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Testing the SM in $B \to D\tau\bar{\nu}$ decay with minimal theory input

Nejc Košnik*
Jožef Stefan Institute, Jamova 39, P. O. Box 3000, 1001 Ljubljana, Slovenia
E-mail: nejc.kosnik@ijs.si

Damir Bečirević
Laboratoire de Physique Théorique (Bât. 210),† Université Paris Sud, F-91405 Orsay-Cedex, France
E-mail: damir.becirevic@th.u-psud.fr

Andrey Tayduganov
Department of Physics, Graduate School of Science, Osaka University, Japan
E-mail: taydugan@lal.in2p3.fr

Recent experimental results for the ratio of the branching fractions of $\bar{B} \to D^{(*)}\tau\nu_\tau$ and $B \to D^{(*)}\mu\nu_\mu$ decays came as a surprise and lead to a discussion of possibility to constraining new physics through these modes. Here we focus on $Br(B \to D\tau\bar{\nu}_\tau)/Br(B \to D\mu\bar{\nu}_\mu)$ and argue that the result is consistent with the Standard Model within 2σ and that the test of compatibility of this ratio with the Standard Model can be done experimentally with a minimal theory input. We also show that these two decay channels can provide us with quite good constraints of the new physics couplings.

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1. Introduction

Recently BaBar collaboration measured the semileptonic branching fractions $B \to D\tau\bar{\nu}$ and $B \to D^*\tau\bar{\nu}$ that are above their Standard Model predictions [1]. The experiment reports

$$R(D) = \frac{\mathscr{B}(\bar{B} \to D\tau\bar{\nu}_\tau)}{\mathscr{B}(\bar{B} \to D\mu\bar{\nu}_\mu)} = 0.440 \pm 0.058_{\text{stat.}} \pm 0.042_{\text{syst.}}\,, \tag{1.1}$$

for the decay with pseudoscalar D and the result was normalized with respect to the decay with light lepton in the final state in order to cancel V_{cb} and form factor parameterization in the theoretical prediction of this observable. Before this result was published a prediction based on the lattice-calculated form factors had been made [2] ,

$$R(D)_{\text{SM}} = 0.296 \pm 0.016\,. \tag{1.2}$$

The discrepancy immediately raised interest in the flavor community to explain it within one of the well-motivated NP models. On the one hand, the SM charged-current contribution to this decay hints that possible NP contributions should be present at tree-level. On the other hand, the SM prediction requires the $B \to D$ form factors whose knowledge from the lattice is limited to high-q^2 region where phase space is small.

In this work we have revisited the theoretical prediction of $B \to D\tau\bar{\nu}$ in the SM in a manner that maximally employs the available experimental information on the form factors from previously measured $B \to D\ell\bar{\nu}$ where ℓ stands for e, μ [3]. Next, we parameterize beyond the Standard Model contributions to this decay mode in the effective Hamiltonian language, focusing on interactions that preserve lepton flavor universality and induce $b \to c$ transitions via scalar and tensor interactions. Assuming presence of either scalar either tensor operator we derive bounds on their respective Wilson coefficients at 1- and 2-σ level, taking into account experimental and theoretical uncertainties.

2. Differential decay width

In the SM the amplitude for hadronic transition $D \to P$ is given in terms of vector and scalar form factors, $F_+(q^2)$ and $F_0(q^2)$, defined as

$$\langle D(p')|\bar{c}\gamma_\mu b|\bar{B}(p)\rangle = \left(p_\mu + p'_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu\right) F_+(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0(q^2)\,. \tag{2.1}$$

The momentum transfer is denoted by $q = p - p'$ and when squared coincides with the invariant mass of the leptons, $q^2 = (p_{\bar{\nu}} + p_\ell)^2$. In the SM, the differential width of $B \to D\ell\bar{\nu}$ decay valid for finite lepton mass m_ℓ is

$$\begin{aligned}\frac{d\mathscr{B}(\bar{B} \to D\ell\bar{\nu}_\ell)}{dq^2} &= \tau_{B^0} \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} \left[c_+^\ell(q^2)|F_+(q^2)|^2 + c_0^\ell(q^2)|F_0(q^2)|^2\right] \\ &= |V_{cb}|^2 \mathscr{B}_0 |F_+(q^2)|^2 \left[c_+^\ell(q^2) + c_0^\ell(q^2)\left|\frac{F_0(q^2)}{F_+(q^2)}\right|^2\right]\,,\end{aligned} \tag{2.2}$$

