PROCEEDINGS OF SCIENCE

Anomalous processes and leading logarithms

Karol Kampf[*†]

Institute of Particle and Nuclear Physics, Faculty of Mathematics and Physics, Charles University in Prague, 18000 Prague, Czech Republic.

E-mail: karol.kampf@mff.cuni.cz

Present status of odd-intrinsic sector of low energy QCD is summarized. The two-photon decay of neutral pion is shortly discussed and its connection with the pion decay constant is analysed. A theoretical tool, the leading-log calculation is also presented, its connection with anomalous sector shown and some new results given.

The 7th International Workshop on Chiral Dynamics,
August 6 -10, 2012
Jefferson Lab, Newport News, Virginia, USA

*Speaker.

†I would like to thank the conference organizers for an enjoyable meeting and my collaborators I had the pleasure to work with on different parts of this report: Johan Bijnens, Marc Knecht, Stefan Lanz, Bachir Moussallam and Jiří Novotný. This work is supported in part by MSM0021620859 of the Ministry of Education of the Czech Republic and by the project UNCE no. 204020/2012 of Charles University.

PoS(CD12)039

1. Introduction

Chiral perturbation theory (ChPT) represents a successful effective field theory of quantum chromodynamics (QCD). For slightly more detailed discussion on its present status and for references see dedicated talk at this conference [1]. The form of its systematic low-energy expansion is govern by the symmetry of underlying QCD. This symmetry, called chiral, acts independently on left and right helicity parts of quark fields. Having N flavours of these quarks the symmetry pattern is $SU(N)_L \times SU(N)_R$. The property of the vacuum of such a system leads to the spontaneous symmetry breaking down to the vector subgroup $SU(N)_V$. The consequence of this symmetry breakdown is the appearance of massless Goldstone boson modes in spectrum. Due to the small but non-zero mass of these N flavour quarks the massless Goldstone bosons will be also massive. In practise we can talk about two possible cases $N = 2$ (for u and d quarks) and $N = 3$ (for u, d and s). At this point everything seems to be prepared for a systematic construction of ChPT Lagrangian which would describe the low-energy dynamics of Goldstone bosons, i.e. three pions for $N = 2$ or altogether eight states (pions, kaons and eta) for $N = 3$. However, there is a problem. The number of these particles would be strictly even in any interaction vertex and it would not be possible to describe e.g. $KK \to 3\pi$ or $\pi^0 \to \gamma\gamma$, i.e. well-established and non-negligible processes. The problem lies in the axial current, as this deserves more detailed study due to the anomaly, in fact two anomalies. One is connected with the so-called $U(1)_A$ problem and will not be considered here. The other one, chiral anomaly, connected with electromagnetic interaction is responsible for the existence of odd-intrinsic parity sector. Its form in ChPT at LO, so-called WZW, can be found in [2] (see also references therein) and for NLO in [3].

2. Anomalous processes

Having formally the Lagrangian at NLO for anomalous processes let us present here a short overview of its applicability in the recent ten-year period. The possible determination of anomalous low energy constants from phenomenology was established in diploma thesis of O. Strandberg [4]. One of the most important anomalous process, the decay of pseudoscalar boson into two photons will be discussed separately in the next subsection. Its tightly connected process, so-called Dalitz decay, $\pi^0 \to e^+e^-\gamma$ was revisited using the Lagrangian of NLO in [5]. LECs were set using the lowest meson dominance model established in [6]. Processes involving kaon, namely radiative kaon decays, $K_{\ell 2\gamma}, K_{\ell 3\gamma}, K_{\ell 4}$ and the contributions of the anomalous sector was considered e.g. in [7]. For an overview not only on these but all kaon decays see also [8]. For the discussion on the relevance of the anomalous sector for radiative pion decays see [9]. The effect of the anomaly in $\pi\gamma \to \pi\pi$ was studied recently [10] (see also [21]). The anomaly is important also for processes with η [11]. As a further possible option to test anomalous contribution one can use hadronic tau decays. These were studied e.g. in [12]. However, the effect of the odd-intrinsic-parity sector was included only via resonances. The systematic study of connection between low-energy odd sector and underlying resonances was subject of [13] (see also [14]).

