Duality invariance for black holes in N=2 gauged Supergravity

Alessandra Gnecchi[*][†]

ITF & Spinoza Institute Utrecht University

E-mail: A.Gnecchi@uu.nl

Electric-magnetic duality invariance of extended Supergravity theories has been a very powerful tool in the study of extremal black holes. A minimal extension is presented to black holes in $N = 2$, $U(1)$-gauged Supergravity theory, where duality covariance is preserved. These solutions are particularly interesting as they exist also in asymptotically AdS_4 space. Their peculiar features are discussed, in particular the supersymmetry preserved by the scalar flow and the appearance of a charged quantization condition between the black hole and gravitini charges.

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[*]Speaker.

[†]Based on work in collaboration with Gianguido Dall'Agata.

1. Introduction

Supersymmetric black holes in gauged Supergravities have been under recent investigation that successfully lead to the construction of static supersymmetric black holes solutions in the so-called $N = 2$ *Fayet-Iliopoulos* gauged theory.

Black holes in String theory and Supergravity represent a rich laboratory in which to test properties of quantum gravity by pushing the semiclassical limits to the edge of their validity. The work of Strominger and Vafa in 1996 [1], for example, showed that the black hole entropy, computed by counting the String theory microscopic constituents, agreed with the semiclassical Bekenstein Hawking entropy, proving how String theory has the chance to account for black holes quantum degrees of freedom. Supergravity BPS (i.e. supersymmetry-preserving) states have revealed to undergo phenomena like wall crossing, and have been useful in understanding the properties of BPS spectrum and moduli space shared by both local and rigid Supersymmetric theories.

However, these results are valid for black holes solutions in asymptotically flat (Minkowski) spacetime. Given the fast progresses made in applications of gauge/gravity duality, it has become of primary importance to direct the attention to black holes in anti de Sitter space, that are one of the main tools for holography studies. In order to construct solutions that asymptote to curved spacetime one has to consider *gauged* Supergravity theories, in which, due to the gauging, a scalar potential appears that mimics a cosmological constant. Many works on black holes in gauged Supergravity are already present in the literature, including results obtained by different groups in the late 90's [2] -[8]. In those works, however, no static BPS solution with regular spherical horizon had been constructed, while it was found that all $\frac{1}{2}$-BPS static black holes with spherical horizon geometry must have a vanishing horizon. In 2009 Cacciatori and Klemm [9] proved that black hole states in $N = 2\ U(1)$-gauged SUGRA with regular $AdS_2 \times S^2$ horizon geometry could exists, if they preserved less supersymmetries, as was shown explicitly by solving for the Killing spinors [10, 11]. They are $\frac{1}{4}$-BPS static geometries that interpolate between a near horizon $AdS_2 \times S^2$ region and an asymptotic "locally" AdS_4 spacetime[1]. We will restrict our attention to black holes with spherical horizon, unless otherwise stated.

As already pointed out, these newly constructed black holes solutions are interesting for applications to various areas: in the context of AdS/CMT correspondence, the existence of solutions with a regular BPS limit opens the investigation of new holographic scenarios. Moreover, due to their stable, solitonic nature, BPS black holes in asymptotically locally AdS_4 spacetime may play a role in the destabilization of vacua in the context of String Theory Landscape [12].

2. Extremal black holes in Supergravity

Let us start by giving an introduction to black holes in Supergravity theories. We first construct black holes in asymptotically flat spacetime, as solutions of *ungauged* Supergravity, and we show further on what modifications are needed to deal with solutions in asymptotically curved space. These additional ingredients will be provided by gauging some isometries of the scalar manifold.

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[1]Notice that, instead, the black hole solutions of [5] have asymptotically *global AdS$_4$* geometries.

2.1 Electric-magnetic dualtiy invariance

The field content of extended Supergravities include in general several abelian vector fields, responsible for the charge of the black hole. They can be graviphotons but also vectors of the matter multiplets[2]. A fundamental property of these theories is their invariance under electric-magnetic duality. This is a generalization of the electric-magnetic duality of the Maxwell field, in which a rotation of the $U(1)$ field strength

$$F'^{\mu\nu} = (\cos\alpha + \sin\alpha *) F^{\mu\nu}, \qquad \alpha \in \mathbb{R}, \tag{2.1}$$

would affect the Lagrangian in such a way that the equations of motions for gravity coupled to electromagnetism

$$\partial_\mu F^{\mu\nu} = 0,$$
$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = -8\pi G T_{\mu\nu} \tag{2.2}$$

remain invariant.