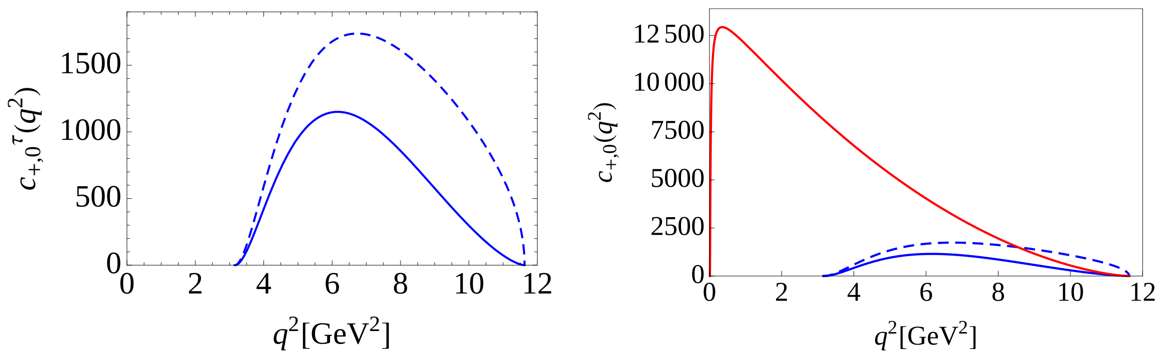

Figure 1: Vector and scalar form factor weight functions for decay $\bar{B} \to D\tau\bar{\nu}_\tau$ are shown in the left plot. Dashed curve corresponds to $c_0^\tau(q^2)$, while the solid curve corresponds to $c_+^\tau(q^2)$. In the right plot the same $c_{+,0}^\tau(q^2)$ are plotted together with $c_+^\mu(q^2)$ (red curve), the weight function in the case of massless muon in the final state.

where

$$c_+^\ell(q^2) = \lambda^{3/2}(q^2, m_B^2, m_D^2)\left[1 - \frac{3}{2}\frac{m_\ell^2}{q^2} + \frac{1}{2}\left(\frac{m_\ell^2}{q^2}\right)^3\right],$$
$$c_0^\ell(q^2) = m_\ell^2\,\lambda^{1/2}(q^2, m_B^2, m_D^2)\frac{3}{2}\frac{m_B^4}{q^2}\left(1 - \frac{m_\ell^2}{q^2}\right)^2\left(1 - \frac{m_D^2}{m_B^2}\right)^2,$$
$$\lambda(q^2, m_B^2, m_D^2) = [q^2 - (m_B + m_D)^2][q^2 - (m_B - m_D)^2]. \tag{2.3}$$

The overall factor $\mathscr{B}_0$ is defined by Eq. (2.2):

$$\mathscr{B}_0 = \frac{G_F^2}{192\pi^3 m_B^3}\tau_B. \tag{2.4}$$

The scalar form factor F_0 is multiplied by $c_0^\ell(q^2)$ that is nonzero only when the lepton in the final state is massive. Therefore only the $B \to D\tau\bar{\nu}$ mode is sensitive to exchanges of charged Higgs in the context of two Higgs doublet models or alternative scenarios that induce scalar operators [4, 5, 6, 7].

The q^2-dependence of the functions $c_+^\ell(q^2)$ and $c_0^\ell(q^2)$ are shown in Fig. 1. On the left-hand side plot one observes that the contribution of scalar form factor is enhanced by large $c_0^\tau(q^2)$ that partially compensates smallness of $F_0(q^2)$.

