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High-Accuracy Extraction of Nucleon Polarisabilities from Proton and Deuteron Compton Scattering

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The Effective Field Theory (EFT) which incorporates the chiral symmetry of QCD is applied to Compton scattering from the proton and deuteron for photon energies into the first resonance region. For photon energies below 300 MeV, the process is parameterised by six dynamical dipole polarisabilities which characterise the two-photon response to a monochromatic photon of fixed frequency and multipolarity; see our recent review [1]. Their zero-energy limit are the static electric and magnetic scalar dipole polarisabilities α_{E1} and β_{M1}, measured in $10^{-4}\,\mathrm{fm}^3$, and the four spin-polarisabilities which are of higher order. A new extraction for the proton from a consistent database below 300 MeV yields [2]:

$$\alpha_{E1}^{(p)} = 10.7 \pm 0.4_{\text{stat}} \pm 0.2_{\text{Baldin}} \pm 0.3_{\text{theory}} \;,\; \beta_{M1}^{(p)} = 3.1 \mp 0.4_{\text{stat}} \pm 0.2_{\text{Baldin}} \mp 0.3_{\text{theory}} \;.$$

Using the deuteron database, which is smaller and suffers from larger errors, we find for the iso-scalar (average) nucleon polarisabilities [1]:

$$\alpha_{E1}^{(s)} = 10.9 \pm 0.9_{\text{stat}} \pm 0.2_{\text{Baldin}} \pm 0.8_{\text{theory}} \;,\; \beta_{M1}^{(s)} = 3.6 \mp 0.9_{\text{stat}} \pm 0.2_{\text{Baldin}} \mp 0.8_{\text{theory}} \;.$$

Combining these results, the neutron polarisabilities are extracted as

$$\alpha_{E1}^{(n)} = 11.1 \pm 1.8_{\text{stat}} \pm 0.2_{\text{Baldin}} \pm 0.8_{\text{theory}} \;,\; \beta_{M1}^{(n)} = 4.2 \mp 1.8_{\text{stat}} \pm 0.2_{\text{Baldin}} \mp 0.8_{\text{theory}} \;.$$

Within the statistics-dominated errors, the proton and neutron polarisabilities are thus identical, i.e. no isospin breaking effects of the pion cloud are seen, as predicted by Chiral EFT.

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1. Theoretical Background

Compton scattering from protons and neutrons provides important insight into the distribution, symmetries and dynamics of the charges and currents inside the nucleon. Experiments to measure this process are therefore presently being pursued at a number of facilities, including MAMI (Mainz) [3], HIγS at TUNL [5], and MAX-Lab at Lund [6]. Chiral effective field theory (χEFT) is one of the main theoretical techniques to analyse and relate them to emerging lattice QCD simulations [4]. χEFT generates the most general Compton amplitude that is consistent with electromagnetic gauge invariance, the pattern of chiral-symmetry breaking in QCD, and Lorentz covariance, to any given order in a small, dimension-less parameter, namely typical low-energy scales like the pion mass m_π and photon energy ω in units of the breakdown scale Λ of the theory.

The pioneering calculations used a variant with only nucleons and pions as dynamical degrees of freedom [7, 8]. This sets Λ to $M_\Delta - M_N \approx 300$ MeV, where the $\Delta(1232)$ enters as dynamical. At leading non-vanishing order, the nucleon scalar dipole polarisabilities are predicted as

$$\alpha_{E1} = 10\beta_{M1} = \frac{10\alpha_{\mathrm{EM}} g_A^2}{192\pi m_\pi f_\pi^2} = 12.6 \times 10^{-4}\ \mathrm{fm}^3. \tag{1.1}$$

The resulting cross sections agree indeed well with experiment up to at least $\omega \sim m_\pi$, but do not capture the rise towards the $\Delta(1232)$ peak. In this variant of χEFT, even a next-to-leading-order calculation cannot describe the data at backward angles once $\omega \gtrsim 180$ MeV [9, 10, 11].

To include the Delta as an active degree of freedom is therefore essential if the full power of the world's Compton data to shed light on fundamental hadron-structure parameters like the polarisabilities is to be realised. The $\Delta(1232)$ adds the ratio $(M_\Delta - M_N)/\Lambda$ as one of χEFT's expansion parameters [12, 13, 14]. Ref. [15] pointed out that $(M_\Delta - M_N)/\Lambda$ is numerically rather similar to the expansion parameter in Δ-less calculations, $m_\pi/(M_\Delta - M_N)$, and denoted both as δ. In this "δ-counting", the Thomson amplitude is $\mathscr{O}(e^2\delta^0)$ and structure effects start with πN loops at $\mathscr{O}(e^2\delta^2)$ in the low-energy region. Further, since $M_\Delta - M_N \sim \delta$, whereas $m_\pi \sim \delta^2$, $\pi\Delta$ loops are suppressed by an additional power of δ, and do not enter the amplitude until $\mathscr{O}(e^2\delta^3)$.