2.1 $\pi^0 \to \gamma\gamma$ and meson decay constant

This process is described by the WZW Lagrangian and at leading order is thus fixed entirely

PoS(CD12)039

by the anomaly and is parameter free. It is interesting to notice that there are no chiral logarithms at one loop level [15]. This motivated the calculation at next-to-leading order [16]. The result can be further rewritten using the modified counting, which is

$$m_u, m_d \sim O(p^2), \qquad m_s \sim O(p)\,. \tag{2.1}$$

It enables to reduce the numbers of odd-intrinsic LECs to two (C_7^W and C_8^W). The amplitude is

$$\begin{aligned} T_{(LO+NLO)_+} = \frac{1}{F_\pi}\Bigg\{ \frac{1}{4\pi^2} - \frac{64}{3}m_\pi^2 C_7^{Wr} + \frac{1}{16\pi^2}\frac{m_d-m_u}{m_s}\Big[1-\frac{3}{2}\frac{m_\pi^2}{16\pi^2F_\pi^2}L_\pi\Big] \\ + 32B(m_d-m_u)\Bigg[\frac{4}{3}C_7^{Wr} + 4C_8^{Wr}\Big(1-3\frac{m_\pi^2}{16\pi^2F_\pi^2}L_\pi\Big) \\ - \frac{1}{16\pi^2F_\pi^2}\Big(3L_7^r + L_8^r - \frac{1}{512\pi^2}(L_K + \frac{2}{3}L_\eta)\Big)\Bigg] - \frac{1}{24\pi^2}\Big(\frac{m_\pi^2}{16\pi^2F_\pi^2}L_\pi\Big)^2\Bigg\}\,, \end{aligned} \tag{2.2}$$

where L_π, L_K and L_η represent chiral logarithms for pion, kaon and eta respectively, e.g. $L_\pi = \log\frac{m_\pi^2}{\mu^2}$. Using the existing phenomenological information we come to the following prediction

$$\Gamma_{\pi^0\to\gamma\gamma} = (8.09 \pm 0.11)\,\mathrm{eV}\,, \tag{2.3}$$

which is in a very good agreement with the existing most precise measurement by PrimEx collaboration [17] (see also contribution of Rory Miskimen at this conference). Naturally the most important phenomenological input is the pion decay constant F_π. Our best estimate led to

$$F_\pi = 92.22 \pm 0.07\ \mathrm{MeV}\,. \tag{2.4}$$

Determination of this value is from the weak charged pion decay, based on the standard $V-A$ interaction. At this point one can ask how reliable is this connection. One can assume some variant of SM, e.g. as proposed in [18] by the existence of right-handed current. Thus in principle we should distinguish $\hat{F}_\pi$ obtained from $\pi^+ \to \mu^+\nu(\gamma)$ and F_π which is the parameter in ChPT. Schematically

$$F_\pi^2 = \hat{F}_\pi^2(1+\varepsilon), \qquad \varepsilon \sim \frac{V_R^{ud}}{V_L^{ud}}\,, \tag{2.5}$$

where we have used notation of the above mentioned right-handed currents, but obviously the meaning can be more general. Now we can turn around the use of the theoretical $\pi^0 \to \gamma\gamma$ decay-width formula. Then F_π is an unknown parameter we want to fix from a π^0 experiment. Using the most precise experimental determination for π^0 decay width [17] we can obtain

$$F_\pi = (93.85 \pm 1.3\,[\mathrm{exp}] \pm 0.6\,[\mathrm{theory}])\ \mathrm{MeV}\,. \tag{2.6}$$

Comparing with the numerical value for $\hat{F}_\pi$ (2.4), one can estimate $\varepsilon \approx (3-4)\%$, which implies roughly 1σ significance for the right-handed currents (or any model beyond SM).

PoS(CD12)039

3. Leading logarithms

In the previous section we have shown the use of ChPT for the real two-loop calculation and discuss one aspect of the importance of such calculation for phenomenology. In this section we will focus on another aspect connected with theoretical ChPT calculations. It is based on arguments introduced in [19] and further developed in [20] and [21]. Short introduction and overview can be also found in Section 6 in [1] of these proceedings.

The leading logs (LL), i.e. the logarithms with highest power at the given order (i.e. LL^1 at one-loop order, LL^2 at two-loop order and so on), are of special interest. They are parameters-free and can be calculate, in principle, to all orders from one-loop diagrams only. Their eventual resummation then leads to the same effect as is the concept of running coupling constant for the renormalizable theories. In the above mentioned articles the procedure of the one-loop calculations was automatized and high orders of LL were presented. This was done for a general $O(N+1)/O(N)$ model which coincides with $SU(2)$ for $N=3$. Another possible group structure $SU(N)\times SU(N)/SU(N)$ with the direct correspondence to ChPT for both $N=2$ and $N=3$ (though only for the equal-mass case) is under development [22].

