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Anomalous AV*V vertex function in the soft-wall holographic model of QCD

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I consider the three-point function of two vector and one axial-vector currents. In the kinematic configuration in which one of the two vector currents corresponds to an on-shell soft photon, such a vertex is described in QCD by two functions, w_L and w_T. After reviewing the properties of these functions in QCD, I describe the result obtained using the soft-wall holographic model of QCD with the addition of the Chern-Simons term and I compare it with the QCD outcome. I also discuss a relation, proposed by Son and Yamamoto, that connects w_L and w_T to the two-point functions Π_{VV} and Π_{AA}.

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1. Introduction

In this paper I summarize the results obtained in [1] using a holographic approach to QCD, the soft-wall model [2], for the three-point function of two vector and one axial vector currents if one of the vector currents corresponds to an on-shell soft photon. This function enters, for example, in the calculation of the massless fermion anomalous triangle diagrams. While the longitudinal part of such diagrams is fixed by the chiral anomaly and receives no kind of corrections, the transverse part is affected by both perturbative and non perturbative corrections to the result obtained through the computation of the leading order diagram. On the other hand, corrections should be included to both longitudinal and transverse parts when massive quarks are considered.

Aiming at understanding to which extent the holographic approach reproduces known QCD features, it is interesting to calculate these quantities in this framework and compare them with the corresponding QCD findings. I also discuss an interesting relation, proposed in [3], connecting the transverse part of the anomalous triangle diagrams and the two-point left-right current correlator.

2. Functions w_L and w_T in QCD

I consider the correlation function of two vector currents $J_\mu = \bar{q} V \gamma_\mu q$ and an axial current $J^5_\nu = \bar{q} A \gamma_\nu \gamma_5 q$, ($V$, A being diagonal matrices acting on the flavour indices of the quark fields q), when one of the two vectors corresponds to a real, soft photon with momentum $k \simeq 0$ and $k^2 = 0$:

$$T_{\mu\nu\sigma}(q,k) = i^2 \int d^4x\, d^4y\, e^{iq\cdot x - ik\cdot y} \langle 0| T[J_\mu(x) J^5_\nu(0) J^{em}_\sigma(y)]\, |0\rangle \ , \tag{2.1}$$

with J^{em}_σ the electromagnetic current. A related quantity is the two-point correlator in an external electromagnetic field $T_{\mu\nu}(q,k) = i \int d^4x\, e^{iq\cdot x} \langle 0| T[J_\mu(x) J^5_\nu(0)]\, |\gamma(k,\varepsilon)\rangle$ since $T_{\mu\nu}(q,k) = e\, \varepsilon^\sigma\, T_{\mu\nu\sigma}(q,k)$, with $\varepsilon^\sigma(k)$ the photon polarization vector and e the electric charge unit. For $k \to 0$, keeping only linear terms in k, $T_{\mu\nu}$ can be written in terms of two structure functions $w_L(q^2)$ and $w_T(q^2)$:

$$T_{\mu\nu}(q,k) = -\frac{i}{4\pi^2} \mathrm{Tr}[QVA] \left\{ w_T(q^2)(-q^2 \tilde{f}_{\mu\nu} + q_\mu q^\lambda \tilde{f}_{\lambda\nu} - q_\nu q^\lambda \tilde{f}_{\lambda\mu}) + w_L(q^2) q_\nu q^\lambda \tilde{f}_{\lambda\mu} \right\} \ . \tag{2.2}$$

Q is the electric charge matrix and $\tilde{f}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} f^{\alpha\beta}$ the dual of the photon field strength $f^{\alpha\beta} = k^\alpha \varepsilon^\beta - k^\beta \varepsilon^\alpha$. The first term in (2.2) is transverse with respect to the axial current index, the second one longitudinal. The calculation of the triangle loop diagram corresponding to (2.1) when it takes contribution from a single quark of mass m, was first performed in [4] with the result:

$$w_L(Q^2) = 2\, w_T(Q^2) = \frac{2N_c}{Q^2} \left[1 + \frac{2m^2}{Q^2} \ln \frac{m^2}{Q^2} + \mathscr{O}\left(\frac{m^4}{Q^4} \right) \right] \ , \tag{2.3}$$