3. $B \to D\ell\bar{\nu}$ with minimal theory input

In this section we demonstrate, how one can maximally employ available experimental data to test for consistency of the SM in observable $R(D)$. Semileptonic decays with light lepton ($\ell = e, \mu$) have enabled extraction of $|V_{cb}|$. Usual approach in the literature has been to make the SM theoretical prediction of the q^2-spectrum at the maximal lepton recoil point $q^2_{\rm max} = (m_B - m_D)^2$, where the two hadrons are both at rest, and where the corrections to the heavy quark limit have been calculated. On the other hand, the experimental data close to $q^2_{\rm max}$ have virtually useless

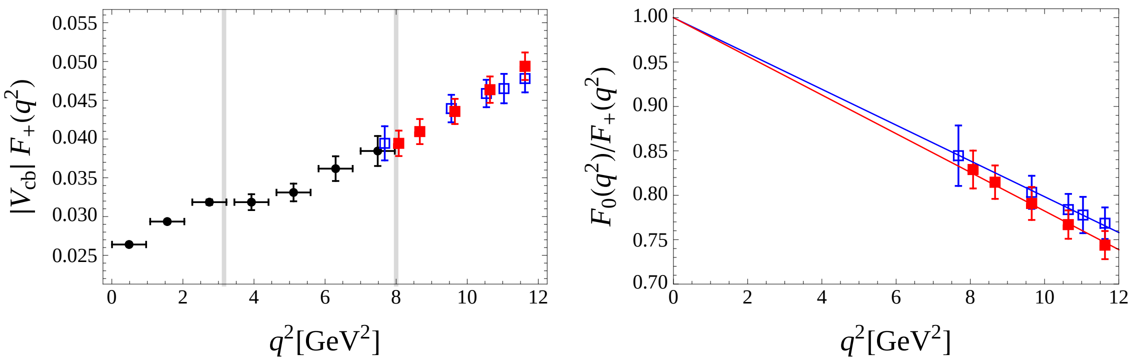

Figure 2: In the left plot we show the binned spectrum of $\bar{B} \to D\mu\bar{\nu}$ below $8\,\mathrm{GeV}^2$ (right vertical grey line) as measured by the BaBar collaboration [9]. Above $8\,\mathrm{GeV}^2$ lattice QCD simulations in quenched approximatian (empty symbols) [10], and those in which $N_\mathrm{f} = 2+1$ dynamical flavors are included (filled symbols) [11] are indicated. First grey line on the left denote the τ threshold at $q^2 = m_\tau^2$. Ratio of $B \to D$ form factors obtained in the same lattice simulations are shown on the right plot.

statistics due to small phase space, and experiments rely on the CLN shape of the form factor $F_+(q^2)$ to fit the differential decay spectrum to data at low to moderate q^2 and extrapolate it to q^2_max [8, 9].

In order to maximally use the experimental data we propose a different path for both light and heavy leptons in the final state of $B \to D\ell\bar{\nu}$. The vector form factor can be extracted from the BaBar measurement of the spectrum with light leptons in the final state. This procedure is viable below $q^2 = 8\,\mathrm{GeV}^2$, where the errors in bins are small [9]. Prediction of $B \to D\tau\bar{\nu}$ in $q^2 \in [8\,\mathrm{GeV}^2, q^2_\mathrm{max}]$ region will rely on the lattice QCD results for the vector form factor [10, 11] as seen in the left panel of Fig. 2. Notice that below the above procedure relies on the input value of V_{cb} only at high q^2 where lattice data is used.

Now we can express both branching fraction entering $R(D)$ as integrals over the three kinematical regions denoted in Fig. 2

$$\begin{aligned}\mathscr{B}(\bar{B} \to D\mu\bar{\nu}_\mu) &= \mathscr{B}_0 \int_{m_\mu^2}^{8\,\mathrm{GeV}^2} c_+^\mu(q^2)|V_{cb}F_+(q^2)|^2_\mathrm{exp}\,dq^2 \\ &\quad + |V_{cb}|^2 \mathscr{B}_0 \int_{8\,\mathrm{GeV}^2}^{q^2_\mathrm{max}} c_+^\mu(q^2)|F_{+,\mathrm{latt}}(q^2)|^2 dq^2\,, \end{aligned} \tag{3.1}$$

where $q^2_\mathrm{max} = (m_B - m_D)^2$. The phase space integral for decay with heavy τ contains the contribution of the scalar form factor and is also split at $q^2 = 8\,\mathrm{GeV}^2$

$$\begin{aligned}\mathscr{B}(\bar{B} \to D\tau\bar{\nu}_\tau) &= \mathscr{B}_0 \int_{m_\tau^2}^{8\,\mathrm{GeV}^2} |V_{cb}F_+(q^2)|^2_\mathrm{exp} \left[c_+^\tau(q^2) + c_0^\tau(q^2)\left|\frac{F_0(q^2)}{F_+(q^2)}\right|^2\right] dq^2 \\ &\quad + |V_{cb}|^2 \mathscr{B}_0 \int_{8\,\mathrm{GeV}^2}^{q^2_\mathrm{max}} |F_{+,\mathrm{latt}}(q^2)|^2 \left[c_+^\tau(q^2) + c_0^\tau(q^2)\left|\frac{F_0(q^2)}{F_+(q^2)}\right|^2\right] dq^2\,. \end{aligned} \tag{3.2}$$