It is clear that main motivation for LLs is theoretical as it can hint to some deeper understanding of calculation within effective field theories. However, their application can be also phenomenological. As already stated, they are not proportional to some unknown parameters. We can define them using the lowest-order parameters of the Lagrangian as

$$L = \frac{M^2}{16\pi^2 F^2}\log\frac{\mu^2}{M^2}\,, \tag{3.1}$$

or using the physical mass and decay constant

$$L_\pi = \frac{M_\pi^2}{16\pi^2 F_\pi^2}\log\frac{\mu^2}{M_\pi^2}\,. \tag{3.2}$$

The high orders of LL of several quantities relevant for ChPT were already calculated. General notation of the expansion is as follows

$$O_{\rm phys} = O_0(1+\sum_i a_i L^i), \qquad O_{\rm phys} = O_0(1+\sum_i c_i L_\pi^i)\,. \tag{3.3}$$

More precisely, it was already calculated: expansion of mass and decay constant, vacuum expectation value, vector and scalar formfactor, $\pi\pi$ scattering and in the forthcoming article [22] also $\gamma\gamma\to\pi\pi$ process. Concerning the anomalous processes, LL for the quantities connected with $\pi\gamma\to\pi\pi$ and $\pi^0\to\gamma\gamma$ were calculated up to the 6th and 7th loop respectively. The calculation of the latter can be used to demonstrate the convergence of the perturbative calculation of π^0 decay width. As stated in the previous section already the leading order is in excellent agreement with experimental measurements. This is a signal of a good convergence of the perturbative series and can be graphically demonstrated using the LL calculation (Fig.1).

Let us conclude this section with a remark on automatized calculation of LL. Following the articles in [20] we can notice that first results for expansion for physical mass and physical decay constant up to 5th loop was presented (in $O(N+1)/O(N)$ model). Approximately one or two years later it was possible to add one order more [21]. Now we are again after similar time

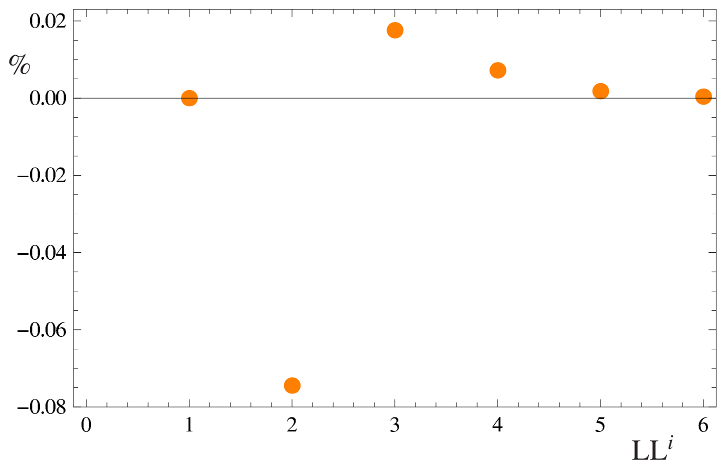

Figure 1: The leading logarithm contribution at individual orders in percent of the leading order for $\Gamma(\pi^0 \to \gamma\gamma)$.

interval and we can present the highest order obtained for this quantities, the 7th loop order. Corresponding tables (Tables 1–4) in [21] can be thus extended by the following formulae, first for the physical meson mass:

$$\begin{aligned} a_7 = {} & 1098817478897/8573040000 - 286907006651/1428840000\,N \\ & + 4533157401977/11430720000\,N^2 - 1986536871797/3429216000\,N^3 \\ & + 436238667943/762048000\,N^4 - 7266210703/21168000\,N^5 + 99977/896\,N^6 - 15N^7 \end{aligned} \tag{3.4}$$

and

$$\begin{aligned} c_7 = {} & 1516884225443/34292160000 - 315684201397/2857680000\,N \\ & + 1125614672041/15240960000\,N^2 + 174975088027/3429216000\,N^3 \\ & - 38300257501/609638400\,N^4 + 1140619717/56448000\,N^5 - 355/21504\,N^6 - 33/2048\,N^7\,. \end{aligned} \tag{3.5}$$

Similarly for the physical decay constant we have obtained

$$\begin{aligned} a_7 = {} & -1560715223869/182891520000 + 15484111239353/548674560000\,N \\ & - 447910156369/7315660800\,N^2 + 63001235724859/548674560000\,N^3 \\ & - 178022410793/1219276800\,N^4 + 9048751829/84672000\,N^5 - 8993/224\,N^6 + 6N^7 \end{aligned} \tag{3.6}$$

and

$$\begin{aligned} c_7 = {} & -5072297701349/182891520000 + 29611209759293/548674560000\,N \\ & + 750961017899/36578304000\,N^2 - 62450118717821/548674560000\,N^3 \\ & + 560319794369/6096384000\,N^4 - 2970744467/112896000\,N^5 + 44777/35840\,N^6 + 429/2048\,N^7 \end{aligned} \tag{3.7}$$

We have checked the correct large N behaviour as calculated in [20]. Note that setting $N = 3$ one can obtain the results relevant for the two-flavour ChPT.

