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Scalar mesons LbL contribution to the (g-2) of muon in NχQM

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The light-by-light contribution from the lightest scalar mesons to the anomalous magnetic moment of muon is calculated in the framework of the nonlocal chiral quark model. This contribution is found to be positive as the contribution of the lightest pseudoscalar mesons. The possible crosscheck of the sign of this contribution with help of the hadronic vaccum polarization is discussed. The combined scalar–pseudoscalar contribution of lightest mesons is estimated as $(6.25 \pm 0.83) \cdot 10^{-10}$.

XXI International Baldin Seminar on High Energy Physics Problems
September 10-15, 2012
JINR, Dubna, Russia

*Speaker.

PoS(Baldin ISHEPP XXI)063

1. Introduction

The description of the muon anomalous magnetic moment (AMM), a_μ, is one of the important problems of the elementary particle physics. The recent precise result for the muon AMM obtained in the experiment E821 at BNL [1] opens possibility for very fine investigation of the contributions from the electromagnetic, weak and strong sectors of the standard model. In near future the two new experiments on measurement of a_μ are planned in J-PARC[2] and Fermilab (E989)[3] laboratories. The aim of these new experiments will be to decreasing the experimental $(g-2)_\mu$ uncertainty and theoretical prediction should be at least at the same level of precision.

The uncertainty of the contribution of strong interactions dominates in the total uncertainty of theoretical prediction of $(g-2)$ of muon. The contribution from the light-by-light (LbL) diagrams can not be estimated from first principles or extracted from experimental data. In this talk we discuss the LbL contribution from the diagrams with intermediate scalar resonances.

2. NχQM and LbL contribution

The basic element for calculations of the hadronic LbL contribution to the muon AMM is the fourth-order light quark hadronic vacuum polarization tensor

$$\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \int \prod_{j=1,3} (d^4x_j) e^{i(q_1x_1+q_2x_2+q_3x_3)} \times \langle 0|T(j_\mu(x_1)j_\nu(x_2)j_\lambda(x_3)j_\rho(0))|0\rangle, \quad (2.1)$$

where $j_\mu(x)$ are light quark electromagnetic currents and $|0\rangle$ is the QCD vacuum state.

The muon AMM can be extracted by using the projection [4]

$$a_\mu^{\mathrm{LbL}} = \frac{1}{48m_\mu}\mathrm{Tr}\left((\hat{p}+m_\mu)[\gamma^\rho,\gamma^\sigma](\hat{p}+m_\mu)\Pi_{\rho\sigma}(p,p)\right), \quad (2.2)$$

where

$$\begin{aligned}\Pi_{\rho\sigma}(p',p) = -ie^6 \int \frac{d^4q_1}{(2\pi)^4}\int\frac{d^4q_2}{(2\pi)^4}\frac{1}{q_1^2q_2^2(q_1+q_2-k)^2}\times \\ \gamma^\mu\frac{\hat{p}'-\hat{q}_1+m_\mu}{(p'-q_1)^2-m_\mu^2}\gamma^\nu\frac{\hat{p}-\hat{q}_1-\hat{q}_2+m_\mu}{(p-q_1-q_2)^2-m_\mu^2}\gamma^\lambda\times \\ \frac{\partial}{\partial k^\rho}\Pi_{\mu\nu\lambda\sigma}(q_1,q_2,k-q_1-q_2),\end{aligned} \quad (2.3)$$

m_μ is the muon mass, $k_\mu = (p'-p)_\mu$ and it is necessary to consider the limit $k_\mu \to 0$.

In the case of the resonance exchanges of the light hadrons in the intermediate pseudoscalar and scalar channels the LbL contribution to the muon AMM consists of three diagrams which are shown in Fig. 1.

The vertices containing the virtual scalar meson with momentum p and two photons with momenta $q_{1,2}$ and the polarization vectors $\varepsilon_{1,2}$ (see [5]) can be written as [6]

$$\mathscr{A}\left(\gamma^*_{(q_1,\varepsilon_1)}\gamma^*_{(q_2,\varepsilon_2)} \to S^*_{(p)}\right) = e^2\varepsilon_1^\mu\varepsilon_2^\nu\Delta_S^{\mu\nu}(p,q_1,q_2) \quad (2.4)$$

