

## Measurement of the strong coupling $\alpha_S$ from the three-jet rate in $e^+e^-$ -annihilation using JADE data

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A measurement of the strong coupling  $\alpha_S$  using the three-jet rate determined with the Durham algorithm in  $e^+e^-$ -annihilation and data taken with the JADE experiment at centre-of-mass energies between 14 and 44 GeV is presented. Improved theoretical predictions of the three-jet rate at next-to-next-to-leading order are matched with next-to-leading logarithmic approximations for the first time. From a comparison between the theoretical predictions and data the value for the strong coupling is determined. The fit returns a combined value of  $\alpha_S(M_{Z^0}) = 0.1199 \pm 0.0010(\text{stat.}) \pm 0.0021(\text{exp.}) \pm 0.0054(\text{had.}) \pm 0.0007(\text{theo.})$ , being consistent with the world average value.

*Xth Quark Confinement and the Hadron Spectrum,  
October 8-12, 2012  
TUM Campus Garching, Munich, Germany*

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## 1. Introduction

Within the Standard Model of particle physics strong interactions are described by Quantum Chromodynamics (QCD), with the strong coupling  $\alpha_S$  being the only free parameter of this fundamental theory. The measurement of  $\alpha_S$  is therefore one of the primary goals of experimental investigations of the Standard Model. The analysis being summarised in this contribution determines the strong coupling  $\alpha_S$  by measuring the three-jet event rate in  $e^+e^-$ -annihilation events. In first order perturbative QCD the radiation of a gluon from a quark is proportional to  $\alpha_S$ , leading to a three-parton final state. This three-parton final state is transformed during the hadronisation phase into a three-jet final state, which can experimentally be observed in the detector. Besides this first order description of radiating a single gluon from a quark higher order corrections significantly disturbs the three-jet rate.

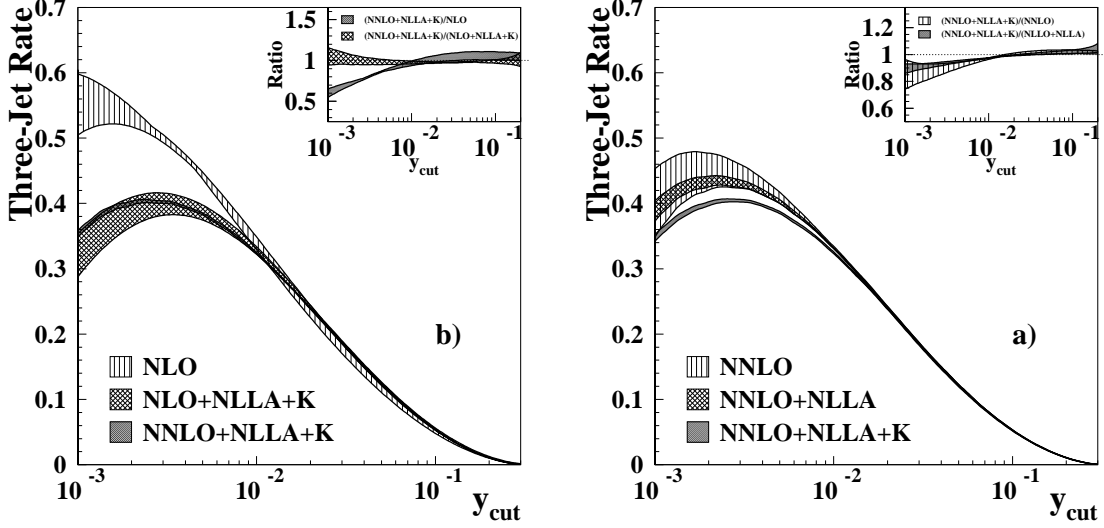
During the past years theoretical improvements were achieved and the three-jet rate can now be predicted with much higher theoretical accuracy. We use next-to-next-to-leading order predictions (NNLO) [1, 2, 3, 4] matched to next-to-leading-logarithm predictions (NLLA) and estimate the missing subleading terms with a K-term. This measurement is the first measurement of  $\alpha_S$  using the three-jet rate together with NNLO+NLLA+K QCD predictions. We use data taken in the years between 1979 and 1986 with the JADE experiment at the PETRA collider at DESY. The centre-of-mass energy varies between 14 and 44 GeV. A full description of the analysis can be found in [5] and references therein.

