PROCEEDINGS OF SCIENCE

Parametrizing the Neutrino sector of the seesaw extension in tau decays

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The Standard Model includes neutrinos as massless particles, but neutrino oscillations showed, that neutrinos are not massless. A simple extension of adding gauge singlet fermions to the particle spectrum allows normal Yukawa mass terms for neutrinos. The smallness of the neutrino masses can be well understood within the seesaw mechanism. We analyse two cases of the minimal extension of the standard model when are added one or two right-handed fields to the three left-handed fields. In this model second Higgs doublet is included. We calculate the one-loop radiative corrections to the mass parameters which produce mass terms for the neutral leptons. In both cases we numerically analyse light neutrino masses as functions of the heavy neutrinos masses. Parameters of the model are varied to find light neutrino masses that are compatible with experimental data of solar $\Delta m^2_{\odot}$ and atmospheric $\Delta m^2_{\rm atm}$ neutrino oscillations for normal and inverted hierarchy.

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1. The model

We discus the extension of the Standard Model (SM) with a second Higgs doublet $\Phi_\alpha, \alpha = 1,2$ with added right-handed neutrino fields to the three left handed neutrino fields. The Yukawa Lagrangian of the leptons is expressed by

$$\mathscr{L}_Y = -\sum_{k=1}^{2}\left(\Phi_k^\dagger \bar{\ell}_R \Gamma_k + \tilde{\Phi}_k^\dagger \bar{\nu}_R \Delta_k\right) D_L + \text{H.c.} \tag{1.1}$$

in a vector and matrix notation, where $\tilde{\Phi}_k = i\tau_2 \Phi_k^*$. In expression (1.1) the ℓ_R, ν_R, and $D_L = (\nu_L\ \ell_L)^T$ are the vectors of the right-handed charged leptons, of the right-handed neutrino singlets, and of the left-handed lepton doublets, respectively. The Yukawa coupling matrices Γ_k are $n_L \times n_L$, while the Δ_k are $n_R \times n_L$.

In this model, spontaneous symmetry breaking of the SM gauge group is achieved by the vacuum expectation values $\langle \Phi_k \rangle_{\text{vak}} = \begin{pmatrix} 0 \\ v_k/\sqrt{2} \end{pmatrix}$. By a unitary rotation of the Higgs doublets, we can achieve $\langle \Phi_1^0 \rangle_{\text{vak}} = v/\sqrt{2} > 0$ and $\langle \Phi_2^0 \rangle_{\text{vak}} = 0$ with $v \simeq 246$ GeV. The charged-lepton mass matrix M_ℓ and the Dirac neutrino mass matrix M_D are

$$M_\ell = \frac{v}{\sqrt{2}}\Gamma_1 \quad \text{and} \quad M_D = \frac{v}{\sqrt{2}}\Delta_1\,, \tag{1.2}$$

respectively with assumption that $M_\ell = \text{diag}\left(m_e, m_\mu, m_\tau\right)$. The mass terms for the neutrinos can be written in a compact form with an $(n_L + n_R) \times (n_L + n_R)$ symmetric mass matrix

$$M_\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & \hat{M}_R \end{pmatrix}, \tag{1.3}$$

where the hat indicates that $\hat{M}_R$ is a diagonal matrix. M_ν can be diagonalized as

$$U^T M_\nu U = \hat{m} = \text{diag}\left(m_1, m_2, \ldots, m_{n_L+n_R}\right), \tag{1.4}$$

where the m_i are real and non-negative. In order to implement the seesaw mechanism [1, 2] we assume that the elements of M_D are of order m_D and those of M_R are of order m_R, with $m_D \ll m_R$. Then, the neutrino masses m_i with $i = 1, 2, \ldots, n_L$ are of order m_D^2/m_R, while those with $i = n_L + 1, \ldots, n_L + n_R$ are of order m_R. It is useful to decompose the $(n_L + n_R) \times (n_L + n_R)$ unitary matrix U as $U = \begin{pmatrix} U_L \\ U_R^* \end{pmatrix}$, where the submatrix U_L is $n_L \times (n_L + n_R)$ and the submatrix U_R is $n_R \times (n_L + n_R)$ [3, 4]. With these submatrices, the left- and right-handed neutrinos are written as linear superpositions of the $n_L + n_R$ physical Majorana neutrino fields χ_i: $\nu_L = U_L P_L \chi$ and $\nu_R = U_R P_R \chi$, where P_L and P_R are the projectors of chirality.