The Delta-pole graph has a special role in δ-counting and leads to a re-ordering of contributions at higher energies: it too is $\mathscr{O}(e^2\delta^3)$ for $\omega \sim m_\pi$, but it becomes enhanced in the region $\omega \sim M_\Delta - M_N$, because of proximity to the Delta's on-shell point [15]. In this régime, the effects that generate the resonance's finite width must be resummed and the dominant Compton scattering mechanism is the excitation of a dressed $\Delta(1232)$ by the magnetic transition from the nucleon state, followed by de-excitation via the same M1 transition. This effect occurs at $\mathscr{O}(e^2\delta^{-1})$ and constitutes the leading-order contribution for $\omega \sim \delta \sim 300$ MeV. At $\mathscr{O}(e^2\delta^0)$, the E2 $N \to \Delta(1232)$ transition must also be considered, as well as the leading-one-loop corrections to the $\gamma N\Delta$ vertex. Relativistic kinematics is of course essential around the Δ resonance. See Refs. [1, 2] for details.

Since the pion cloud around the nucleon is in χEFT at $\mathscr{O}(e^2\delta^2)$ isospin symmetric, Eq. (1.1) is also the prediction for the *neutron* polarisabilities. To access $\alpha_{E1}^{(\mathrm{n})}$ and $\beta_{M1}^{(\mathrm{n})}$ experimentally, a nuclear target is required. The deuteron is the simplest nucleus, and it is accurately described by χEFT (see, e.g., Refs. [16, 17]). Deuteron Compton scattering was first calculated in (Δ-less) χEFT in Ref. [18] at the first order in which nucleon polarisabilities enter. Since nuclear binding is mediated by charged meson-exchange currents to which the photons can couple, these need to be described accurately to obtain reasonable agreement with data. A plus of χEFT is that such binding effects can be disentangled model-independently. For $\omega \sim 100$ MeV, this calculation was improved to higher orders and augmented with $\Delta(1232)$ degrees of freedom in Ref. [10, 11, 19]. Ref. [20] extended this to a χEFT treatment of deuteron Compton scattering which is valid from the Thomson limit up to about 120 MeV, including the $\Delta(1232)$ degree of freedom.

2. A new analysis of proton Compton scattering in χEFT

The calculation in Ref. [2] includes the nucleon Born graph and the t-channel π^0 pole graph (both calculated covariantly). The Δ-pole graphs (s- and u-channel) are dealt with as described in Refs. [1, 2, 15, 21]—covariantly, with a finite width stemming from πN loops, and with $\gamma N\Delta$ vertex corrections. With Compton πN and $\pi\Delta$ loop graphs also added, the amplitude is complete up to order $\mathcal{O}(e^2\delta^0)$ (NLO) in the high-energy régime $\omega \sim M_\Delta - M_N$ where the contributions are re-ordered as described above, with a predicted accuracy of $\lesssim 25\%$. At lower energies, $0 \leq \omega \lesssim m_\pi$, all effects at $\mathcal{O}(e^2\delta^4)$ (N^{4}LO) are included, with a predicted accuracy of $\sim 2\%$. At this order, two contact interactions encode the short-distance ($r \ll 1/m_\pi$) contributions to the scalar polarisabilities. Their coefficients (or, equivalently, the static values $\alpha_{E1}^{(\mathrm{p})}$ and $\beta_{M1}^{(\mathrm{p})}$) are fit to data.

The parameters of the πN sector take standard values (see Refs. [1, 2]). The $\Delta(1232)$ parameters $M_\Delta - M_N = 293$ MeV and $g_{\pi N\Delta} = 1.425$ are obtained from the Breit-Wigner peak and width via the relativistic formula. We adopt $b_2/b_1 = -0.34$ for the ratio of E2 and M1 couplings [23].

Three EFT parameters, b_1, $\alpha_{E1}^{(\mathrm{p})}$ and $\beta_{M1}^{(\mathrm{p})}$, are fit to the data base established in Refs. [1, 2]. For the reasons explained there, we include in the low-energy region data from Refs. [24, 25, 26, 27, 28, 29, 30, 31, 32] and float the normalisation of each within the quoted normalisation uncertainty. In the medium-energy region, the data of Refs. [30, 33] are in significant disagreement with those from MAMI (most notably Refs. [34, 35]), so that a consistent fit to all sets simultaneously cannot be obtained. We have chosen to use the MAMI data for our fits in this region [2].