## 2. Observable and Theoretical Predictions

Several possibilities exist to bundle particles reconstructed in the detector to jets in order to classify a multi-jet event. An approach commonly used in  $e^+e^-$ -annihilation studies, the Durham clustering algorithm [6], clusters particles with respect to their relative transverse momentum. The three-jet event rate is predicted as a function of the transition parameter  $y_{\text{cut}}$ . The Durham clustering approach is collinear and infrared safe, which allows a secure theoretical treatment. The three-jet event rate evaluated with the Durham clustering scheme can be predicted with next-to-next-to-leading order QCD calculations (NNLO). For low  $y_{\text{cut}}$ -values the smallness of the expansion parameter is no longer given. An all order resummation of leading and next-to-leading logarithm (NLLA) contributions is applied to overcome this problem. Missing sub-leading logarithms can be controlled and added with an additional K-term. The fixed order prediction is matched to the resummation calculation resulting in NNLO+NLLA+K QCD predictions. Missing higher order terms are taken into account by the renormalisation scale parameter  $x_\mu = \mu/\sqrt{s}$ . The QCD predictions for the quarks and gluons (parton level) are shown in Fig. 1.

## 3. Data and Monte Carlo

The data used in this analysis were collected with the JADE detector located at the PETRA  $e^+e^-$ -collider at DESY. We analyse hadronic events recorded at centre-of-mass energies of 14, 22, 34.6, 35, 38.3 and 43.8 GeV. Overall about 43000 hadronic events are used with the data being collected at a centre-of-mass energy of 34.6 and 35 GeV dominating the statistics of the sample.



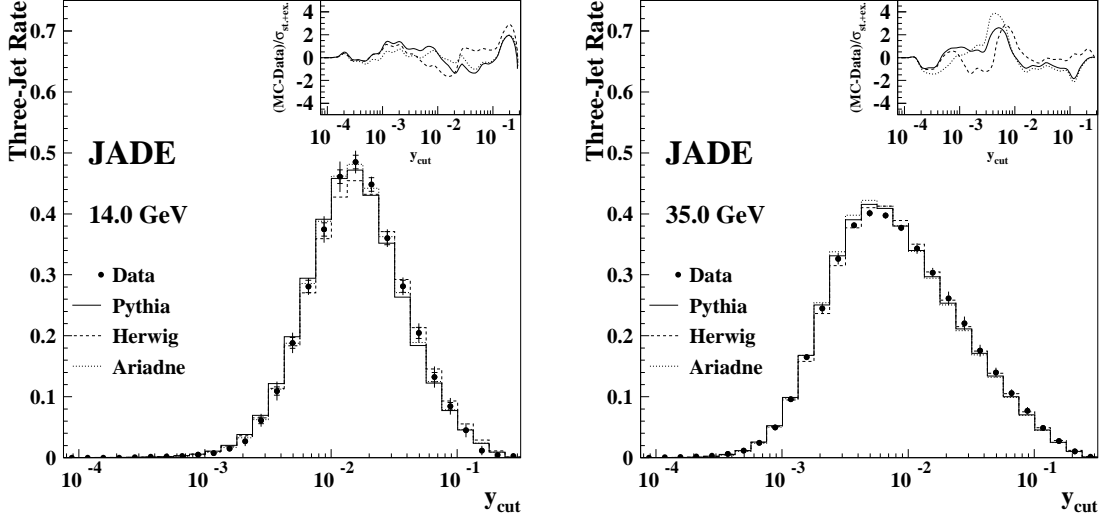
**Fig. 1:** The QCD prediction is shown for the three-jet rate using the Durham clustering scheme for different levels of precision evaluated for  $\alpha_S=0.1180$  at a centre-of-mass energy of 35 GeV. The uncertainty band indicates the change in the prediction originating from setting the renormalisation scale parameter  $x_\mu=\mu/\sqrt{s}$  to 0.5 and 2.0. The left plot shows the three-jet rate calculated with next-to-leading order prediction (NLO), matched next-to-leading order prediction (NLO+NLLA+K) and matched next-to-next-to-leading order calculations (NNLO+NLLA+K). The right plot shows the predictions using next-to-next-to-leading order prediction (NNLO) compared to matched next-to-next-to-leading order calculation with (NNLO+NLLA+K) and without K-term (NNLO+NLLA).