We can express the couplings of the model in terms of mass eigenfields, where three neutral particles are coupling to neutrinos. The interaction of the Z boson with the neutrinos is given by

$$\mathscr{L}_{\text{nc}}^{(\nu)} = \frac{g}{4c_w} Z_\mu \bar{\chi}\gamma^\mu \left[P_L\left(U_L^\dagger U_L\right) - P_R\left(U_L^T U_L^*\right)\right]\chi\,, \tag{1.5}$$

where g is the SU(2) gauge coupling constant and c_w is the cosine of the Weinberg angle.

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Full formalism for the scalar sector of the multi-Higgs-doublet SM is given in Ref. [3, 4]. The Yukawa couplings of the Higgs bosons H_b^0 to the neutrinos are given by

$$\mathscr{L}_{\mathrm{Y}}^{(\nu)}\left(H^0\right) = -\frac{1}{2\sqrt{2}}\sum_b H_b^0\,\bar{\chi}\left[\left(U_R^\dagger \Delta_b U_L + U_L^T \Delta_b^T U_R^*\right)P_L + \left(U_L^\dagger \Delta_b^\dagger U_R + U_R^T \Delta_b^* U_L^*\right)P_R\right]\chi\,, \quad (1.6)$$

with $\Delta_b = \sum_k b_k \Delta_k$, where b are two-dimensional complex unit vectors. The neutral Goldstone boson $G^0_{b_Z}$ is given by the vector b_Z with $b_Z = (i,0)$.

Once the one-loop corrections are taken into account the neutral fermion mass matrix is given by

$$M_\nu^{(1)} = \begin{pmatrix} \delta M_L & M_D^T + \delta M_D^T \\ M_D + \delta M_D & \hat{M}_R + \delta M_R \end{pmatrix} \approx \begin{pmatrix} \delta M_L & M_D^T \\ M_D & \hat{M}_R \end{pmatrix}, \quad (1.7)$$

where the $0_{3\times 3}$ matrix appearing at tree level (1.3) is replaced by the contribution δM_L. This correction a symmetric matrix, it dominates among all the sub-matrices of corrections. Neglecting the sub-dominant pieces in (1.7), one-loop corrections to the neutrino masses originate via the self-energy function $\Sigma_L^S(0) = \Sigma_L^{S(Z)}(0) + \Sigma_L^{S(G^0)}(0) + \Sigma_L^{S(H^0)}(0)$, where the $\Sigma_L^{S(Z,G^0,H^0)}(0)$ functions arise from the self-energy Feynman diagrams and are evaluated at zero external momentum squared. In the calculation of the self energies the neutrino couplings to the Z boson as well as the Higgs and Goldstone bosons are determined by eqs. (1.5) and (1.6). Each diagram contains a divergent piece but when summing up the three contributions the result turns out to be finite.

The final expression for one-loop corrections is given by [5]

$$\begin{aligned}\delta M_L = &\sum_b \frac{1}{32\pi^2}\Delta_b^T U_R^* \hat{m}\left(\frac{\hat{m}^2}{m^2_{H_b^0}} - \mathbb{1}\right)^{-1}\ln\left(\frac{\hat{m}^2}{m^2_{H_b^0}}\right)U_R^\dagger \Delta_b \\ &+ \frac{3g^2}{64\pi^2 m_W^2} M_D^T U_R^* \hat{m}\left(\frac{\hat{m}^2}{m_Z^2} - \mathbb{1}\right)^{-1}\ln\left(\frac{\hat{m}^2}{m_Z^2}\right)U_R^\dagger M_D, \end{aligned}\quad (1.8)$$

where sum $\sum_b$ runs over all neutral physical Higgses H_b^0 [1].