In 1981 Gaillard and Zumino [13] showed that, for a theory of n abelian vector fields F^a, $a = 1,..,n$, coupled to scalar or fermionic fields χ^i, and whose Lagrangian only depends on the abelian field strengths, the scalars and fermions and their derivatives, namely

$$\mathcal{L} = \mathcal{L}(F^a, \chi^i, \partial_\mu \chi^i), \tag{2.3}$$

it is possible to define a field strength dual to F^a, as

$$\widetilde{G}^a_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}G^{a\rho\sigma} \equiv 2\frac{\partial L}{\partial F^{a\mu\nu}}. \tag{2.4}$$

In this way one constructs a $2n$-vector of field strengths that transform linearly under the action of duality transformations, while the other bosonic or fermionic fields transform nonlinearly

$$\delta \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}, \tag{2.5}$$

$$\delta\chi^i = \xi^i(\chi),$$

$$\delta(\partial_\mu\chi^i) = \partial_\mu\xi^i = \partial_\mu\chi^j\frac{\partial\xi^i}{\partial\chi^j}. \tag{2.6}$$

The most general matrix acting infinitesimally as a duality transformation on the vector (F, G) is a symplectic transformation

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in sp(2n, \mathbb{R}). \tag{2.7}$$

The field strengths F^a and their generalized dual G_a thus form a symplectic vector

$$\mathbf{V} \equiv \begin{pmatrix} F^a \\ G_a \end{pmatrix},$$

and the duality transformations are implemented by a matrix $S \in Sp(2n, \mathbb{R})$, that acts on the field strengths as

$$\mathbf{V}' = S\mathbf{V}. \tag{2.8}$$

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[2]Matter coupling is restricted to $\mathcal{N} \leq 4$ Supergravity theories.

2.2 Black holes in extended Supergravity

Supersymmetric black holes carry quantized charges given by the fluxes of the abelian field strengths on a sphere at infinity

$$p^\Lambda = \frac{1}{4\pi}\int_{S^2} F^\Lambda\,, \qquad q_\Lambda = \frac{1}{4\pi}\int_{S^2} G_\Lambda\,, \qquad Q = \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}\,. \tag{2.9}$$

They are solution of the equations of motion obtained from the bosonic sector of $\mathcal{N} \geq 2$ Supergravity theories

$$S = \int \sqrt{-g}\,d^4x \left(-\frac{1}{2}R + \mathrm{Im}\mathcal{N}_{\Lambda\Gamma}F^\Lambda_{\mu\nu}F^{\Gamma,\,\mu\nu} + \frac{1}{2\sqrt{-g}}\mathrm{Re}\mathcal{N}_{\Lambda\Gamma}\varepsilon^{\mu\nu\rho\sigma}F^\Lambda_{\mu\nu}F^\Gamma_{\rho\sigma} + \right.$$
$$\left. +\frac{1}{2}g_{rs}(\phi)\partial_\mu\phi^r\partial^\mu\phi^s\right)\,, \tag{2.10}$$

which includes the Einstein-Hilbert term, the kinetic and axionic term of the vector fields, and the kinetic term of the scalar fields ϕ^x describing a nonlinear sigma model. The fermionic equations of motion, indeed, decouple from the bosonic ones, and we can restrict to solutions on a zero fermions background. The scalars couple to the vector fields through the matrix $\mathcal{N}_{\Lambda\Sigma}(\phi)$. It is important to notice that, in $\mathcal{N} = 2$ Supergravity, scalar fields are present both in the vector multiplets as in the hypermultiplets. However, the symplectic matrix $\mathcal{N}_{\Lambda\Sigma}$ only depends on the scalars of the vector multiplets, so when dealing with black holes in $\mathcal{N} = 2$ theories we can consistently set the hypermultiplets to zero.

In the above action there is no term that provide a potential responsible for the curvature of spacetime at infinity. This means that any solution we can derive from the extended *ungauged* Supergravity theory will asymptote to flat spacetime.

In particular, the action (2.10), under the assumptions of staticity and spherical symmetry, can be dimensionally reduced upon the metric ansatz

$$ds^2 = -e^{2U(\tau)}dt^2 + e^{-2U(\tau)}\left[\frac{c^4 d\tau^2}{\sinh^4(c\tau)} + \frac{c^2}{\sinh^2(c\tau)}(d\theta^2 + \sin\theta^2 d\phi^2)\right]\,, \tag{2.11}$$

that captures the solution of a general (extremal and non-extremal) black hole. In this way, the system reduces to a 1-dimensional one, described by the Lagrangian

$$\mathscr{L} = \left(\frac{dU}{d\tau}\right)^2 + G_{ab}\frac{d\phi^a}{d\tau}\frac{d\phi^b}{d\tau} + e^{2U}V_{BH} - c^2\,, \qquad c^2 = 4S^2T^2\,. \tag{2.12}$$