We emphasize again that the partial decay rates at low-q^2 do not rely on the input value of V_{cb}, since $|V_{cb}F_+(q^2)|$ is available experimentally. A suitable observable with this pleasing property should be defined as a ratio of partial decay rates, i.e.,

$$R(D)\big|_{q^2\le 8\,\mathrm{GeV}^2} = \frac{\mathscr{B}(\bar{B}\to D\tau\bar{\nu}_\tau)}{\mathscr{B}(\bar{B}\to D\mu\bar{\nu}_\mu)}\bigg|_{q^2\le 8\ \mathrm{GeV}^2}, \tag{3.3}$$

and would depend only on $|F_0/F_+|$ that is well under control theoretically as we show below. The theoretical prediction of $R(D)$, as defined in Eq. (1.1), requires in addition the SM value of V_{cb} and calculation of the vector form factor at high q^2. For the latter we take the lattice results of Refs. [10, 11] and fit them to a dipole parameterization. The global fits of the SM to flavor observables yield $|V_{cb}| = 0.0411(16)$, a value that we use above $8\,\mathrm{GeV}^2$ [12].

The ratio of scalar-to-vector form factor is constrained to be 1 at $q^2 = 0$ by construction and indicates, together with high q^2 results of lattice QCD, a linear behaviour (see Fig. 2)

$$\frac{F_0(q^2)}{F_+(q^2)} = 1 - \alpha\, q^2\,. \tag{3.4}$$

We use the value $\alpha = 0.020(1)\,\mathrm{GeV}^{-2}$ that is consistent with different theoretical approaches (see e.g. [10, 11, 13, 14, 15]). The above procedure gives finally

$$R(D) = 0.31 \pm 0.02\,, \tag{3.5}$$

which is less than 2σ below the BaBar result (1.1).

4. Constraint on NP effective Hamiltonian

The SM effective Hamiltonian is extended to include operators that are scalars, tensors, or vectors, their dimensionless couplings labelled as g_S, g_T, and g_V, respectively. We assume that the lepton flavor universality is respected by all couplings on the Lagrangian level. We do not invoke operators that contain a right-handed neutrino.

$$\begin{aligned}\mathscr{H}_{\mathrm{eff}} = -\sqrt{2}G_F V_{cb} \quad & \big[(\bar{c}\gamma_\mu b)(\bar{\ell}_L\gamma^\mu \nu_L) + g_V(\bar{c}\gamma_\mu b)(\bar{\ell}_L\gamma^\mu\nu_L) \\ & + g_S(\mu)(\bar{c}b)(\bar{\ell}_R\nu_L) + g_T(\mu)(\bar{c}\sigma_{\mu\nu}b)(\bar{\ell}_R\sigma^{\mu\nu}\nu_L)\big] + \mathrm{h.c.}\end{aligned} \tag{4.1}$$

All couplings scale as $g_{V,S,T} \propto m_W^2/m_{\mathrm{NP}}^2$, and m_{NP} is the new physics scale. The differential decay width in the presence of NP operators is

$$\begin{aligned}\frac{d\mathscr{B}(\bar{B}\to D\ell\bar{\nu}_\ell)}{dq^2} = |V_{cb}|^2\ \mathscr{B}_0\ |F_+(q^2)|^2 \Bigg\{ & |1+g_V|^2 c_+^\ell(q^2) + |g_T(\mu)|^2 c_T^\ell(q^2)\left|\frac{F_T(q^2,\mu)}{F_+(q^2)}\right|^2 \\ & +\ c_{TV}^\ell(q^2)\,\mathrm{Re}\left[(1+g_V)g_T^*(\mu)\,\frac{F_T(q^2,\mu)}{F_+(q^2)}\right] \\ & +\ \left|(1+g_V) - \frac{q^2}{m_\ell}\frac{g_S(\mu)}{m_b(\mu)-m_c(\mu)}\right|^2 c_0^\ell(q^2)\left|\frac{F_0(q^2)}{F_+(q^2)}\right|^2\Bigg\}\,,\end{aligned} \tag{4.2}$$