where $Q^2 = -q^2$. Both perturbative and nonperturbative corrections modify this result. Nevertheless, a non-renormalization theorem protects w_L from receiving perturbative corrections [5], while, in the case of w_T, in [6] it has been shown that in the kinematic condition in which one of the photons is on shell and soft ($k \to 0$), and for $Q^2 \gg m^2$, also w_T does not receive perturbative corrections to any order. Therefore, in the chiral limit $m = 0$ one has

$$w_L(Q^2) = \frac{2N_c}{Q^2} \ . \tag{2.4}$$

Furthermore, discarding nonperturbative corrections, the relation holds:

$$w_L(Q^2) = 2\,w_T(Q^2)\,. \tag{2.5}$$

In the chiral limit nonperturbative corrections to w_L are also absent since the dependence $w_L \propto 1/Q^2$ reflects the contribution of the pion pole at $Q^2 = 0$ to the longitudinal part of $T_{\mu\nu}$. Nonperturbative corrections to w_T exist and start at $\mathscr{O}\left(Q^{-6}\right)$. For $m \neq 0$, using the Operator Product Expansion (OPE) at large Euclidean Q^2, the operator $\hat{T}_{\mu\nu}$ can be written as

$$\hat{T}_{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\, T[J_\mu(x) J^5_\nu(0)] = \sum_i c^i_{\mu\nu\alpha_1\alpha_2\dots\alpha_i}(q)\, O_i^{\alpha_1\alpha_2\dots\alpha_i}\,., \tag{2.6}$$

where O_i are local operators and c^i coefficients computed perturbatively. The dimension of the O_i matches the dependence of the c^i on the inverse powers of Q^2. Keeping only linear terms in the photon momentum k, the structure of the OPE for $\hat{T}_{\mu\nu}$ is

$$\hat{T}_{\mu\nu} = \sum_i \left\{ c^i_T(q^2)(-q^2 O^i_{\mu\nu} + q_\mu q^\lambda O^i_{\lambda\nu} - q_\nu q^\lambda O^i_{\lambda\mu}) + c^i_L(q^2) q_\nu q^\lambda O^i_{\lambda\mu} \right\}\,, \tag{2.7}$$

so that, writing $\langle 0|\, O_i^{\alpha\beta}\, |\gamma(k,\varepsilon)\rangle = -\frac{ie}{4\pi^2}\kappa_i \tilde{f}^{\alpha\beta}$, one finds: $w_{L,T}(Q^2) = \sum_i c^i_{L,T}(Q^2)\,\kappa_i$. Including operators with dimension up to $D = 3$, the OPE result for w_L and w_T reads:

$$w_L(Q^2) = 2\,w_T(Q^2) = \frac{2N_c}{Q^2}\left[1 + \frac{2m^2}{Q^2}\ln\frac{m^2}{Q^2} - \frac{8\pi^2 m\langle\bar{q}q\rangle\chi}{N_c Q^2} + \mathscr{O}\left(\frac{m^4}{Q^4}\right)\right] \tag{2.8}$$

at large Q^2 (with $\mathscr{O}(\alpha_s)$ corrections computed in [7]). In (2.8), $\langle\bar{q}q\rangle$ denotes the vacuum quark condensate and χ the so-called magnetic susceptibility of the quark condensate.

As for higher order terms, the dimension $D = 4$ operators can be reduced to the $D = 3$ ones using the quark equation of motion, while both $D = 5$ and $D = 6$ terms contribute to $\mathscr{O}\left(\frac{1}{Q^6}\right)$ order. Remarkably, the contribution of the dimension $D = 6$ operators does not vanish in the chiral limit and is responsible of the difference between w_L and $2\,w_T$. Indeed, for $m_q = 0$, w_L remains $w_L(Q^2) = \dfrac{2N_c}{Q^2}$, while w_T, including the leading nonperturbative correction, reads [8, 9]:

$$w_T(Q^2) \;=\; \frac{N_c}{Q^2} \;+\; \frac{128\pi^3\alpha_s\,\chi\,\langle\bar{q}q\rangle^2}{9\,Q^6} \;+\; \mathscr{O}\left(\frac{1}{Q^8}\right). \tag{2.9}$$