Since the power counting confirms that the high-energy amplitudes are most sensitive to Δ parameters, we determine the γNΔ M1 coupling b_1 from the MAMI data for $\omega_{\mathrm{lab}} =$200–325 MeV. Sensitivity to the polarisabilities is greater up to 170 MeV, where the amplitudes are also known with higher accuracy. We thus fit $\alpha_{E1}^{(\mathrm{p})}$ and $\beta_{M1}^{(\mathrm{p})}$ concurrently to these low-energy data. This procedure is iterated betwixt both regions until convergence is reached. Since the $\chi^2/$d.o.f. of the low-energy Hallin data is hard to accept, we prefer to quote our best results without them. We then obtain a solution with a $\chi^2/\mathrm{d.o.f.} = 113.2/135$, $b_1 = 3.61 \pm 0.02$ and the values of $\alpha_{E1}^{(\mathrm{p})}$ and $\beta_{M1}^{(\mathrm{p})}$ quoted in the abstract. Special care was taken to reproducibly justify a theoretical uncertainty to this result from neglecting higher-order contributions. We determined it as ± 0.3, from the most conservative of several estimates. An acceptable χ^2 and description of the cross sections at intermediate angles can only be reached when one of the spin polarisabilities is also treated as fit parameter, albeit the corresponding counter term strictly speaking enters only at one order higher: $\gamma_{M1M1} = 2.2 \pm 0.5_{\mathrm{stat}}$. The corresponding cross sections are displayed in Fig. 1. A fit to α_{E1} and β_{M1} independently is highly consistent with the Baldin sum rule $\alpha_{E1}^{(\mathrm{p})} + \beta_{M1}^{(\mathrm{p})} = 13.8 \pm 0.4$ [32], so that the numbers quoted above are those when this constraint is implemented. All fits are stable against reasonable variations in the procedure [2], and agree with the data of Refs. [34, 35, 36, 37, 38, 39, 40] well beyond the region in which the parameters are determined.