The warp factor $U(r)$ appear as an additional scalar field, and the dynamics of the 1d system is governed by the effective black hole potential $V_{BH}(\phi^a, p^\Lambda, q_\Lambda)$. This can be written in a symplectic covariant way as

$$V_{BH} = -\frac{1}{2}Q^T \mathscr{M}(\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}, \mathrm{Im}\mathcal{N}_{\Lambda\Sigma})Q\,, \tag{2.13}$$

where the $2n \times 2n$ matrix $\mathscr{M}$ depends only on the scalars through the symplectic matrix $\mathcal{N}_{\Lambda\Sigma}(\phi^a)$. The one-dimensional Lagrangian has to be supplemented by a Hamiltonian constraint

$$\left(\frac{dU}{d\tau}\right)^2 + G_{ab}\frac{d\phi^a}{d\tau}\frac{d\phi^b}{d\tau} - e^{2U}V_{BH} - c^2 = 0\,. \tag{2.14}$$

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The extremal solution has zero temperature, and thus $c = 0$. In this case the near horizon geometry is $AdS_2 \times S^2$ and because of the presence of an infinite throat, the AdS_2 space, the horizon r_{hor} becomes an *attractor* point for the scalars, forcing them to lose memory of their values at infinity [14, 15, 16]. The scalar flow has a critical point at $r = r_{hor}$ and this translates into a criticality condition on the black hole potential in moduli space

$$\left(\frac{\partial V_{BH}}{\partial \phi^a} \right)_{r_{hor}} = 0 \ . \tag{2.15}$$

By the definition (2.13), this corresponds to a set of algebraic equations allowing to fix the scalars at the horizon in terms of electric and magnetic charges. In this way, in particular, the entropy of the black hole only depends on the quantize electric and magnetic charges.

3. Gauged $\mathcal{N} = 2$ Supergravity

The scalar manifold of the $\mathcal{N} = 2$ Supergravity is a product of a Special Kähler manifold and a Quaternionic Kähler manifold

$$\mathcal{SM} \times \mathcal{QM} \ , \tag{3.1}$$

parametrized respectively by complex scalars of the vector multiplets z^i and the quaternionic scalars in the hypermultiplets q^u. In general, let g be the Kähler metric of a Kähler manifold $\mathcal{M}$; if g has a a non trivial group of continuous isometries $G \in \mathcal{M}$ generated by Killing vectors, then the kinetic Lagrangian admits G as a group of global space-time symmetries. The appearance of local isometries is described by a procedure called *momentum map*. We are going to present, in this section, how the gauging affects the theory and the search for black holes, and we refer for a detailed review to [18].

With respect to the ungauged theory, the connections of the relevant bundles are modified as:

$$\begin{array}{llll}
\mathcal{T}SM & : \text{ tangent bundle } & \Gamma^i{}_j \to & \widehat{\Gamma}^i{}_j = \Gamma^i{}_j + gA^\Lambda \partial_j k^i_\Lambda \\
\mathcal{L} & : \text{ line bundle } & \mathcal{Q} \to & \widehat{\mathcal{Q}} = \mathcal{Q} + gA^\Lambda \mathcal{P}^0_\Lambda \\
\mathcal{SU} & : SU(2) \text{ bundle } & \omega^x \to & \widehat{\omega}^x = \omega^x + gA^\Lambda \mathcal{P}^x_\Lambda \\
\mathcal{SU}^{-1} \otimes \mathcal{T}HM & : Sp(2m) \text{ bundle } & \Delta^{\alpha\beta} \to & \widehat{\Delta}^{\alpha\beta} = \Delta^{\alpha\beta} + gA^\Lambda \partial_u k^v_\Lambda \mathcal{U}^{u|\alpha A} \mathcal{U}^\beta_{v|A}
\end{array} \tag{3.2}$$

thus, in presence of local isometries, the fields are charged under combinations of the gauge fields defined by the function $\mathcal{P}^0_\Lambda(z^i)$ and the $SU(2)$ vector $\mathcal{P}^x_\Lambda(q^u)$. These $\mathcal{P}$'s functions are called respectively holomorphic and triholomorphic *momentum maps* and define the Killing vectors associated to the gauged isometries.

Apart from the modification of gauge connections, in order for the full Lagrangian to be invariant under supersymmetry variations, a scalar potential and fermionic mass terms have to be added. Let us comment on how the new terms will affect the solutions, keeping in mind we are looking for charged, possibly supersymmetric black holes in a zero fermion background.

- Scalars are charged

$$\partial_\mu z^i \to \nabla_\mu z^i \ , \qquad \partial_\mu q^u \to \nabla_\mu q^u \ . \tag{3.3}$$

Simple derivatives are replaced with covariant derivatives, according to (3.2). The vector fields now appear in the Langrangian whose field dependence becomes $\mathcal{L}(F^a, A^a, \chi^i, \partial_\mu \chi^i)$, meaning that the hypothesis of Gaillard and Zumino are no more satisfied: **duality invariance is broken**[3].