χ arises here factorizing the matrix element of four-quark operators in the external field $F^{\alpha\beta}$. There might be other $\mathscr{O}(1/Q^6)$ contributions in the OPE; however, they stem from operators contributing at one loop with small coefficients, while the $1/Q^6$ term in (2.9) comes from tree-level diagrams.

3. Functions w_L and w_T in the soft-wall AdS/QCD model

The AdS/CFT correspondence was formulated as a duality between a type IIB string theory defined on AdS$_5$ $\times S^5$ space and a $\mathscr{N}$=4 super Yang-Mills theory with gauge group $SU(N_c)$, for large N_c [10]. Subsequently, the correspondence has been generalized as an equivalence between a theory defined on AdS$_{d+1} \times \mathscr{C}$ ($\mathscr{C}$ being a compact manifold) and a conformal field theory defined

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on the flat boundary $\mathscr{M}_d$ of the AdS space [11]; on this basis the challenging attempt of describing strong interaction processes by this approach has been undertaken. Two ways are followed to this aim. The first one is so-called top-down approach in which, starting from a string theory, one tries to derive a low-energy QCD-like theory on $\mathscr{M}_d$ through compactifications of the extra dimensions [12]. The second one is the bottom-up approach in which, starting from $4d$ QCD, one tries to construct its higher dimensional dual [13].

In both cases it is necessary to break conformal invariance, since QCD is not a conformal theory [14], and to account for confinement. In the bottom-up approach, conformal invariance can be broken by allowing the fifth coordinate z to vary up to a maximum value z_{max} of $\mathscr{O}(\frac{1}{\Lambda_{QCD}})$ [13, 15] (hard-wall model), or by introducing in the $5d$ AdS space a background dilaton field (soft-wall model) [16, 17]. I consider the soft-wall model, defined in a five dimensional AdS space with line element $ds^2 = g_{MN}dx^M dx^N = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$ with $M,N = 0,1,2,3,5$, $\eta_{\mu\nu} = diag(+1,-1,-1,-1)$ and R is the AdS curvature radius (set to unity). The fifth coordinate z runs in the range $\varepsilon \le z < +\infty$, with $\varepsilon \to 0^+$, and one introduces a background dilaton-like field $\Phi(z) = (cz)^2$. With this choice, linear Regge trajectories for light vector mesons are obtained; c is a dimensionful parameter numerically fixed to $c = \frac{M_\rho}{2}$ from the analysis of the spectrum of the light vector mesons. In this framework light vector, axial-vector and pseudoscalar mesons can be described, with a mechanism of chiral symmetry breaking related to the presence of a scalar field; the light scalar meson sector has also been studied in [18].

To study the considered three-point function one introduces the left and right gauge fields $\mathscr{A}^a_{L\mu}$ and $\mathscr{A}^a_{R\mu}$, dual to the $SU(N_f)_L$ and $SU(N_f)_R$ flavour currents, $\bar{q}_L\gamma^\mu T^a q_L$ and $\bar{q}_R\gamma^\mu T^a q_R$, with T^a the generators of $SU(N_f)$ [19, 20, 2]. Since we want to describe the electromagnetic current that contains both isovector and isoscalar components, we enlarge the gauge group to $U(N_f)_L \times U(N_f)_R$ to describe its dual. The gauge fields $\mathscr{A}^M_{L,R}$ are then combined into a vector $V^M = \dfrac{\mathscr{A}^M_L + \mathscr{A}^M_R}{2}$ and an axial-vector field $A^M = \dfrac{\mathscr{A}^M_L - \mathscr{A}^M_R}{2}$, and the corresponding field strength tensors $F^{MN}_{V,A}$ are introduced: $F^{MN}_V = \partial^M V^N - \partial^N V^M - i[V^M,V^N] - i[A^M,A^N]$, $F^{MN}_A = \partial^M A^N - \partial^N A^M - i[V^M,A^N] - i[A^M,V^N]$. A scalar bulk field, dual to the quark bifundamental field $\bar{q}^\alpha_R q^\beta_L$, is also introduced: $X = X_0 e^{2i\pi}$, where $X_0 = \dfrac{v(z)}{2}$ is a background field that depends only on z. It provides chiral symmetry breaking, being dual to the QCD quark condensate $<\bar{q}q>$. $\pi(x,z)$ represents the pseudoscalar meson field. The definition of X can be further modified to $(X_0+S)e^{2i\pi}$, including a scalar field $S(x,z)$ describing light scalar mesons [18]. The 5d action for the fields V,A and X reads