References

[1] J. Bijnens, PoS(CD12)002, arXiv:1301.6953 [hep-ph].

[2] J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 465; R. Kaiser, Phys. Rev. D **63** (2001) 076010 [hep-ph/0011377].

[3] T. Ebertshauser, H. W. Fearing and S. Scherer, Phys. Rev. D **65** (2002) 054033 [hep-ph/0110261]; J. Bijnens, L. Girlanda and P. Talavera, Eur. Phys. J. C **23** (2002) 539 [hep-ph/0110400].

[4] O. Strandberg, J. Bijnens hep-ph/0302064.

[5] K. Kampf, M. Knecht and J. Novotny, Eur. Phys. J. C **46** (2006) 191 [hep-ph/0510021].

[6] M. Knecht and A. Nyffeler, Eur. Phys. J. C **21** (2001) 659 [hep-ph/0106034].

[7] J. Gasser, B. Kubis, N. Paver and M. Verbeni, Eur. Phys. J. C **40** (2005) 205 [hep-ph/0412130]; J. -M. Gerard, C. Smith and S. Trine, Nucl. Phys. B **730** (2005) 1 [hep-ph/0508189].

[8] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich and J. Portoles, Rev. Mod. Phys. **84** (2012) 399 [arXiv:1107.6001 [hep-ph]].

[9] V. Mateu and J. Portoles, Eur. Phys. J. C **52** (2007) 325 [arXiv:0706.1039 [hep-ph]]; R. Unterdorfer and H. Pichl, Eur. Phys. J. C **55** (2008) 273 [arXiv:0801.2482 [hep-ph]].

[10] M. Hoferichter, B. Kubis and D. Sakkas, Phys. Rev. D **86** (2012) 116009 [arXiv:1210.6793 [hep-ph]].

[11] B. Borasoy and R. Nissler, Eur. Phys. J. A **19** (2004) 367 [hep-ph/0309011]; F. Stollenwerk, C. Hanhart, A. Kupsc, U. G. Meissner and A. Wirzba, Phys. Lett. B **707** (2012) 184 [arXiv:1108.2419 [nucl-th]]; F. -K. Guo, B. Kubis and A. Wirzba, Phys. Rev. D **85** (2012) 014014 [arXiv:1111.5949 [hep-ph]]; J. Bijnens and K. Kampf, Nucl. Phys. Proc. Suppl. **207-208** (2010) 220 [arXiv:1009.5493 [hep-ph]].

[12] M. Gonzalez-Alonso, A. Pich and J. Prades, Phys. Rev. D **78** (2008) 116012 [arXiv:0810.0760 [hep-ph]]; D. G. Dumm, P. Roig, A. Pich and J. Portoles, Phys. Rev. D **81** (2010) 034031 [arXiv:0911.2640 [hep-ph]].

[13] K. Kampf and J. Novotny, Phys. Rev. D **84** (2011) 014036 [arXiv:1104.3137 [hep-ph]].

[14] P. D. Ruiz-Femenia, A. Pich and J. Portoles, JHEP **0307** (2003) 003 [hep-ph/0306157].

[15] J. F. Donoghue, B. R. Holstein and Y. C. R. Lin, Phys. Rev. Lett. **55** (1985) 2766 [Erratum-ibid. **61** (1988) 1527]. J. Bijnens, A. Bramon and F. Cornet, Phys. Rev. Lett. **61** (1988) 1453.

[16] K. Kampf and B. Moussallam, Phys. Rev. D **79** (2009) 076005 [arXiv:0901.4688 [hep-ph]].

[17] I. Larin *et al.* [PrimEx Collaboration], Phys. Rev. Lett. **106** (2011) 162303 [arXiv:1009.1681 [nucl-ex]].

[18] V. Bernard, M. Oertel, E. Passemar and J. Stern, JHEP **0801** (2008) 015 [arXiv:0707.4194 [hep-ph]].

[19] S. Weinberg, Physica A **96** (1979) 327; M. Buchler and G. Colangelo, Eur. Phys. J. C **32** (2003) 427 [hep-ph/0309049].

[20] J. Bijnens and L. Carloni, Nucl. Phys. B **827** (2010) 237 [arXiv:0909.5086 [hep-ph]]; J. Bijnens and L. Carloni, Nucl. Phys. B **843** (2011) 55 [arXiv:1008.3499 [hep-ph]].

[21] J. Bijnens, K. Kampf and S. Lanz, Nucl. Phys. B **860** (2012) 245 [arXiv:1201.2608 [hep-ph]].

[22] J. Bijnens, K. Kampf and S. Lanz, "Leading logarithms in N-flavour mesonic Chiral Perturbation Theory," *in prep.*