- A scalar potential appears in the Lagrangian, whose general form is

$$V_g = (g_{ij^\star} k^i_\Lambda k^{j^\star}_\Sigma + 4 h_{uv} k^u_\Lambda k^v_\Sigma)\bar{L}^\Lambda L^\Sigma + (U^{\Lambda\Sigma} - 3\bar{L}^\Lambda L^\Sigma)\mathcal{P}^x_\Lambda \mathcal{P}^x_\Sigma \ . \tag{3.4}$$

Two consequences arise

1. This term couples the dynamics of the scalar fields of the vector multiplets, parametrized by the covariantly holomorphic sections $\mathcal{V} = (L^\Lambda, M_\Lambda)$, to the scalars of the hypermultiplets, q^u, so that **we cannot neglect hypermultiplets anymore when looking for black hole geometries**.

2. This potential can be nonvanishing at asymptotic infinity, so that a geometry that is a solution of the gauged theory will asymptote in general to a curved space, according to the particular value of the scalar fields at infinity. Depending on the model under consideration and the choice of gauged isometries, **it will be possible to find asymptotic AdS_4 geometries**.

- For a generic gauging, the $U(1)^n$ fields A^Λ_μ will be promoted to non abelian gauge fields of the group G, depending on the choice of the momentum maps. In particular, $\mathcal{P}^0_\Lambda$, which defines the gauging of the isometries of the Special Kähler manifold, has an explicit expression in terms of the symplectic sections

$$\mathcal{P}^0_\Lambda = e^{\mathcal{K}/2} \left(M_\Delta f^\Delta_{\Lambda\Sigma} \bar{L}^\Sigma + \overline{M}_\Delta f^\Delta_{\Lambda\Sigma} L^\Sigma \right)$$

with $f^\Delta_{\Lambda\Sigma}$ being the structure constants of G. It is clear, then, that a nonzero $\mathcal{P}^0_\Lambda$ would correspond to non abelian gaugings, and **the black hole would acquire non abelian charge**.

If we are looking for generalization of black holes solutions from the ungauged to gauged theory, it is convenient to consider a minimal departure from the setup of the ungauged $\mathcal{N} = 2$ Supergravity: scalars coupled to hypermultiplets, non abelian BPS black hole states and loss of duality invariance cannot be handled in full generalization, yet. A way out exists, that allows us to keep a nonvanishing potential in the Lagrangian (needed to study black holes in curved asymptotic geometries), while leaving the scalars neutral and the gauge fields abelian: the so-called *Fayet-Iliopoulos gauging*.

3.1 Fayet-Iliopoulos gauging

Consider the momentum maps

$$\mathcal{P}^0_\Lambda = 0 \ , \qquad \mathcal{P}^x_\Lambda = \xi^x_\Lambda \ , \tag{3.5}$$

---

[3] Broken duality invariance in gauged Supergravities means that different choices of gauging may correspond to physically inequivalent theories. This has been studied recently in the work [17], in the context of gauged $\mathcal{N} = 8$ Supergravity, in which a classification of duality orbits for different gaugings has been discussed.

where the ξ_Λ's are constant parameters. Without loss of generality, it is possible to perform an $SU(2)$ rotation on the $\mathcal{P}_\Lambda^x$ vector to align the momentum map in one direction, that we can take $x = 2$. This fixes the gauging parameters as $\xi_\Lambda^2 \equiv \xi_\Lambda$. The choice (3.5) corresponds to an abelian gauging of the isometries of the quaternionic scalar manifold $\mathfrak{QM}$, and precisely to the *R*-symmetry subgroup

$$U(1)_R \in SU(2)_R \subset \mathfrak{QM} . \tag{3.6}$$

The Killing vectors satisfy the following relations [18]

$$\begin{aligned} k_\Lambda^i &= ig^{i\bar{j}}\partial_{\bar{j}}\mathcal{P}_\Lambda , \\ K_{uv}^x k_\Lambda^u k_\Sigma^v &= \frac{1}{2}f^\Delta_{\Lambda\Sigma}\mathcal{P}_\Delta^x , \end{aligned} \tag{3.7}$$

and the Fayet-Iliopoulos gauging does not generate any Killing vector. The only contribution appearing in the scalar potential (3.4) is then

$$V_{FI} = (U^{\Lambda\Sigma} - 3\bar{L}^\Lambda L^\Sigma)\xi_\Lambda \xi_\Sigma , \tag{3.8}$$

which does not introduce any mixing between the scalars of the vectors and those of the hypermultiplets. From (3.2), with this choice of gauging, the only fields that acquires a charge are the sections of the $SU(2)$ *R*-symmetry bundle, in particular gravitini. The black hole is no more the only state of the theory carrying conserved charges, and a Dirac quantization condition has to be imposed, among the black hole and the gravitini[4].

