\pi^2<t<16m_\pi^2$ form factor respects so-called elastic unitary condition $Im\Gamma_\pi(t)=M_0^0\Gamma_\pi^*(t)$, where M_0^0 stands for $I=J=0$ partial wave $\pi\pi$ scattering amplitude. Its phase representation $M_0^0=e^{i\delta_0^0}\sin\delta_0^0$ leads to $Im\Gamma_\pi=e^{i\delta_0^0}\sin\delta_0^0\Gamma_\pi^*$, which implies $\delta_0^0\equiv\delta_\pi$.

The method we propose is based on dispersion relation derived from Cauchy formula. The dispersion relation has to be considered without subtraction or with one subtraction

$$\Gamma_\pi(t)=\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{Im\Gamma_\pi(t')}{t'-t}dt', \qquad \Gamma_\pi(t)=1+\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{Im\Gamma_\pi(t')}{t'(t'-t)}dt. \tag{2.2}$$

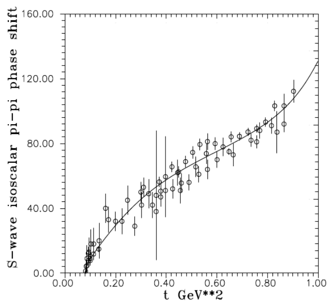
Together with the elastic unitary condition this relations lead to the so-called Omnes-Muskelishvili integral equations. The solution are known:

$$\Gamma_\pi(t)=P_n(t)\exp\left[\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_0^0(t')}{t'-t}dt'\right], \quad \text{or} \quad \Gamma_\pi(t)=P_n(t)\exp\left[\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_0^0(t')}{t'(t'-t)}dt'\right], \tag{2.3}$$

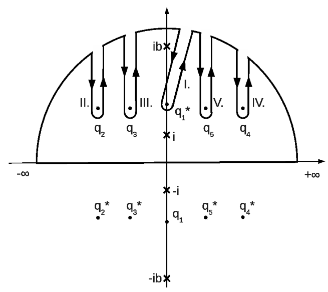
with $P_n(t)$ an arbitrary but normalized polynomial (for $t=0$). If one knows the phase shift δ_0^0 one can predict the behavior of the pion scalar form factor.

We describe the experimental data on δ_0^0 (Fig. 1a) by performing a conformal mapping into the q variable $q=\sqrt{(t-4)/4}$ (assuming $m_\pi=1$). In the q-plane form factor $\Gamma_\pi(t)$ possesses poles and zeros only and therefore an appropriate description can be achieved by a rational function (Padè type approximation) $\Gamma_\pi(t)=[\sum_{n=0}^{M}a_nq^n]/[\Pi_{i=1}^{N}(q-q_i)]$.

Poles q_i of this function lie on the imaginary axis or are positioned in pairs symmetrically with respect to the imaginary axis. From the reality condition it can be shown that the coefficients a_{2i} are real and the coefficients a_{2i+1} are purely imaginary. Taking into account the threshold behavior of δ_0^0, one can rewrite the expression in the form $\delta_0^0(t)=\arctan\frac{A_1q+A_3q^3+A_5q^5+A_7q^7+\ldots}{1+A_2q^2+A_4q^4+A_6q^6+\ldots}$, where A are real coefficients (A_1 actually corresponds to the S-wave iso-scalar $\pi\pi$ scattering length a_0^0). A good fit ($\chi^2/ndf=1.41$) can be achieved with five coefficients only: $A_1=0.2351\pm0.0107$, $A_2=0.2137\pm0.0283$, $A_3=0.2706\pm0.0162$, $A_4=-0.0443\pm0.0048$ and $A_5=-0.0248\pm0.0007$.



(a)



(b)

Figure 1: a) Data points on δ_0^0 and their description. b) Integration contour.

The resulting behavior $lim_{q\to\infty}\delta_0^0(t) = \pi/2$ then implies that the phase representation (2.3) with one subtraction needs to be used.

Insertion of δ_0^0 in an equivalent form of logarithm [$\arctan(z) = \frac{1}{2i}\ln\frac{1+iz}{1-iz}$] leads to the integrand, which is, in addition, a symmetric function. Thus the integral can be stretched over the whole real axis

$$\Gamma_\pi(t) = P_n(t)\exp\left[\frac{q^2+1}{2\pi i}\frac{t}{\pi}\int_{-\infty}^{\infty}\frac{q'\ln\frac{(1+A_2q'^2+A_4q'^4)+i(A_1q'+A_3q'^3+A_5q'^5)}{(1+A_2q'^2+A_4q'^4)-i(A_1q'+A_3q'^3+A_5q'^5)}}{(q'^2+1)(q'^2-q^2)}dq'\right] \tag{2.4}$$

and the theory of residua is used for its calculation. Logarithm generates branch points and so the roots of the polynomial it contains need to be found. Numerical analysis leads to: $q_1 = -1.863i$, $q_2 = -3.583+0.283i$, $q_3 = -1.333+1.280i$, $q_4 = 3.583+0.283i$ and $q_5 = 1.333+1.280i$. Integral (I) can be split and evaluated separately for the upper and lower half-planes $I = I_1 + I_2$:

$$I_1 = \int_{-\infty}^{\infty}\frac{q'\ln\frac{(q'-q_2)(q'-q_3)(q'-q_4)(q'-q_5)}{q'-q_1^*}}{(q'+i)(q'-i)(q'+ib)(q'-ib)}dq',\; I_2 = \int_{-\infty}^{\infty}\frac{q'\ln\frac{q'-q_1}{(q'-q_2^*)(q'-q_3^*)(q'-q_4^*)(q'-q_5^*)}}{(q'+i)(q'-i)(q'+ib)(q'-ib)}dq'. \tag{2.5}$$

The contour related to the first integral is closed around the upper half-plane (Fig. 1b), for the second integral, it is closed around the lower half-plane. In order to evaluate the first integral I_1, we have to evaluate contributions of cuts generated by branch points q_i and to calculate residua in the poles (i, ib). The final result can be formally written $I_1 = 2\pi i\sum_n Res_n - [-\int_{1^*} + \int_2 + \int_3 + \int_4 + \int_5]$ (similarly for I_2).

3. Results

We arrive to the result $\Gamma_\pi(t) = P_n(t)\frac{q-q_1}{(q+q_2)(q+q_3)(q+q_4)(q+q_5)}\frac{(i+q_2)(i+q_3)(i+q_4)(i+q_5)}{i-q_1}$ by inserting the previously evaluated integral into the expression for the pion scalar form factor.

We get our final results by identifying the $-q_3$ and $-q_2$ poles of this expression as the scalar mesons $f_0(500)$ and $f_0(980)$ with parameters [in MeV]:

$$m_{f_0(500)} = 360\pm 33,\ \Gamma_{f_0(500)} = 587\pm 85;\quad m_{f_0(980)} = 957\pm 72,\ \Gamma_{f_0(980)} = 164\pm 142. \tag{3.1}$$

References

[1] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86** (2012) 010001.