2. Case $n_R = 1$

First we consider the minimal extension of the standard model adding only one right-handed field ν_R to the three left-handed fields contained in ν_L.

We use the parametrization of $\Delta_1 = \frac{\sqrt{2}m_D}{v}\vec{a}_1{}^T$ and $\Delta_2 = \frac{\sqrt{2}m_D}{v}\vec{a}_2{}^T$ with $|\vec{a}_1| = 1$ and $|\vec{a}_2| = 1$. Diagonalization of the symmetric mass matrix M_ν (1.3) in block form is

$$U^T M_\nu U = U^T \begin{pmatrix} 0 & m_D\vec{a}_1 \\ m_D\vec{a}_1{}^T & \hat{M}_R \end{pmatrix} U = \begin{pmatrix} \hat{M}_l & 0 \\ 0 & \hat{M}_h \end{pmatrix}. \quad (2.1)$$

The non zero masses in $\hat{M}_l$ and $\hat{M}_h$ are determined analytically by finding eigenvalues of the hermitian matrix $M_\nu M_\nu^\dagger$. These eigenvalues are the squares of the masses of the neutrinos $\hat{M}_l = \mathrm{diag}(0,0,m_l)$ and $\hat{M}_h = m_h$. Solutions $m_D^2 = m_h m_l$ and $m_R^2 = (m_h - m_l)^2 \approx m_h^2$ correspond to the seesaw mechanism.

[1] In our analysis we fix $m_{H_1^0} = 125$ GeV but $m_{H_2^0}$ and $m_{H_3^0}$ we generate randomly in the range 1 to 1000 GeV.

We can construct the diagonalization matrix U for the tree level from two diagonal matrices of phases and three rotation matrices $U_{\rm tree} = U_\phi(\phi_i)U_{12}(\alpha_1)U_{23}(\alpha_2)U_{34}(\beta)U_i$, where the angle β is determined by the masses m_l and m_h. The values of ϕ_i and α_i can be chosen to cover variations in M_D.

For calculation of radiative corrections we use following set of orthogonal complex vectors: $b_Z = (i,0)$, $b_1 = (1,0)$, $b_2 = (0,i)$ and $b_3 = (0,1)$. Diagonalization of the mass matrix after calculation of one-loop corrections is performed with a unitary matrix $U_{\rm loop} = U_{\rm egv}U_\varphi(\varphi_1,\varphi_2,\varphi_3)$, where $U_{\rm egv}$ is an eigenmatrix of $M_\nu^{(1)}M_\nu^{(1)\dagger}$ and U_φ is a phase matrix. The second light neutrino obtains its mass from radiative corrections. The third light neutrino remains massless.

It is possible to estimate masses of the light neutrinos from experimental data of solar and atmospheric neutrino oscillations [6] assuming that the lightest $m_{l_3} = 0$. Considering the normal ordering of the light neutrinos we receive $m_{l_1} = 5.0 \pm 0.2 \times 10^{-11}$ GeV and $m_{l_2} = 8.7 \pm 0.3 \times 10^{-12}$ GeV. Numerical analysis shows that we can reach those values for a heavy singlet with the mass bigger than 830 GeV, see Fig. 1.