Finally, the action for the bosonic sector of $\mathcal{N} = 2$ $U(1)$-gauged Supergravity is

$$S = \int d^4x \left(-\frac{R}{2} + g_{i\bar{j}}\partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} + \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F_{\mu\nu}^\Lambda F^{\Lambda\mu\nu} + \frac{1}{2\sqrt{-g}}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^\Lambda F_{\rho\sigma}^\Sigma - V_g \right) , \tag{3.9}$$

where the scalar potential

$$V_g = g^2 V_{FI} \tag{3.10}$$

is the only modification generated by the Fayet-Iliopoulos gauging.

4. Duality covariant derivation of BPS flow

It is possible to rewrite the scalar potential in a symplectic covariant form. Let us define from now on the gravitini charges as $g\xi_\Lambda \equiv g_\Lambda$. If we introduce a set of *magnetic* charges g^Λ, and a symplectic vector

$$\mathcal{G} = \begin{pmatrix} g^\Lambda \\ g_\Lambda \end{pmatrix} , \tag{4.1}$$

we can extend the form of the gauging potential to

$$V_g(\mathcal{G}, \mathcal{V}) = -3|\mathcal{L}|^2 + |D_i\mathcal{L}|^2 . \tag{4.2}$$

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[4]For BPS solutions this condition is provided by supersymmetry, while for extremal non-BPS ones this has to be imposed by hand, further constraining the parameters of the black hole solution [10, 11, 19]

This is a symplectic covariant potential that reduces to (3.10) when $g^\Lambda \equiv 0$. The scalar quantity $\mathcal{L} = \langle \mathcal{G}, \mathcal{V} \rangle$[5] is a section of the line bundle of the scalar manifold, defined analogously to the central charge $\mathcal{Z} = \langle Q, \mathcal{V} \rangle$.

We are interested in black hole geometries that are solutions of the equations of motion derived from (3.9), static and spherically symmetric. The metric ansatz that capture solutions in asymptotically locally Anti de Sitter spacetime is

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}\left[dr^2 + e^{2\psi(r)}(d\theta^2 + \sin\theta^2 d\phi^2)\right] , \tag{4.3}$$

which is however valid only for the extremal case. This ansatz allows to reduce the action (3.9) to the one dimensional system

$$S_{1d} = \int dr \left\{ e^{2\psi} \left[U'^2 - \psi'^2 + g_{i\bar{j}}z^{i\prime}\bar{z}^{\bar{j}\prime} + e^{2U-4\psi}V_{BH} + e^{-2U}V_g \right] - 1 \right\} + \tag{4.4}$$
$$+ \int dr \frac{d}{dr}\left[e^{2\psi}(2\psi' - U') \right] ,$$

that yields the second order equations in terms of the τ parameter $e^\psi \partial_\tau \equiv \partial_r$:

$$-\frac{d^2}{d\tau^2}U(\tau) = e^{2U}V_{BH}(p^\Lambda, q_\Lambda, z, \bar{z}) + e^{4\psi-2U}V_g(z, \bar{z})$$
$$\frac{d^2}{d\tau^2}\psi(\tau) = e^{2\psi} - 2e^{4\psi-2U}V_g(z, \bar{z})$$
$$e^{-U}\frac{d^2}{d\tau^2}e^{U(\tau)} - e^{-\psi}\frac{d^2}{d\tau^2}e^{\psi(\tau)} = g_{i\bar{j}}\partial_\tau z^i \partial_\tau \bar{z}^{\bar{j}} . \tag{4.5}$$

Supersymmetric solutions, however, solve first order equations corresponding to the requirement that the supersymmetry variation of the fermionic fields remains zero

$$\delta_\varepsilon \psi_\mu^A = 0 , \qquad \delta_\varepsilon \lambda^{iA} = 0 . \tag{4.6}$$

Looking for a BPS solution, it is convenient to re-write the action (4.4) using a *BPS*-trick, i.e. as a sum of squares. This is possible if we introduce an additional phase α, as

$$S_{1d} = \int dr \left\{ -\frac{1}{2}e^{2(U-\psi)}\mathcal{E}^T \mathcal{M}\mathcal{E} - e^{2\psi}\left[(\alpha' + \mathcal{A}_r) + 2e^{-U}\mathrm{Re}(e^{-i\alpha}\mathcal{L}) \right]^2 \right.$$
$$-e^{2\psi}\left[\psi' - 2e^{-U}\mathrm{Im}(e^{-i\alpha}\mathcal{L}) \right]^2 - (1 + \langle \mathcal{G}, Q \rangle) \tag{4.7}$$
$$\left. -2\frac{d}{dr}\left[e^{2\psi-U}\mathrm{Im}(e^{-i\alpha}\mathcal{L}) + e^U \mathrm{Re}(e^{-i\alpha}\mathcal{Z}) \right] \right\} ,$$