$$S_{YM} = \frac{1}{k_{YM}}\int d^5x\sqrt{g}e^{-\Phi}Tr\left\{|DX|^2 - m_5^2|X|^2 - \frac{1}{2g_5^2}(F_V^2 + F_A^2)\right\} , \tag{3.1}$$

where $D^M X = \partial^M X - i[V^M,X] - i\{A^M,X\}$ is the covariant derivative, g the determinant of the metric tensor g_{MN}, $\Phi(z)$ the dilaton, and k_{YM} a parameter. Matching the two-point function of the vector field V, and that of the scalar field S, with the corresponding leading order perturbative QCD results fixes $k_{YM} = \frac{16\pi^2}{N_c}$ and $g_5^2 = \frac{3}{4}$ [19, 18].

To compute the functions $w_{L,T}$, following [11] and [21]-[24], [3] a Chern-Simons contribution to S_{YM} is added:

$$S_{CS}(\mathscr{A}) = k_{CS}\int d^5x\, Tr\left[\mathscr{A}F^2 - \frac{i}{2}\mathscr{A}^3F - \frac{1}{10}\mathscr{A}^5\right] . \tag{3.2}$$

Terms in the Chern-Simons action S_{CS} proportional to higher odd powers of $\mathscr{A}_{L,R}$ do not contribute to the AV^*V vertex considered here, so that they can be neglected, keeping in (3.2) only the terms $Tr\left[\mathscr{A}_{L,R}F^2_{L,R}\right] = \varepsilon_{ABCDE}\, Tr\left[\mathscr{A}^A_{L,R}F^{BC}_{L,R}F^{DE}_{L,R}\right]$, with $A,\dots,E$ indices of the $5d$ coordinates. Moreover, since the Chern-Simons actions are invariant only up to a boundary term, in [1] a boundary term has been included to make explicit the invariance under a vector gauge transformation, obtaining: $S_{CS+b} = 3\,k_{CS}\,\varepsilon_{ABCDE}\int d^5x\, Tr\left[A^A\left\{F^{BC}_V, F^{DE}_V\right\}\right]$. The constant k_{CS} is fixed below.

Our starting point is the effective action $S^{eff}_{5d} = S_{YM} + S_{CS+b}$. Exploiting the AdS/QCD correspondence, the 5d action is dual to the QCD generating functional relative to a given operator $O(x)$ provided that the source of $O(x)$ coincides with the $z=0$ boundary value, $f_0(x) = f(x,0)$, of the dual field $f(x,z)$ in the 5d action: $\left\langle e^{i\int d^4x\, 0(x)\, f_0(x)}\right\rangle_{QCD} = e^{iS^{eff}_{5d}[f(x,z)]}$. According to this prescription, the functions w_L and w_T are computed by a functional derivation of the 5d action. One has first to define $\tilde G^a_\mu(q,z)$ as the Fourier transform with respect to the 4d coordinates x^μ of a generic gauge field $G^a(x,z) = V^a(x,z)$ and $A^a(x,z)$ (a flavour index) and then introduce the bulk-to-boundary propagator $G(q,z)$ as: $\tilde G^a_\mu(q,z) = G(q,z)G^a_{\mu 0}(q)$, where $G^a_{\mu 0}(q)$ is the source field. In the case of the vector and axial-vector fields of momentum q we consider two projectors $P^\perp_{\mu\nu} = \eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$, $P^{\|}_{\mu\nu} = \frac{q_\mu q_\nu}{q^2}$, in such a way that the vector and axial-vector bulk-to boundary propagators are expressed in terms of the transverse and longitudinal parts:

$$\tilde V^a_\mu(q,z) = V_\perp(q,z)P^\perp_{\mu\nu}V^{a\nu}_0(q) \quad , \quad \tilde A^a_\mu(q,z) = A_\perp(q,z)P^\perp_{\mu\nu}A^{a\nu}_0(q) + A_{\|}(q,z)P^{\|}_{\mu\nu}A^{a\nu}_0(q)\,, \tag{3.3}$$

imposing as boundary conditions $V_\perp(q,0)=1$ and $A_\perp(q,0) = A_{\|}(q,0) = 1$ (the behaviour at $z\to\infty$ is discussed later) and accounting for the fact that the (conserved) vector field is transverse. The longitudinal component of $\tilde A$ is written as $\tilde A^{a\|}_\mu(q,z) = A_{\|}(q,z)P^{\|}_{\mu\nu}A^a_{\nu 0}(q) = i\,q_\mu\tilde\phi^a$.

From the 5d action a set of equations of motion are obtained in the gauge $V_z = A_z = 0$:

$$\partial_y\left(\frac{e^{-y^2}}{y}\,\partial_y V_\perp\right) - \tilde Q^2\frac{e^{-y^2}}{y}V_\perp = 0 \tag{3.4}$$

$$\partial_y\left(\frac{e^{-y^2}}{y}\,\partial_y A_\perp\right) - \tilde Q^2\frac{e^{-y^2}}{y}A_\perp - \frac{g_5^2 v^2(y)e^{-y^2}}{y^3}A_\perp = 0 \tag{3.5}$$

$$\partial_y\left(\frac{e^{-y^2}}{y}\,\partial_y\tilde\phi^a\right) + \frac{g_5^2 v^2(y)e^{-y^2}}{y^3}(\tilde\pi^a - \tilde\phi^a) = 0 \tag{3.6}$$

$$\tilde Q^2(\partial_y\tilde\phi^a) + \frac{g_5^2 v^2(y)}{y^2}\partial_y\tilde\pi^a = 0 \tag{3.7}$$

where $y = cz$ and $\tilde Q^2 = \dfrac{Q^2}{c^2}$, with $Q^2 = -q^2 > 0$ (the notation $V = V_\perp$, $A = A_\perp$ has been adopted). From the relation $\tilde\phi^a(q,y) = -i\frac{q^\mu}{q^2}A_{\|}(q,y)P^{\|}_{\mu\nu}A^a_{\nu 0}(q)$ and writing $\tilde\pi^a(q,y) = -i\dfrac{q^\mu}{q^2}\pi(q,y)A^a_{\mu 0}(q)$, it turns out that $\pi(q,y)$ and $A_{\|}(q,y)$ obey the same equations (3.6) and (3.7) as $\tilde\pi^a$ and $\tilde\phi^a$.

An equation can also be derived for the field $X_0 = \frac{1}{2}v$: $\partial_y\left(\frac{e^{-y^2}}{y^3}\,\partial_y v(y)\right) + \frac{3e^{-y^2}}{y^5}v(y) = 0$ the regular solution of which reads $v(y)\sim\Gamma\left(\frac{3}{2}\right)yU\left(\frac{1}{2},0,y^2\right)$ and can be expanded for $y\to 0$: $v(y)\to C_1 y + C_2 y^3$. On the basis of the holographic dictionary [19], one argues that the quark mass, responsible of explicit chiral symmetry breaking, enters in the coefficient C_1, and the quark condensate, governing the spontaneous chiral symmetry breaking, enters in C_2: $m_q \propto C_1$, $\sigma \propto \langle \bar{q}q \rangle \propto C_2$. However, since C_1 and C_2 are related, a proportionality between m_q and $\langle \bar{q}q \rangle$ is implied, which does not hold in QCD. This feature of the soft-wall model could be corrected adding potential terms $V(|X|)$ to the action. However, in the following the form $v(y) = \frac{m_q}{c}\, y + \frac{\sigma}{c^3}\, y^3$ is assumed [25].