where

$$\begin{aligned} c_T^\ell(q^2,\mu) &= \lambda^{3/2}(q^2,m_B^2,m_D^2)\frac{2q^2}{(m_B+m_D)^2}\left[1-3\left(\frac{m_\ell^2}{q^2}\right)^2+2\left(\frac{m_\ell^2}{q^2}\right)^3\right], \\ c_{TV}^\ell(q^2,\mu) &= \frac{6m_\ell}{m_B+m_D}\,\lambda^{3/2}(q^2,m_B^2,m_D^2)\left(1-\frac{m_\ell^2}{q^2}\right)^2.\end{aligned} \tag{4.3}$$

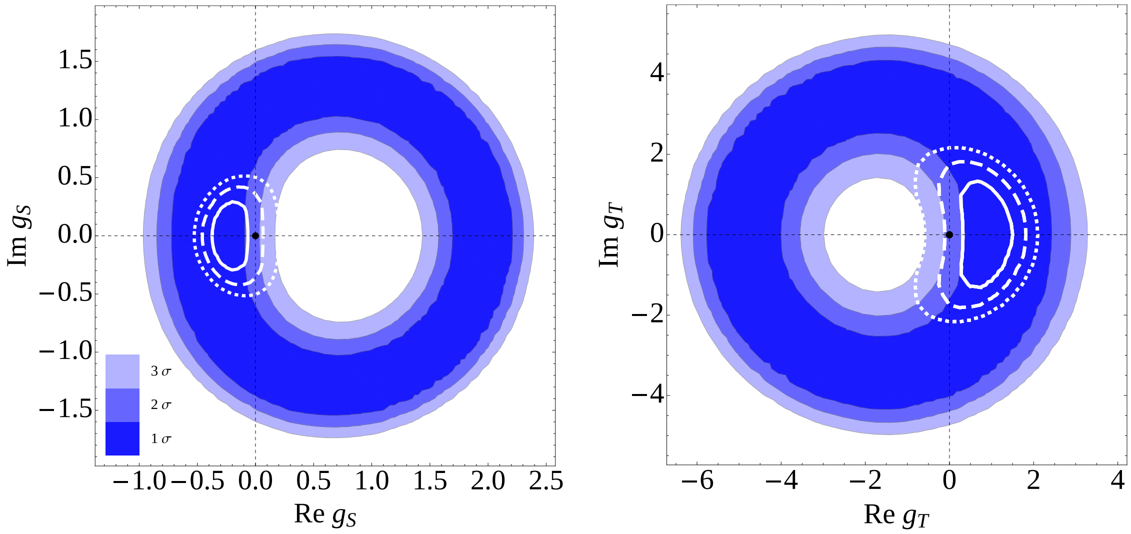

Figure 3: Regions of allowed values for $g_S(m_b)$ and $g_T(m_b)$, compatible with experimentally measured $R(D)$. The small region within the solid, dashed and dot-dashed white curves correspond to the respective 1-, 2- and 3-σ compatibility with both $\mathscr{B}^{(\mathrm{exp})}(\bar{B} \to D\tau\bar{\nu}_\tau)$ and $\mathscr{B}^{(\mathrm{exp})}(\bar{B} \to D\mu\bar{\nu}_\mu)$. The thick dot represents the Standard Model, namely $g_{S,T}(m_b) = 0$.

The tensor form factor $F_T(q^2,\mu)$ is defined as

$$\langle D(p')|\bar{c}\sigma_{\mu\nu}b|\bar{B}(p)\rangle = -i\left(p_\mu p'_\nu - p'_\mu p_\nu\right)\frac{2\,F_T(q^2,\mu)}{m_B+m_D}\,. \tag{4.4}$$

While the vector nature of weak currents has been thouroughly tested and are compatible with $g_V = 0$, scalar $g_S(\mu)$ and tensor $g_T(\mu)$ operators are not as constrained. $g_S(\mu) \neq 0$ induces the left-right operator and lifts the helicity suppression in $\bar{B} \to D\tau\bar{\nu}$ and $\bar{B} \to D\mu\bar{\nu}$ decays. Same operator is enhanced by the factor m_D^2/m_c^2 with respect to the left-left (SM) contribution to the $D^0 - \bar{D}^0$ mixing amplitude. Furthermore a noticable effect could also be seen in $D \to V\gamma$ decays that are governed by loops containing the down-type quarks and are therefore sensitive to $g_S(\mu) \neq 0$ [16].