where $\mathcal{M}(\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}\mathrm{Im}\mathcal{N}_{\Lambda\Sigma})$ is the matrix defining the black hole potential in (2.13), and $\mathcal{E}$ is the symplectic vector

$$\mathcal{E}^T \equiv 2e^{2\psi}\left(e^{-U}\mathrm{Im}(e^{-i\alpha}\mathcal{V}) \right)^{\prime\,T} - e^{2(\psi-U)}\mathcal{G}^T \Omega \mathcal{M}^{-1} + 4e^{-U}(\alpha' + \mathcal{A}_r)\mathrm{Re}(e^{-i\alpha}\mathcal{V})^T + Q^T . \tag{4.8}$$

---

[5]We define symplectic product of two vectors $\mathcal{A} = (a^\Lambda, a_\Lambda)$ and $\mathcal{B} = (b^\Lambda, b_\Lambda)$ as $\langle \mathcal{A}, \mathcal{B} \rangle = \mathcal{A}^T \Omega \mathcal{B} = a_\Lambda b^\Lambda - a^\Lambda b_\Lambda$, being $\Omega = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

Since the action is a sum of squares, the requirement that each of the squared quantity vanishes ensures that the variation of the action is zero on shell. One can check that these equations are equivalent to those obtained by Supersymmetry variations of the fermions, and they are, precisely

$$
\begin{aligned}
U' &= -e^{U-2\psi}\mathrm{Re}(e^{-i\alpha}\mathcal{Z}) + e^{-U}\mathrm{Im}(e^{-i\alpha}\mathcal{L}) \\
\psi' &= 2e^{-U}\mathrm{Im}(e^{-i\alpha}\mathcal{L}) \\
\dot{z}^i &= -e^{i\alpha}g^{i\bar{j}}(e^{U-2\psi}\overline{D}_{\bar{j}}\overline{\mathcal{Z}} + ie^{-U}\overline{D}_{\bar{j}}\overline{\mathcal{L}}) \\
\alpha' + \mathcal{A}_r &= -2e^{-U}\mathrm{Re}(e^{-i\alpha}\mathcal{L}) \ .
\end{aligned}
\tag{4.9}
$$

They are supplemented by the constraint

$$
e^{2U-2\psi}\mathrm{Im}(e^{-i\alpha}\mathcal{Z}) = \mathrm{Re}(e^{-i\alpha}\mathcal{L}) \ .
\tag{4.10}
$$

that identifies the phase as

$$
e^{2i\alpha} = \frac{\mathcal{Z} - ie^{2(\psi-U)}\mathcal{L}}{\overline{\mathcal{Z}} + ie^{2(\psi-U)}\overline{\mathcal{L}}} \ ,
\tag{4.11}
$$

in terms of the fields of the theory. It is easy to see that the BPS flow (4.9) can be derived from a superpotential

$$
\mathcal{W} = e^U|\mathcal{Z} - ie^{2(\psi-U)}\mathcal{L}| \ ,
\tag{4.12}
$$

which interestingly reduces to the known superpotential of the ungauged $\mathcal{N}=2$ theory, $\mathcal{W}_{g=0} = e^U|\mathcal{Z}|$, if we turn off $\mathcal{G}$.

However, what has no analogue in the ungauged theory is the constraint

$$
\langle \mathcal{G}, Q \rangle = -1 \ ,
\tag{4.13}
$$

that has to be imposed for a Supersymmetric solution, as we see from (4.7). This constraint acts as a quantization condition relating among them the black hole and the gravitino charges. It is analogous to the quantization condition that Romans found for his monopole solutions [20]. Actually the black holes, solutions of the BPS equations we presented above, have even more in common with the Romans' monopole: they are 1/4-BPS states. In order to solve the Supersymmetry variations of the gravitino and gaugino fields in the Fajet-Iliopoulos theory, in fact, one has to impose two projection conditions on the Killing spinor ε^A

$$
\begin{aligned}
\gamma^0 \varepsilon_A &= \pm i e^{i\alpha}\varepsilon_{AB}\varepsilon^B \ , \\
\gamma^1 \varepsilon_A &= e^{i\alpha}\delta_{AB}\varepsilon^B \ ,
\end{aligned}
\tag{4.14}
$$

each one reducing by $1/2$ the independent degrees of freedom of the four dimensional spinorThe choice of $+$ or $-$ in (4.14) gives rise to two different BPS branches of solutions. The flow equations obtained in this section, by squaring the action, refer to the choice of $+$ sign. Solution of the other branch are obtained by changing the sign of the vector of black hole charges $Q \rightarrow -Q$, in the flow equations and the constraints.