Determining w_L and w_T, by the AdS/CFT prescription requires a functional derivation of the effective 5d action. The Chern-Simons action is written as $S_{CS+b} = 48\, k_{CS}\, d^{ab}\, \tilde{F}^{\mu\nu}_{em} \int d^5x A^b_\nu\, \partial_z V^a_\mu$, with $d^{ab} = \frac{1}{2} Tr[Q\{T^a, T^b\}]$, and $\tilde{F}^{\mu\nu}_{em}$ the external photon field strength. Performing the functional derivation one has

$$d^{ab} (2\pi)^{-4} \delta^4(q_1+q_2) \langle J^V_\mu J^A_\nu \rangle^{\perp\perp(\|)}_{\tilde{F}} = \frac{\delta^2 S_{CS+b}}{\delta V^{a\perp}_{\mu 0}(q_1)\, \delta A^{b\perp(\|)}_{\nu 0}(q_2)} \,. \tag{3.8}$$

On the other hand, the correlation function of a vector and an axial vector current in the external electromagnetic background field reads, in terms of s w_L and w_T,

$$d^{ab} \langle J^V_\mu J^A_\nu \rangle_{\tilde{F}} \equiv i \int d^4x\, e^{iqx} \langle T\{J^{Va}_\mu(x) J^{Ab}_\nu(0)\}\rangle_{\tilde{F}} = d^{ab} \frac{Q^2}{4\pi^2} P^\perp_{\mu\alpha} \left[P^\perp_{\nu\beta} w_T(Q^2) + P^\|_{\nu\beta} w_L(Q^2) \right] \tilde{F}^{\alpha\beta} \,, \tag{3.9}$$

so that comparison of (3.9) with (3.8) gives

$$w_L(Q^2) = -\frac{2N_c}{Q^2} \int_0^\infty dy A_\|(Q^2,y) \partial_y V(Q^2,y) \quad , \quad w_T(Q^2) = -\frac{2N_c}{Q^2} \int_0^\infty dy A_\perp(Q^2,y) \partial_y V(Q^2,y) \,. \tag{3.10}$$

The choice $k_{CS} = -\dfrac{N_c}{96\,\pi^2}$ reproduces the leading term in the QCD OPE (2.5).

To compare these results with the QCD findings one needs to solve the equations of motion for V, $A_\perp$, $A_\|$. Eq.(3.4) for $V(Q^2,y)$ can be exactly solved with the boundary conditions $V(Q^2,0)=1$ and $V(Q^2,\infty)=0$, with the result $V(Q^2,y) = \Gamma\left(1+\frac{Q^2}{4c^2}\right) U\left(\frac{Q^2}{4c^2}, 0, y^2\right)$, where U is the Tricomi confluent hypergeometric function. The calculation is more difficult for $A_\perp$ and $A_\|$ since Eqs.(3.5) and (3.7) involve the function $v(y)$. Expanding in the inverse powers of Q^2, the result is found [1]:

$$w_T(Q^2) = \frac{N_c}{Q^2} \left(1 - \frac{g_5^2 m_q^2}{3Q^2} - \frac{2 g_5^2 m_q^2 c^2}{5Q^4} + \frac{g_5^4 m_q^4}{6Q^4} - \frac{8 g_5^2 m_q \sigma}{5Q^4} \right) + \mathcal{O}\left(\frac{1}{Q^8}\right) , \tag{3.11}$$

$$w_L(Q^2) = \frac{2N_c}{Q^2} - \left[1-\pi(Q^2,0)\right] N_c \left[\frac{g_5^2 m_q^2}{Q^4} + \frac{4 g_5^2 m_q \sigma}{Q^6} - \frac{2 g_5^4 m_q^4}{3Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right) \right] , \tag{3.12}$$