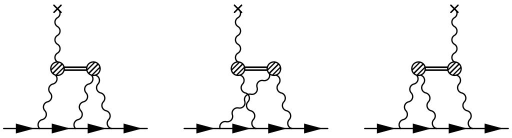

Figure 1: LbL contribution from intermediate meson exchanges.

with

$$\Delta_S^{\mu\nu}(p,q_1,q_2) = \mathrm{A}_S\left(p^2;q_1^2,q_2^2\right) P_A^{\mu\nu}(q_1,q_2) + \mathrm{B}_S(p^2;q_1^2,q_2^2) P_B^{\mu\nu}(q_1,q_2),$$

where

$$P_A^{\mu\nu}(q_1,q_2) = \left(g^{\mu\nu}(q_1q_2) - q_1^\nu q_2^\mu\right); \qquad P_B^{\mu\nu}(q_1,q_2) = \left(q_1^2 q_2^\mu - (q_1q_2)q_1^\mu\right)\left(q_2^2 q_1^\nu - (q_1q_2)q_2^\nu\right);$$

and $p = q_1 + q_2$. Note, that the scalar form factor B_S is singular in the limit when one photon is real and the virtuality of the second photon equals to the virtuality of the scalar meson $p^2 \to q_1^2$, $q_2^2 \to 0$. For convenience we also define an additional function

$$\mathrm{B}'_S(p^2;q_1^2,q_2^2) = \mathrm{B}_S(p^2;q_1^2,q_2^2)\left((q_1q_2)^2 - q_1^2 q_2^2\right), \tag{2.5}$$

which is regular in this limit.

The corresponding polarization tensor $\Pi^{\mu\nu\lambda\rho}$ for the exchange of meson with mass M is

$$\Pi^{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \quad i\frac{\Delta^{\mu\nu}(q_1+q_2,q_1,q_2)\Delta^{\lambda\rho}(q_1+q_2,q_3,q_4)}{(q_1+q_2)^2 - \mathrm{M}^2} +$$
$$i\frac{\Delta^{\mu\rho}(q_2+q_3,q_1,q_4)\Delta^{\nu\lambda}(q_2+q_3,q_2,q_3)}{(q_2+q_3)^2 - \mathrm{M}^2} + i\frac{\Delta^{\mu\lambda}(q_1+q_3,q_1,q_3)\Delta^{\nu\rho}(q_1+q_3,q_2,q_4)}{(q_1+q_3)^2 - \mathrm{M}^2},$$

where q_i are momenta of outgoing photons, $q_4 = -(q_1+q_2+q_3)$, and one should take $\Delta_P^{\mu\nu}$ for pseudoscalar and $\Delta_S^{\mu\nu}$ for scalar mesons, respectively. Details for the pseudoscalar exchange can be found in [8] (Eqs. (3.1) and (3.3)). The low-energy expansion of the derivative of the polarization tensor $\Pi^{\mu\nu\lambda\rho}$ for the scalar meson is given by

$$\begin{aligned}
\frac{\partial}{\partial k^\rho}\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,k-q_1-q_2) &= i\frac{\Delta^{\mu\nu}(q_1+q_2,q_1,q_2)}{(q_1+q_2)^2-\mathrm{M}^2}\frac{\partial}{\partial k^\rho}\Delta^{\lambda\sigma}(q_1+q_2,-q_1-q_2,-k)\\
&+ i\frac{\Delta^{\nu\lambda}(-q_1,q_2,-q_1-q_2)}{q_1^2-\mathrm{M}^2}\frac{\partial}{\partial k^\rho}\Delta^{\mu\sigma}(-q_1,q_1,-k)\\
&+ i\frac{\Delta^{\mu\lambda}(-q_2,q_1,-q_1-q_2)}{q_2^2-\mathrm{M}^2}\frac{\partial}{\partial k^\rho}\Delta^{\nu\sigma}(-q_2,q_2,-k) + O(k).
\end{aligned}$$

and the low-energy expansion for the derivative of $\Delta_S^{\mu\nu}$ is

$$\frac{\partial}{\partial k^\rho}\Delta_S^{\mu\nu}(-q,q,k) = \mathrm{A}_S(q^2,q^2,0)\left(g^{\mu\nu}q^\rho - q^\nu g^{\mu\rho}\right) + \mathrm{B}'_S(q^2,q^2,0)q^\nu\left(\frac{q^\mu q^\rho}{q^2} - g^{\mu\rho}\right) + O(k).$$