4.1 Attractors in the gauged theory

Supersymmetric black holes in gauged Supergravities are geometries, described by a metric ansatz of the form (4.3), that interpolates between an asymptotic AdS_4 region and a near horizon Bertotti-Robinson geometry $AdS_2 \times S^2$. This means that, in addition to the attractor condition at the horizon (a consequence of the infinite throat of AdS_2), in order for the geometry at infinity to be supersymmetric a D-term condition has to be imposed:

$$D_i \mathcal{L}|_\infty = 0 \ . \tag{4.15}$$

This further constrains the dynamics of the scalars, and restricts the solutions that can be actually found.

Given the superpotential (4.12), defining $A = \psi - U$, the attractor equations for the BPS black hole are given by $\partial_i \mathcal{W}|_{r_h} = 0$, $\mathcal{W}|_{r_h} = 0$, that translate to

$$Q + e^{2A} \Omega \mathcal{M} \mathcal{G} = -2\mathrm{Im}(\overline{\mathcal{Z}} \mathcal{V}) + 2 e^{2A} \mathrm{Re}(\overline{\mathcal{L}} \mathcal{V}) \ ,$$
$$e^{2A} = -i \frac{\mathcal{Z}}{\mathcal{L}} = R_S^2 \ . \tag{4.16}$$

The first one reduces to the attractor equations of ungauged Supergravity as $\mathcal{G} \to 0$, while the second one is proper of gauged Supergravity and contains the information on the area of the horizon, i.e. the entropy. By projecting the first equation on the symplectic scalar sections $\mathcal{V}$ one obtains two interesting expressions for the area of the black hole

$$e^{-2A} = 2 \left(|D_i \mathcal{L}|^2 - |\mathcal{L}|^2 \right) \ ,$$
$$e^{2A} = 2 \left(|D_i \mathcal{Z}|^2 - |\mathcal{Z}|^2 \right) \ . \tag{4.17}$$

These correspond in fact to the second symplectic invariants $I_2(\mathcal{G})$ and $I_2(Q)$ of $\mathcal{N} = 2$ Supergravity, defined for a generic symplectic vector of charges as

$$I_2(\Gamma) = |Z(\Gamma)|^2 - |D_i Z(\Gamma)|^2 \ , \qquad Z(\Gamma) \equiv \langle \Gamma, \mathcal{V} \rangle \ . \tag{4.18}$$

Its absolute value gives the entropy of the Q-charged black hole solution in ungauged Supergravity, so we interpret the appearence of the symplectic invariant in the gauged theory as the sign of an underlying duality structure in the BPS spectrum, also in presence of gauging.

5. Constructing the black hole solution

Given the attractor equations and BPS flow, one can analyse the model of Fajet-Iliopoulos gauged Supergravity in search of black holes solutions. In the rest of this section we discuss two examples with running scalars. Solutions with constant scalars cannot be found [10].

5.1 Single modulus theory

The theory with prepotential $F = iX^0 X^1$ has one complex modulus z and Kähler potential $\mathcal{K} = -2\log(z + \bar{z})$, defining the moduli space as $\mathrm{Re}\,z > 0$. For general gauge charges (g^0, g^1, g_0, g_1), the requirement of AdS_4 vacuum at infinity fixes

$$z_\infty = \frac{g_0 g_1 + g^0 g^1 + i (g_0 g^0 - g_1 g^1)}{(g_1)^2 + (g^0)^2} \ , \tag{5.1}$$

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which is a value allowed in the moduli space if $g_0g_1 + g^0g^1 > 0$. We have seen that the attractor equations give the entropy as

$$e^{2A} = -(2I_2(\mathcal{G}))^{-1} , \tag{5.2}$$

where $I_2(\mathcal{G}) = |\mathcal{G}|^2 - |D_i\mathcal{G}|^2 = g_0g_1 + g^0g^1$. Thus, it is impossible to have positive entropy and at the same time asymptotic AdS_4, and we can conclude that no supersymmetric regular solutions in AdS_4 can be found in the one-modulus theory with prepotential $F = iX^0X^1$.

5.2 Electric $\frac{1}{4}$-BPS black hole in the t^3-model

As an example, consider the t^3-model, with prepotential

$$F = \frac{(X^1)^3}{X^0} . \tag{5.3}$$

The theory has one complex modulus t and Kähler potential $\mathcal{K} = -\log[-i(t - \bar{t})^3]$. We look for solutions with zero axionic fields, meaning the modulus is purely imaginary $t = -i\lambda$, with $\lambda > 0$, and we fix the charges for the abelian fields and the gauging to be

$$Q = \begin{pmatrix} p^0 \\ 0 \\ 0 \\ q_1 \end{pmatrix} , \qquad \mathcal{G} = \begin{pmatrix} 0 \\ g^1 \\ g_0 \\ 0 \end{pmatrix} . \tag{5.4}$$