The events are corrected for acceptance and resolution effects originating from the incompleteness of the JADE detector. We use Monte Carlo (MC) events including a complete simulation of the detector. The Monte Carlo generators (PYTHIA, HERWIG and ARIADNE) are tuned with data taken by the OPAL detector at a centre-of-mass energy of 91 GeV. Fig. 2 shows a comparison of the three-jet event rate corrected for resolution and acceptance effects compared to the three different MC generators used in this analysis. In general the data is very well reproduced by the predictions from the various MC generators. Besides the correction for acceptance and resolution effects simulated events are used to determine the impact on the three-jet event rate due to the transition from partons to hadrons. The experimental measured event rate distribution is obtained with hadrons, while the QCD NNLO+NLLA+K calculation predicts the event rate at parton level only.

#### 4. Results

Hadronic events are selected with a cut based method using the number of charged tracks, the momentum imbalance and the total visible energy. This selection collects a very large fraction of all hadronic events keeping the number of hadronic tau events and events from photon-photon scattering at an insignificant level. The expected contribution to the three-jet event rate from  $b\bar{b}$ -events is removed using simulated events.

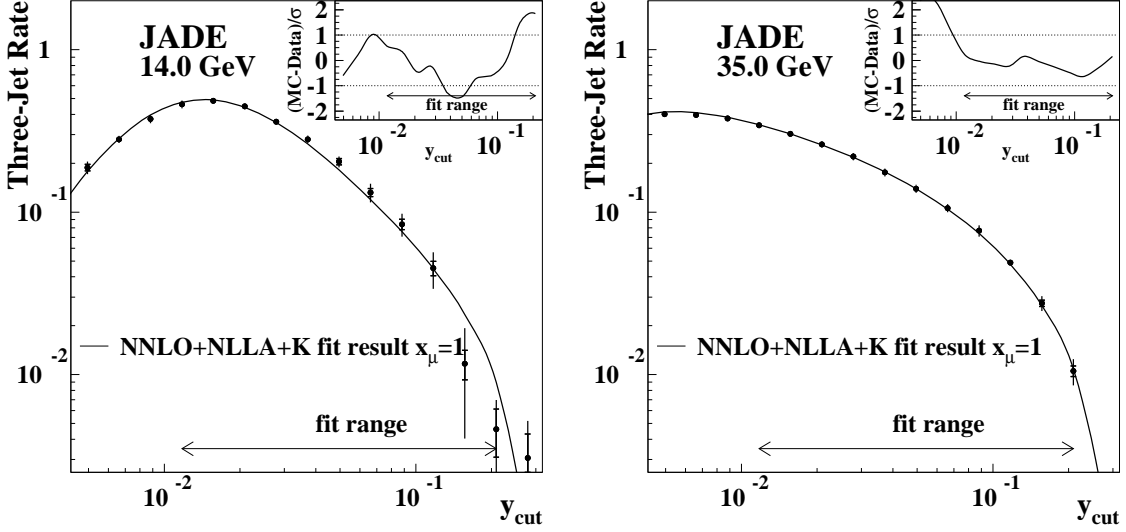
The strong coupling  $\alpha_S$  is determined by minimising a  $\chi^2$ -value obtained from the difference