where the boundary condition of the chiral field $\pi(Q^2,0)$ appears in the expression of w_L. For $m_q=0$, at leading order in the $1/Q^2$ expansion, the QCD results in (2.4-2.5) are recovered. The result for w_L holds even for $\sigma \neq 0$, while that for w_T is modified. Considering Eq. (3.11) for $m_q=0$ and $\sigma \neq 0$, one can observe that the first correction to the leading term is of $\mathcal{O}(\frac{1}{Q^8})$. On the other hand, Eq. (2.9), obtained in QCD in the same limit, shows the first correction of $\mathcal{O}(\frac{1}{Q^6})$. This could be an indication that in the considered holographic model the susceptibility of the chiral condensate χ vanishes. Furthermore, a mismatch is found also comparing the solutions in the most general case $m_q \neq 0$, $\sigma \neq 0$ since the logarithmic term in the QCD result (2.8) is missing in the soft-wall model result.

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4. Two-point functions and Son-Yamamoto relation

Let us consider the left-right correlator $\Pi_{LR} = \Pi_{\perp}^{VV} - \Pi_{\perp}^{AA}$, where $\Pi_{\perp}^{VV}$ and $\Pi_{\perp}^{AA}$ are the transverse invariant structures that appear in the two-point functions of the vector and axial-vector currents $J_\mu^a = \bar{q}\gamma_\mu T^a q$ and axial-vector currents $J_\mu^{5a} = \bar{q}\gamma_\mu\gamma_5 T^a q$

$$\Pi_{\mu\nu}^{ab}(q) = i\int d^4x\, e^{iqx}\, \langle 0|T\{J_\mu^a(x) J_\nu^b(0)\}|0\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\,\delta^{ab}\,\Pi_\perp(q^2) + q_\mu q_\nu\,\delta^{ab}\,\Pi_\parallel(q^2)\ . \tag{4.1}$$

In [3] a relation has been derived that links this quantity to the pion decay constant F_π and the structure function w_T in massless QCD and for any positive and negative Q^2:

$$w_T(Q^2) = \frac{N_c}{Q^2} + \frac{N_c}{F_\pi^2}\,\Pi_{LR}(Q^2)\ . \tag{4.2}$$

Computing Π_{LR} in the soft wall model and expanding in the inverse powers of Q^2 one gets [1]

$$\Pi_{LR}(Q^2) = -\frac{N_c\,\sigma^2}{10\pi^2\,Q^6} + \mathscr{O}\left(\frac{1}{Q^8}\right). \tag{4.3}$$

The first contribution to Π_{LR} is of $\mathscr{O}\left(1/Q^6\right)$, and has the same negative sign as the corresponding dimension six condensate in QCD [26, 27]. On the other hand, for $m_q = 0$, the leading power correction to w_T is $w_T(Q^2) = \frac{N_c}{Q^2}\left(1 + \mathscr{O}(\frac{1}{Q^6})\right)$, so that the Q^2 dependencies of the two sides of (4.2) do not match, similarly to what is found in [3]. Other discussions about this relation can be found in and in other holographic models [28, 29, 30, 31].

5. Conclusions

The functions w_L and w_T can be determined in the soft-wall model with the Chern-Simons term in the action, providing the results (3.10) . In the chiral limit, the result (2.4) for w_L is recovered and holds also for non vanishing chiral condensate. As for w_T, when $m_q = 0$ and $\sigma = 0$ the described calculation reproduces the QCD result and the relation (2.5).

Away from the chiral limit, mismatches are found in the $1/Q^2$ expansion when comparing the holographic results for $w_{L,T}$ to the QCD outcome, a consequence of the choice of the simplest inclusion of the quark mass in the holographic framework through the expression of $v(y)$. In particular, in the expansion of w_T for large Q^2, the next-to-leading contribution in w_T is $\mathscr{O}(1/Q^8)$ in the holographic model, while it is $\mathscr{O}(1/Q^6)$ in QCD. Since in QCD this correction involves the magnetic susceptibility χ of the quark condensate, a simple interpretation of this result could be that, in the soft-wall model, χ vanishes. Another explanation could be that operators like $O_{\mu\nu} = \bar{q}\sigma_{\mu\nu}q$ should be included in the dual approach [32]. Finally, this calculation eveils a violation of a proposed relation between w_T and Π_{LR}, Eq.(4.2).