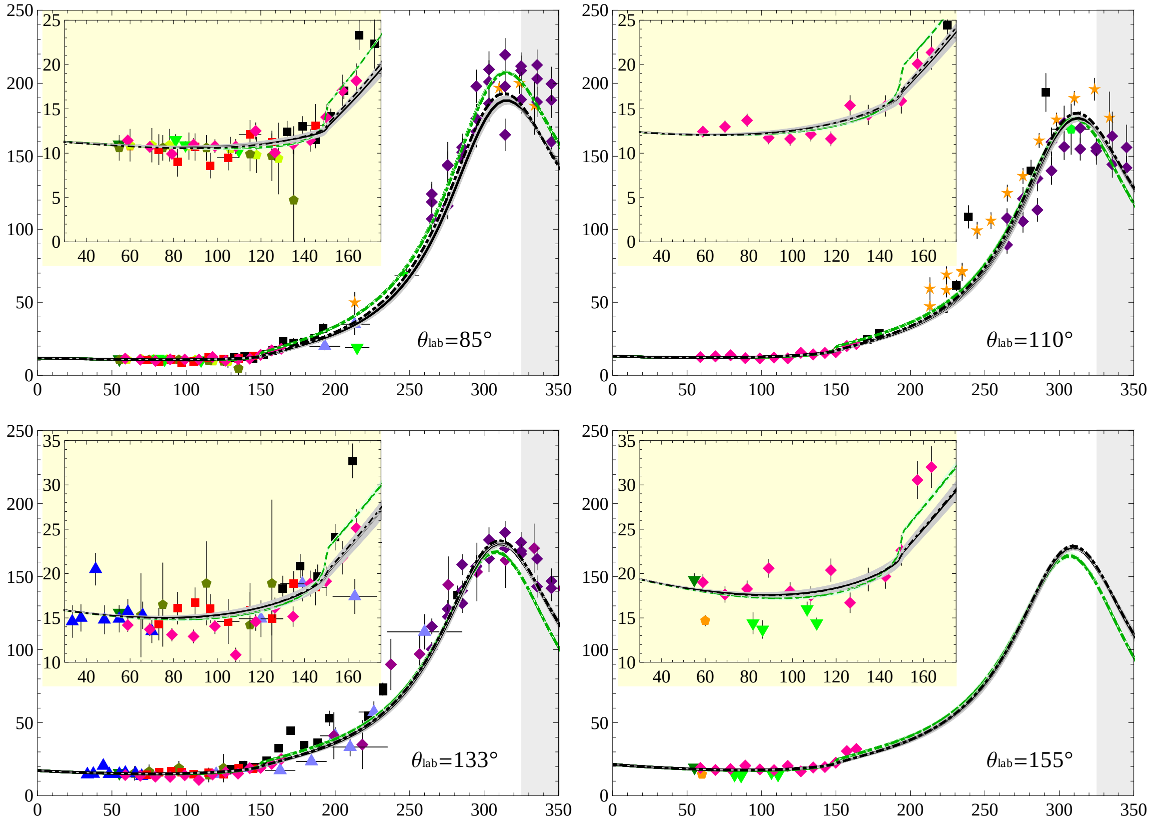

Figure 1: Comparison of our χEFT result for γp scattering with data: lab cross section in nb/sr in 10° bins for θ_{lab} as function of ω_{lab} in MeV, with insets of the fit region. The grey band shows the variation within the statistical error of the one-parameter fit. Adapted from Ref. [2], where the detailed legend can be found.

3. A new analysis of deuteron Compton scattering in χEFT

The χEFT treatment of γd scattering developed in Ref. [20] is valid from threshold to $\omega_{\mathrm{lab}} \gtrsim 100$ MeV and represents a complete $\mathcal{O}(e^2\delta^3)$ calculation in both the two- and one-nucleon sectors. It has the added virtue that the dependence of cross sections on the choice of deuteron wave function is $< 1\%$ [1]. We employ the χEFT deuteron wave function at NNLO [16] to extract $\alpha_{E1}^{(\mathrm{s})}$ and $\beta_{M1}^{(\mathrm{s})}$ from the elastic data of Refs. [41, 42, 43]. This data base has significantly larger statistical error-bars and numbers only a tenth of the proton data, lying at relatively few angles and energies. While the amplitudes we use are one order lower than in the proton extraction, the statistical errors are still larger than the increased estimated theoretical uncertainties, ± 0.8.

The fit to the isoscalar, scalar dipole polarisabilities yields the results quoted in the abstract, with $\chi^2/\mathrm{d.o.f.} = 24.3/25$; see Fig. 2. In contrast to the proton case, the data base is consistent: each experiment contributes roughly equally to the χ^2, and the extracted polarisabilities are largely insensitive to the elimination of any one data set. The isoscalar polarisabilities we obtain are close to the proton ones, so isovector effects in α_{E1} and β_{M1} are small, as predicted by χEFT at $\mathcal{O}(e^2\delta^3)$. An independent fit to α_{E1} and β_{M1} is again consistent with the (isoscalar) Baldin sum rule, so that the latter constraint reduces statistical uncertainties (see Fig. 2). For further details, see Ref. [1].

The future of Compton scattering lies in unpolarised, single-polarised and doubly-polarised

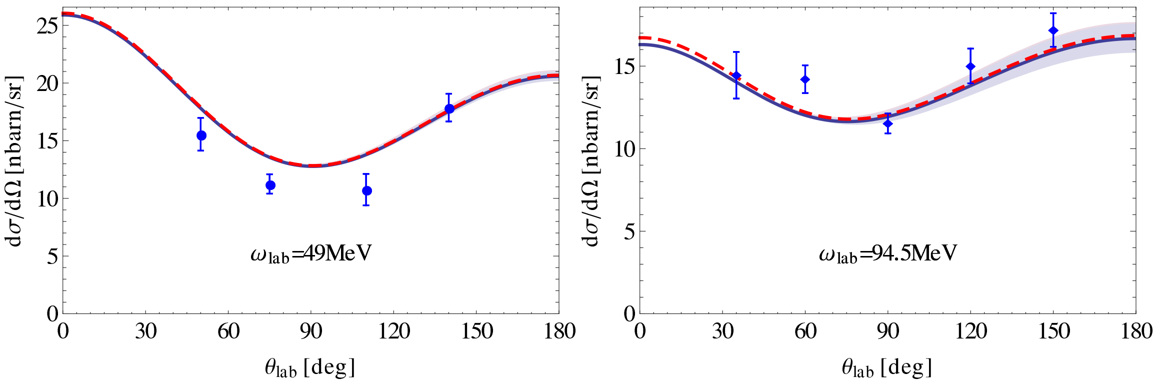

Figure 2: γd cross sections at 49 and 94.5 MeV in the two-parameter (dashed) and one-parameter (solid) determinations of the isoscalar spin-independent dipole polarisabilities. Bands: statistical error of the Baldin constrained fit. Data at 49 (94.5) MeV from Ref. [41] ([42]). Adapted from Ref. [1].

data on the proton, deuteron and ^{3}He of high accuracy, with reproducible systematic uncertainties. At this point, only such experiments will improve the database. Extracting the so-far nearly untested spin-polarisabilities as well as isospin breaking effects of the pion cloud is a top priority of experiment and theory alike.

Acknowledgments

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