**Fig. 2:** The three-jet event rate at a centre-of-mass energy of 14 and 35 GeV corrected for acceptance and resolution effects compared to the prediction from simulated events using PYTHIA, HERWIG or ARIADNE. The expected contribution from  $b\bar{b}$  is subtracted.

between the observed three-jet rate and the QCD prediction, with the strong coupling  $\alpha_S$  being the only free parameter. The fit range is determined by requiring the correction for acceptance and resolution effects and the correction for hadronisation effects to be small (less than 30%). Additionally we ask the leading logarithm terms in the low  $y_{\text{cut}}$ -region to be well below unity, leading to the lower limit of the fit range to be 0.01. The correlation between the  $y_{\text{cut}}$ -bins is taken into account by the correlation matrix determined with simulated events. The NNLO+NLLA+K calculation predicts the three-jet event rate for a quarks and gluon final state only. The expected change in the three-jet event rate originating from the transition from partons to hadrons is estimated with simulated events and applied to the QCD prediction. A  $\alpha_S$ -value is determined for each centre-of-mass energy separately. A comparison between the obtained fit result and the measured three-jet event rate is shown in Fig. 3. In general the data is very well described by the QCD prediction evaluated with the  $\alpha_S$ -value obtained from the minimisation.

The systematic uncertainties are subdivided into three different categories: uncertainties from experimental effects, incomplete theoretical predictions and effects from a wrong simulation of the transition from partons to hadrons. The experimental uncertainty is dominated by comparing the result obtained with two different data sets based on a different calibration and by evaluating the correction from resolution and acceptance effects with two different MC generators. The theoretical uncertainty is calculated by repeating the fit with the renormalisation scale parameter  $x_\mu$  in the QCD predictions being set to 0.5 and 2.0. The uncertainty originating from modelling the transformation from partons to hadrons is estimated by using different MC generators. For the default fit the transition is modelled with events simulated with PYTHIA, while for the systematic uncertainty the fit is repeated with events simulated by HERWIG or ARIADNE. The variation using HERWIG returns the larger deviation from the default fit and also dominates the overall uncertainty.



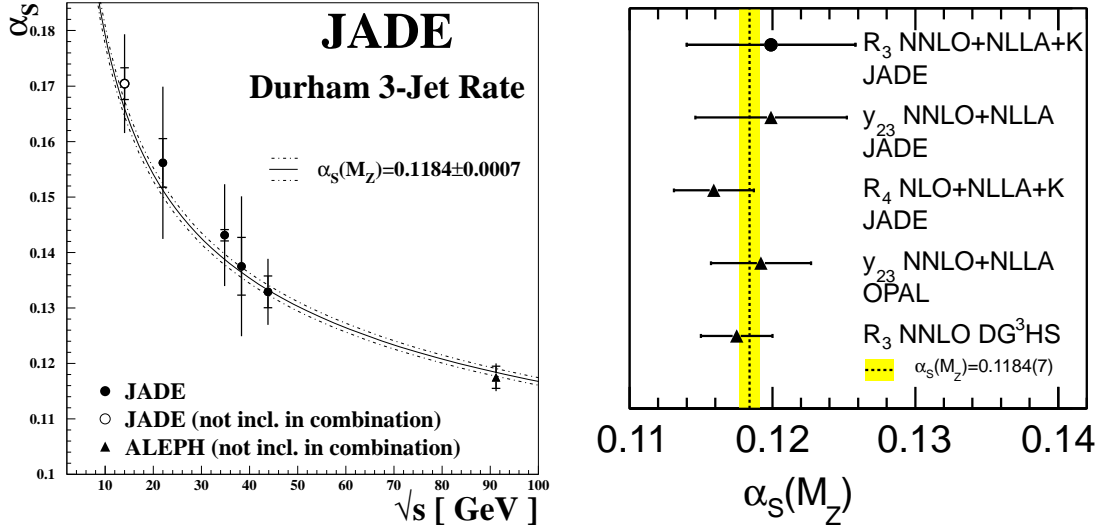
**Fig. 3:** The data in comparison to the fit result is shown for the three-jet event rate measured at a centre-of-mass energy of 14 and 35 GeV.