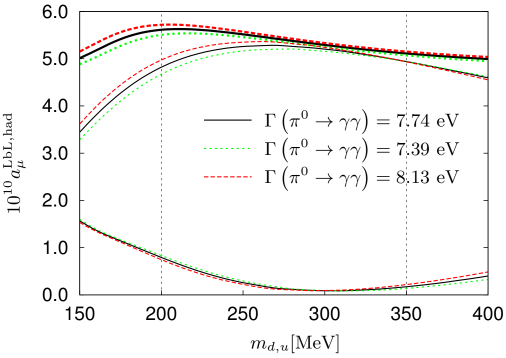

Figure 2: LbL contribution to the muon AMM from the neutral pion and σ exchanges as a function of the dynamical quark mass. Bunch of three lower lines correspond to the σ contribution, the π^0 contribution is in the middle, and the upper lines are the combined contribution. The band along the thick line between dashed and dotted lines corresponds to the error interval for the pion two-photon width. Vertical thin dashed lines denote the interval of dynamical quark masses used for the estimation of the error band for $a_\mu^{\rm LbL}$.

As a result the numerator of the two-loop integrand for the $a_\mu^{\rm LbL}$ contains the combination of two form-factors and is a polynomial in momenta. At next step, the expression for LbL can be averaged [7] over directions of the muon momentum p

$$\langle ... \rangle = \frac{1}{2\pi^2} \int d\Omega\,(\widehat{p}) \ldots \tag{2.6}$$

After averaging the expression for the LbL contribution to the muon AMM from the light scalar meson exchange can be written in the form of integral over Euclidean momenta

$$a_\mu^{\rm LbL,S} = -\frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1^2 \int_0^\infty dQ_2^2 \int_{-1}^1 dt \sqrt{1-t^2}\,\frac{1}{Q_3^2} \times \sum_{S=a_0^0,\sigma,f_0} \left[2\frac{\mathcal{N}_{\mathbf{1}}^S}{Q_2^2+\mathrm{M}_S^2} + \frac{\mathcal{N}_{\mathbf{2}}^S}{Q_3^2+\mathrm{M}_S^2} \right], \tag{2.7}$$
$$\mathcal{N}_{\mathbf{1}}^S = \sum_{\mathrm{X=A,B'}} \sum_{\mathrm{Y=A,B}} \mathrm{X}_S\left(Q_2^2;Q_2^2,0\right) \mathrm{Y}_S\left(Q_2^2;Q_1^2,Q_3^2\right) \mathrm{Ts}_{\mathbf{1}}^{\rm XY},$$
$$\mathcal{N}_{\mathbf{2}}^S = \sum_{\mathrm{X=A,B'}} \sum_{\mathrm{Y=A,B}} \mathrm{X}_S\left(Q_3^2;Q_3^2,0\right) \mathrm{Y}_S\left(Q_3^2;Q_1^2,Q_2^2\right) \mathrm{Ts}_{\mathbf{2}}^{\rm XY},$$

where $Q_3 = -(Q_1+Q_2)$ and capital letters are introduced for Euclidean momenta, i.e. $Q_l^2 = -q_l^2$. One should note that $\mathrm{Ts}_{\mathbf{i}}^{\rm B'A} = \frac{1}{2}\mathrm{Ts}_{\mathbf{i}}^{\rm AA}$, $\mathrm{Ts}_{\mathbf{i}}^{\rm B'B} = \frac{1}{2}\mathrm{Ts}_{\mathbf{i}}^{\rm AB}$. The functions $\mathrm{Ts}_{\mathbf{i}}^{\rm XY}$ can be found in [5].

We calculate the LbL contribution in framework of nonlocal chiral quark model (NχQM). The SU(2)×SU(2) and SU(3)×SU(3) version of the model are considered.