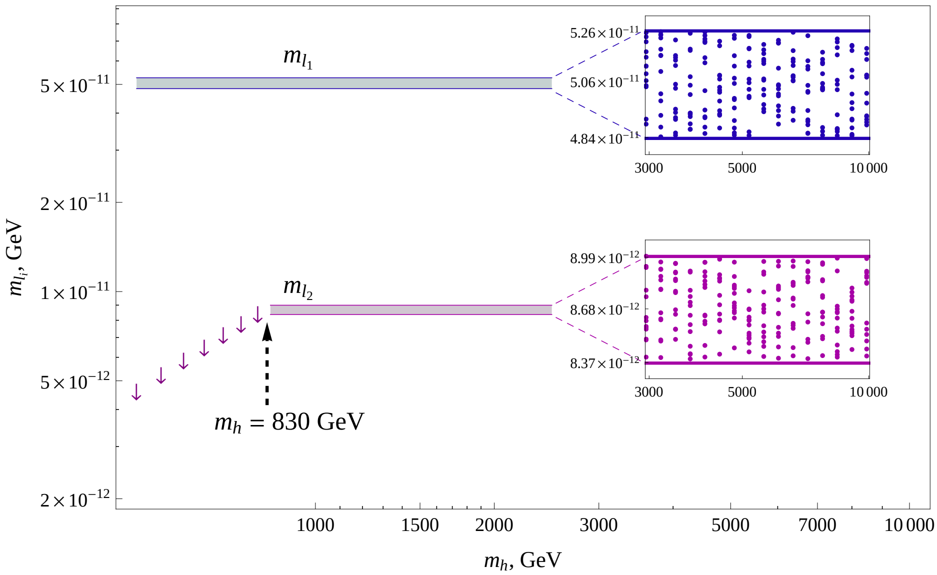

Figure 1: Calculated masses of two light neutrinos as a function of the heavy neutrino mass m_h. The mass of the third light neutrino is zero, when $n_R = 1$. Solid lines show the boundaries of allowed neutrino mass ranges when the model parameters are constrained by the experimental data on neutrino oscillations. The purple arrows indicates the values of m_{l_2} neutrino mass which don't satisfy allowed experimental neutrino mass ranges. Due to the scale, the band of the allowed m_{l_1} and m_{l_2} values are close to each other accordingly their values are shown separately in the right plots.

3. Case $n_R = 2$

If we add two singlet fields ν_R to the three left-handed fields ν_L, the radiative corrections give masses to all three light neutrinos.

Now we parametrize $\Delta_1 = \frac{\sqrt{2}}{v}\begin{pmatrix} m_{D_2}\vec{a}_1^{\,T} \\ m_{D_1}\vec{b}_1^{\,T} \end{pmatrix}$ and $\Delta_2 = \frac{\sqrt{2}}{v}\begin{pmatrix} m_{D_2}\vec{a}_2^{\,T} \\ m_{D_1}\vec{b}_2^{\,T} \end{pmatrix}$ with $|\vec{a}_1| = 1$, $|\vec{b}_1| = 1$, $|\vec{a}_2| = 1$ and $|\vec{b}_2| = 1$. Diagonalizing the symmetric mass matrix M_ν (1.3) in block form we write:

$$U^T M_\nu U = U^T \begin{pmatrix} 0_{3\times 3} & m_{D_2}\vec{a} \;\; m_{D_1}\vec{b} \\ m_{D_2}\vec{a}^T & \\ m_{D_1}\vec{b}^T & \hat{M}_R \end{pmatrix} U = \begin{pmatrix} \hat{M}_l & 0 \\ 0 & \hat{M}_h \end{pmatrix}. \tag{3.1}$$

The non zero masses in $\hat{M}_l$ and $\hat{M}_h$ are determined by the seesaw mechanism: $m_{D_i}^2 \approx m_{h_i} m_{l_i}$ and $m_{R_i}^2 \approx m_{h_i}^2$, $i = 1,2$. Here we use $m_1 > m_2 > m_3$ ordering of masses. The third light neutrino is massless at tree level.

The diagonalization matrix for tree level $U_{\rm tree} = U_{12}(\alpha_1, \alpha_2) U_{\rm egv}(\beta_i) U_\phi(\phi_i)$ is composed of a rotation matrix, an eigenmatrix of $U_{12}^T M_\nu M_\nu^\dagger U_{12}^*$ and a diagonal phase matrix, respectively.