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References

[1] P. Colangelo, F. De Fazio, F. Giannuzzi, S. Nicotri, J.J. Sanz-Cillero, Phys. Rev. D **85** (2012) 035013.

[2] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74** (2006) 015005.

[3] D. T. Son and N. Yamamoto, arXiv:1010.0718 [hep-ph].

[4] S. L. Adler, Phys. Rev. **177** (1969) 2426.

[5] S. L. Adler and W. A. Bardeen, Phys. Rev. **182** (1969) 1517.

[6] A. Vainshtein, Phys. Lett. B **569** (2003) 187.

[7] K. Melnikov, Phys. Lett. **B639** (2006) 294.

[8] M. Knecht, S. Peris, M. Perrottet, E. De Rafael, JHEP **0211** (2002) 003.

[9] A. Czarnecki et al., Phys. Rev. D **67** (2003) 073006 [Erratum-ibid. D **73** (2006) 119901].

[10] J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231 [Int. J. Theor. Phys. **38** (1999) 1113].

[11] E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253; ibidem **2** (1998) 505; S. S. Gubser *et al.*, Phys. Lett. B **428** (1998) 105.

[12] For a review see: J. Erdmenger et al., Eur. Phys. J. A **35** (2008) 81 and references therein.

[13] J. Polchinski *et al.*, Phys. Rev. Lett. **88** (2002) 031601.

[14] For a discussion see S. J. Brodsky and G. F. de Teramond, arXiv:0802.0514.

[15] J. Erlich et al., Phys. Rev. Lett. **95** (2005) 261602; L. Da Rold et al., Nucl. Phys. B**721** (2005) 79.

[16] O. Andreev, Phys. Rev. D **73** (2006) 107901.

[17] A. Karch *et al.*, Phys. Rev. D **74** (2006) 015005.

[18] P. Colangelo, F. De Fazio, F. Giannuzzi, F. Jugeau and S. Nicotri, Phys. Rev. D **78** (2008) 055009.

[19] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95** (2005) 261602.

[20] L. Da Rold and A. Pomarol, Nucl. Phys. B **721** (2005) 79.

[21] C. T. Hill, Phys. Rev. D **73** (2006) 126009.

[22] H. Grigoryan and A. Radyushkin, Phys. Rev. D**77** (2008) 115024; Phys. Rev. D **78** (2008) 115008.

[23] A. Gorsky and A. Krikun, Phys. Rev. D **79** (2009) 086015.

[24] S. J. Brodsky, F. -G. Cao and G. F. de Teramond, Phys. Rev. D **84** (2011) 075012.

[25] H. J. Kwee and R. F. Lebed, JHEP **0801** (2008) 027.

[26] M. Shifman, A. Vainshtein, V. Zakharov, Nucl. Phys. B**147** (1979) 385; Nucl. Phys. B**147** (1979) 448.

[27] J. Bijnens, E. Gamiz and J. Prades, JHEP **0110** (2001) 009; S. Narison, Phys. Lett. B **624** (2005) 223.

[28] P. Colangelo, J. J. Sanz-Cillero and F. Zuo, JHEP **1211** (2012) 012.

[29] M. Knecht, S. Peris, E. de Rafael, JHEP **1110**, 048 (2011).

[30] I. Iatrakis and E. Kiritsis, JHEP **1202** (2012) 064.

[31] A. Gorsky, P. N. Kopnin, A. Krikun and A. Vainshtein, Phys. Rev. D **85** (2012) 086006.

[32] L. Cappiello et al., Phys. Rev. **D82** (2010) 095008; O. Cata, AIP Conf. Proc. **1317** (2011) 328; S. K. Domokos et al., JHEP **1105** (2011) 107; R. Alvares et al., Phys. Rev. D **84** (2011) 095020.

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