The  $\alpha_S$ -values obtained at five different centre-of-mass energies are combined to a single  $\alpha_S$ -value. The  $\alpha_S$ -value measured with data taken at 14 GeV is not used for the combination due to large corrections necessary to model the transition from partons to hadrons. The correlation between the  $\alpha_S$ -values determined at the various energy points is taken into account by using a minimum overlap method. The combined result is  $\alpha_S(M_{Z^0}) = 0.1199 \pm 0.0010(\text{stat.}) \pm 0.0021(\text{exp.}) \pm 0.0054(\text{had.}) \pm 0.0007(\text{theo.})$ . The value for  $\alpha_S$  is consistent with the world average value [7]. The  $\alpha_S$ -value obtained at the different centre-of-mass energies together with the  $\alpha_S$  world average value of  $\alpha_S$  is shown in Fig 4.

## 5. Consistency check

The default fit is performed with matched NNLO+NLLA+K calculations and the renormalisation scale parameter  $x_\mu$  being fixed to the natural choice  $x_\mu=1$ . For a consistency check the fit is repeated with the strong coupling  $\alpha_S$  and the renormalisation scale parameter both being varied within the fit. The combined measurement of  $\alpha_S$  returns  $\alpha_S(M_{Z^0}) = 0.1204 \pm 0.0009(\text{stat.}) \pm 0.0021(\text{exp.}) \pm 0.0059(\text{had.}) \pm 0.0008(\text{theo.})$ , consistent with the result obtained from the default method. The value for the renormalisation scale parameter returned by the fit is consistent with the range used in the default fit to estimate the theoretical uncertainty.

The measurement is also repeated with NNLO QCD predictions only, both with the renormalisation scale parameter  $x_\mu$  set to one and being varied within the fit. The fit with a fixed renormalisation scale parameter returns significant worse  $\chi^2$ -values and shows a larger sensitivity to the fit range. In the fit where  $\alpha_S$  as well as  $x_\mu$  is being varied, the description of the data is significantly improved compared to a fit with fixed order predictions only. However, the  $x_\mu$ -values returned from the fit are small and not consistent with the range used in the  $x_\mu$ -variation for estimating the theoretical uncertainty in the default fit.



**Fig. 4:** Left: The  $\alpha_S$ -values obtained at different centre-of-mass energies are shown together with the world average value [7]. The data point at 14 GeV is not used for the  $\alpha_S$  combination and the result obtained at 91 GeV is taken from [8]. For clarity the results obtained at 34.6 and 35 GeV are combined to a single entry. Right: Results obtained with measurements using the Durham clustering scheme in hadronic  $e^+e^-$ -annihilation events compared to this  $\alpha_S$ -measurement.

## 6. Summary

We present a measurement of the strong coupling  $\alpha_S$  using the three-jet event rate together with matched NNLO+NLLA+K QCD predictions. The three-jet event rate is determined at centre-of-mass energies of 14, 22, 34.6, 35, 38.3 and 43.8 GeV. The measured distributions are well described by simulated events. The result obtained at the different energy points are combined to a single value  $\alpha_S(M_{Z^0}) = 0.1199 \pm 0.0010(\text{stat.}) \pm 0.0021(\text{exp.}) \pm 0.0054(\text{had.}) \pm 0.0007(\text{theo.})$ , being consistent with the world average value [7]. A comparison of this measurement, together with previously performed measurements of the strong coupling using the Durham clustering scheme is shown in Fig. 4. The overall statistical and systematical uncertainty is dominated by applying different MC models to estimate the transition from partons to hadrons. The large available statistics together with the good experimental and theoretical control of the three-jet rate allows to use these kind of measurements to test and to validate models describing the hadronisation from partons into hadrons.

## Acknowledgments

This research was supported by the DFG cluster of excellence 'Origin and Structure of the Universe'.

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