The SU(2)× SU(2) version of the model has three different parameters: current quark mass $m_{c,u}$, dynamical quark mass $m_{d,u}$ and the nonlocality parameter Λ. In order to understand the stability of the model predictions with respect to changes of the model parameters one may vary one parameter in rather wide physically acceptable interval, while fix other parameters by using as input the pion mass and the two-photon decay constant of the neutral pion. Thus, we take the values of the dynamical quark mass in the typical interval of model values 200–350 MeV and other parameters are fitted by the above physical observables within the error range given in [9]. The results are shown in Fig. 2. We see that the contribution of the σ meson is small, positive and has

set	$a_0(980)$	σ	$f_0(980)$	S	$\pi^0+\sigma$	PS+S
G_I	0.0064	0.100	0.0035	0.110	5.15	5.98
G_{II}	0.0079	0.100	0.0038	0.110	5.15	6.24
G_{III}	0.0058	0.100	0.0034	0.109	5.15	5.87
G_{IV}	0.0060	0.115	0.0038	0.126	5.25	5.97

Table 1: Contribution scalar mesons into AMM of muon for different parametrization [10] of model and general contribution with pseudoscalar mesons. All number are given in 10^{-10}.

very small minimum value around the value of 300 MeV for the dynamical mass. It is interesting to note that the total result for the pion and σ meson contributions is rather stable to variation of the dynamical mass in the tested interval. Our estimates for the π^0 and the sum of π^0 and σ contributions (here and below in 10^{-10}) are

$$a_\mu^{\mathrm{LbL},\pi^0} = 5.01 \pm 0.37, \quad a_\mu^{\mathrm{LbL},\pi^0+\sigma} = 5.40 \pm 0.33. \tag{2.8}$$

The SU(3)$\times$ SU(3) version of the model has two additional parameters: current $m_{c,s}$ and dynamical mass $m_{d,s}$ of strange quark. We use G_I–G_{IV} parameterizations of model parameters from [10]. Results are given in table 1. We estimate the combined contribution from the $a_0(980)$ and $f_0(980)$ mesons as

$$a_\mu^{\mathrm{LbL,a_0+f_0}} \sim 0.01. \tag{2.9}$$

3. Sign of scalar contribution

As suggested in [11] the independent cross-check of the sign of LbL contribution can be performed with help of the estimation of hadronic vacuum polarization (HVP) with meson and photon in the intermediate state, see Fig. 3. This HVP contribution can be related to the cross section electron-positron and therefore should be positive.

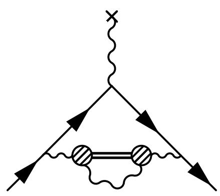

Figure 3: Hadronic vacuum polarization with meson and photon in the intermediate state.

Since we are interested only in the sign of scalar contribution we perform this cross check with the VMD form-factors, as described in [11], in the artificial case when $M_\sigma = M_\pi$. As a result for VMD model we obtain positive contribution for LbL and HVP cases:

$$a_{\mu;VMD}^{LbL;\pi^0} = +5.64 \cdot 10^{-10}; \qquad a_{\mu;VMD}^{LbL;\sigma} = +4.76 \cdot 10^{-10};$$

$$a_{\mu;VMD}^{HVP;\pi^0\gamma} = +0.368 \cdot 10^{-10}; \qquad a_{\mu;VMD}^{HVP;\sigma\gamma} = +0.265 \cdot 10^{-10}.$$

We have a coincidence of sign as well as numerical result for pion contribution with [11] using our program code.

4. Conclusion

It is found that in the framework of NχQM the neutral pseudoscalar meson contributions to $(g-2)$ of muon are systematically lower than the results obtained in the other works (see discussion in [12]). The full kinematic dependence of the vertices on the pion virtuality diminishes the result by about 20-30% in contrast the case where this dependence is neglected. For η and η' mesons the results are reduced by factor about 3 in comparison with the results obtained in other models where the kinematic dependence was neglected (see details in [12]). The total contribution of pseudoscalar exchanges

$$a_\mu^{\rm LbL,PS} = (5.85 \pm 0.87) \cdot 10^{-10} \tag{4.1}$$

is approximately by factor 1.5 less than the most of previous estimates [6, 8, 13, 14] .

The scalar mesons contribution is positive. The combined contribution of scalar–pseudoscalar is estimated as

$$a_\mu^{\rm LbL,PS+S} = (6.25 \pm 0.83) \cdot 10^{-10}. \tag{4.2}$$

We thank C. Fischer, T. Goecke for critical remarks and our discussions. A. S. Zh. is grateful Prof. C. Fischer and Prof. M. V. Polyakov for the hospitality during visits at the ITP of Giessen and TP2 of Bochum. This work is supported in part by the Heisenberg-Landau program (JINR), the Russian Foundation for Basic Research (projects No. 12-02-31874, No. 10-02-00368, No. 11-02-00112), the Ministry of Education and Science of the Russian Federation by Federal Target Program (Agreement No. 14.B37.21.0910).

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