The potential of the gauging is

$$V_g = -3g^1\left(\frac{g_0}{\lambda} + g^1\lambda\right) , \tag{5.5}$$

it fixes the value of the scalar at infinity $\lambda_\infty = \sqrt{\frac{g_0}{g^1}}$, for a solution with Anti de Sitter asymptotics. In order to construct the supersymmetric black hole, we have to solve the flow equations [10]

$$2e^{2\psi}\left(e^{-U}\mathrm{Re}\mathcal{V}\right)' + e^{2(\psi-U)}\Omega\mathcal{M}\mathcal{G} + Q = 0 ,$$
$$(e^\psi)' = 2e^{\psi-U}\mathrm{Re}\mathcal{L} , \tag{5.6}$$

and impose on the charges the constraint (4.13), which in this case is $g_0p^0 - g^1q_1 = -1$. It is convenient to re-define the symplectic sections in terms of the positive functions

$$H^0 = L^0e^{-U} , \quad H_1 = -\frac{1}{3}M_1e^{-U} , \tag{5.7}$$

so that the equations of motion reduce to

$$2e^{2\psi}\begin{pmatrix} \partial_rH^0 + 4g_0(H^0)^2 \\ -3\partial_rH_1 - 12g^1(H_1)^2 \end{pmatrix} = \begin{pmatrix} -p^0 \\ -q_1 \end{pmatrix} , \qquad \psi' = 2(g_0H^0 + 3g^1H_1) . \tag{5.8}$$

Notice that $e^{-2U} = 8\sqrt{H^0(H_1)^3}$, and $\lambda = \sqrt{\frac{H_1}{H^0}}$. Following the assumptions of [9] we make the following ansatz

$$H^0 = e^{-\psi}(\alpha^0r + \beta^0) , \quad H_1 = e^{-\psi}(\alpha_1r + \beta_1) , \quad \psi = \log(r^2 - r_h^2) . \tag{5.9}$$

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The equations (5.8) now become algebraic equations, that can be solved in terms of the coefficients in the H's and ψ ansatz as

$$p^0 = \frac{g^1 q_1 - 1}{g_0} \qquad \beta = -\frac{\sqrt{1 - 4 g^1 q_1/3}}{8 g^1} \qquad \beta^0 = \frac{3}{8 g^0}\sqrt{1 - 4 g^1 q_1/3} \qquad r_h = \frac{\sqrt{1 - 4 g^1 q_1}}{2}$$

(5.10)

The solution is then parametrized by $q_1 < 0$, $g_0 > 0$ and $g^1 > 0$, the scalar field and the warp factor $U(r)$ are

$$\lambda = \sqrt{\frac{H_1}{H^0}} = \lambda_\infty \sqrt{\frac{2r - \sqrt{1 - 4 g^1 q_1/3}}{2r + 3\sqrt{1 - 4 g^1 q_1/3}}} \ , \tag{5.11}$$

$$e^{2U} = \frac{2\sqrt{g^0 (g^1)^3}(r^2 - r_h^2)^2}{\left(r - \frac{1}{2}\sqrt{1 - 4 g^1 q_1/3}\right)^{3/2}\left(r + \frac{3}{2}\sqrt{1 - 4 g^1 q_1/3}\right)^{1/2}} \ . \tag{5.12}$$

The entropy $S = e^{2A}|_h = e^{2\psi(r_h) - 2U(r_h)}$ is

$$e^{2A(r_h)} = \frac{1}{8\sqrt{g_0 (g^1)^3}}\left(\sqrt{1 - 4 g^1 q_1} - \sqrt{1 - 4 g^1 q_1/3}\right)^{3/2}\sqrt{\sqrt{1 - 4 g^1 q_1} + 3\sqrt{1 - 4 g^1 q_1/3}} \ . \tag{5.13}$$

It is interesting how the duality invariant expression $g_0 (g^1)^3$ appears in the entropy. This however multiplies a quantity whose relation to duality invariance has not yet been explained.

6. Conclusions

1/4-BPS solutions of $\mathcal{N} = 2$ Supergravity with Fayet-Iliopoulos gauging are the first example of regular, supersymmetric, spherical black holes in asymptotically AdS_4 space. They represent a new branch that does not reduce to the ungauged Supergravity black holes, as one can read from the quantization condition that constrains the charges of the black hole and the gravitini.

It is possible to extend the study of black holes in Fayet-Iliopoulos gauged Supergravity departing from the supersymmetric solutions. In fact, black holes at finite temperature have been recently constructed [21, 22] as well as extremal non-supersymmetric ones [23, 19]. However, in order to reach more realistic scenarios in the context of flux compactifications or applied holography, it would be desirable to have an analogous derivation of black hole solutions in the case where the hypermultiplets do not decouple, as well as for models of Supergravity with non-abelian gauging.

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