For calculation of radiative corrections we use same set of orthogonal complex vectors b_i as in first case. Diagonalization of the mass matrix including the one-loop correction is performed with a unitary matrix $U_{\rm loop} = U_{\rm egv} U_\varphi(\varphi_i)$, where $U_{\rm egv}$ is the eigenmatrix of $M_\nu^{(1)} M_\nu^{(1)\dagger}$ and U_φ is a phase matrix.

In numerical calculations the model parameters as well as the derived masses of the light neutrinos are obtained in several steps. First, the diagonal mass matrix for tree level is constructed. The lightest neutrino is massless, and the masses of other two light neutrinos are estimated from experimental data on solar and atmospheric neutrino oscillations. The masses of the heavy neutrinos are input parameters. This diagonal matrix is used to constrain the parameters α_i and ϕ_i that enter the tree-level mass matrix M_ν and its diagonalization matrix $U_{\rm tree}$. Then the diagonalization matrix is used to evaluate one-loop corrections to the mass matrix. Diagonalization of the corrected mass matrix yields masses for three light neutrinos. If the calculated mass difference is compatible with the experimental neutrino mass difference, the parameter set is kept. Otherwise, another set of the parameters is generated. Figure 2 illustrate the obtained results. Both normal and inverted neutrino mass orderings are considered.

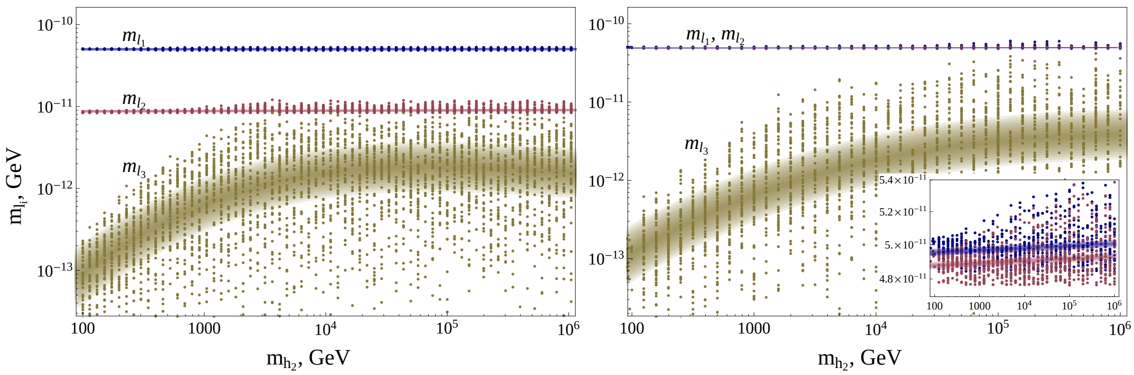

Figure 2: The masses m_{l_i} of the light neutrinos as functions of the heaviest right-handed neutrino mass m_{h_1}, for the case $n_R = 2$. The value of second heaviest right-handed neutrino mass is fixed at $m_{h_2} = 100$ GeV. Plot in the left represent normal hierarchy than the plot in the right represent inverted hierarchy of the light neutrinos. The wide solid lines indicate the place of the most frequent values of the scatter data. In inverted hierarchy case the nearly degenerate masses m_{l_1} and m_{l_2} are shown separately in the lower right plot.

4. Conclusions

For the case $n_R = 1$ we can match the differences of the calculated light neutrino masses to $\Delta m^2_{\odot}$ and Δm^2_{atm} with the mass of a heavy singlet bigger than 830 GeV. Only normal ordering of neutrino masses is possible.

In the case $n_R = 2$ we obtain three non vanishing masses of light neutrinos for normal and inverted hierarchies. The numerical analysis shows that the values of light neutrino masses (especially of the lightest mass) depend on the choice of the heavy neutrinos masses. The radiative corrections generate the lightest neutrino mass and have a big impact on the second lightest neutrino mass.

In future we plan to apply our parametrization to study the τ polarization coming from the decay of a W boson in the data of the CMS experiment at LHC and thus determine restrictions to the parameters of the neutrino sector.

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