Using $R(D)$ alone we get a very loose constraint on $g_S(m_b)$ while we require $g_V = g_T(m_b) = 0$ (contours on the left-hand plot in Fig. 3). Requiring in addition the compatibility of the theoretical expression for $\mathscr{B}(\bar{B} \to D\mu\bar{\nu}_\mu)$ obtained by using Eq. (4.2) and the measured value [9], restricts the allowed $g_S(m_b)$ to a small region also indicated in Fig. 3. For example, when $g_S(m_b)$ is real then the 1σ compatibility with experiment allows $-0.37 \leq g_S(m_b) \leq -0.05$, while the 3σ compatibility amounts to $-0.53 \leq g_S(m_b) \leq +0.20$. The statistical error in Eq. (1.1) is treated as Gaussian, while the systematic errors and uncertainties with respect to the form factors are treated as uniform.

If we allow for $g_T(m_b) \neq 0$ in Eq. (4.2) then the possible values that are compatible with $R(D)$ are those in the contour plot shown in the right-hand plot of Fig. 3. The needed tensor form factor has not been computed, to our knowledge, on the lattice nor in the QCD sum rules. The computation in the model of ref. [13] shows that $F_T(q^2)/F_+(q^2) = 1.03(1)$ is a constant, in agreement with naive expectations based on the pole dominance. As before, $R(D)$ alone is not constraining strongly the possible values of $g_T(m_b)$ whereas taking into account the constraint from measured $\mathscr{B}(\bar{B} \to D\mu\bar{\nu}_\mu)$ [9] shrinks the allowed region, as shown in the right-hand plot in Fig. 3.

If Im $g_T(m_b) = 0$, we obtain $0.3 \le g_T(m_b) \le 1.5$ and $-0.6 \le g_T(m_b) \le 2.1$, from the requirement of respective 1- and 3σ compatibility with both experimental $R(D)$ and $\mathscr{B}(\bar{B} \to D\mu\bar{\nu}_\mu)$.

5. Conclusions

The result for $R(D)$ could be an indication of new physics should the significance of incompatibility with the Standard Model raise to at least 3σ. The compatibility with the Standard Model can be tested experimentally, with a minimal hadronic input, as discussed in this letter. Here we used the lattice QCD results for $F_+(q^2)$ at larger q^2's because the full branching fractions were reported in Ref. [1]. Thus we have found $R(D) = 0.31 \pm 0.02$ where the significance of the discrepancy with Eq. (1.1) to be below 2σ.

If, instead of comparing the full branching fractions of both decay modes, the experimenters cut at about $q^2 \approx 8$ GeV^2, then the shape of the needed vector form factor including the factor of $|V_{cb}|$ could be reconstructed from the differential branching fraction of $\bar{B} \to D\mu\bar{\nu}$ [4, 5]. The only theoretical hadronic quantity needed then is the slope of the form factor ratio (3.4), which is quite accurately known from lattice QCD with the values that agree with quark models and with recent QCD sum rule studies. By using the vector form factor multiplied by $|V_{cb}|$ data from Ref. [9] only, and by integrating the decay rates up to $q^2_{\rm cut} = 8$ GeV^2, we obtain

$$\left.\frac{\mathscr{B}(\bar{B} \to D\tau\bar{\nu}_\tau)}{\mathscr{B}(\bar{B} \to D\mu\bar{\nu}_\mu)}\right|_{q^2 \le 8\ \mathrm{GeV}^2} = 0.20 \pm 0.02\,. \tag{5.1}$$

Allowing for departures from the Standard Model, while keeping lepton flavor universality which has been experimentally verified to a very good accuracy [17], the measured $R(D)$ and $\mathscr{B}(\bar{B} \to D\mu\bar{\nu}_\mu)$ impose quite strong constraints on the new physics scalar and tensor effective couplings $g_{S,T}(m_b)$.

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