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Endpoint Logarithms in $e^+e^- \to J/\psi + \eta_c$

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We investigate the origin of the double logarithms of Q^2/m_c^2 that appear in the calculation of the cross section for $e^+e^- \to J/\psi + \eta_c$ at next-to-leading order in the strong coupling α_s. Here, Q^2 is the square of the center-of-momentum energy, and m_c is the charm-quark mass. We find that, diagram-by-diagram, the double logarithms are accounted for by Sudakov double logarithms and endpoint double logarithms. The Sudakov double logarithms cancel in the sum over all diagrams, but the endpoint double logarithms do not. We reinterpret the endpoint double logarithms in terms of a leading region of loop integration in which a spectator fermion line becomes soft and collinear. This reinterpretation may simplify the process of establishing an all-orders factorization theorem for this helicity-flip process, which, in turn, might allow one to resum logarithms of Q^2/m_c^2 to all orders in α_s.

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1. Introduction

The exclusive double-charmonium production cross section $\sigma(e^+e^- \to J/\psi + \eta_c)$, which has been measured by the Belle [1, 2] and *BABAR* [3] collaborations, has provided serious challenges to the nonrelativistic QCD (NRQCD) factorization formalism [4, 5, 6, 7, 8]. The theoretical value for the cross section at leading order (LO) in the strong coupling α_s and the heavy-quark velocity v [9, 10] is almost an order of magnitude smaller than the measured rate. The discrepancy between theory and experiment seems to have been resolved by the inclusion of higher-order corrections, which include QCD corrections of next-to-leading order (NLO) in α_s [11, 12] and corrections of higher order in v (relativistic corrections) [13, 14, 15]. Here, v is the velocity of the heavy quark (Q) or heavy antiquark ($\bar{Q}$) in the quarkonium rest frame.

As the authors of Ref. [16] have pointed out, the large NLO QCD corrections involve double logarithms of Q^2/m_c^2, where Q^2 is the square of the e^+e^--center-of-momentum energy and m_c is the charm-quark mass. These logarithms are sufficiently large that it may be necessary to resum them to all orders in α_s in order to obtain a reliable theoretical prediction. Typically, the resummation of large logarithms is carried out through a factorization of the contributions that arise from small momentum scales from the contributions that arise from large momentum scales. A first step in deriving such a factorization is to identify the regions of loop-momentum integrals that produce the large logarithms at the leading nonzero power of Q^2 [17].

In this paper, we identify the leading loop-momentum regions that give rise to the double logarithms in the NLO QCD corrections to the process $e^+e^- \to J/\psi + \eta_c$. According to the NRQCD factorization formalism [4], the amplitude for this process at LO in v, can be written as a product of a short-distance coefficient with the NRQCD long-distance matrix elements (LDMEs) for the evolution of $Q\bar{Q}$ pairs into the J/ψ and the η_c. The same short-distance coefficient appears in the amplitude for the production of two $Q\bar{Q}$ pairs that have the same quantum numbers as the corresponding quarkonia. The short-distance coefficient can be obtained perturbatively by comparing the full-QCD amplitude $i\mathscr{A}[e^+e^- \to Q\bar{Q}_1(^3S_1) + Q\bar{Q}_1(^1S_0)]$ with the NRQCD amplitude, which consists of the short-distance coefficient times the NRQCD LDMEs for the $Q\bar{Q}$ states. (Here the subscripts 1 indicate that the $Q\bar{Q}$ pairs are in color-singlet states.) The double logarithms of Q^2/m_c^2 arise solely from the full-QCD amplitude because the NRQCD LDMEs for the $Q\bar{Q}$ states are insensitive to momentum scales of order m_c or larger. In Sec. 2, we carry out the calculation of the double logarithms of Q^2/m_c^2 at NLO in α_s for the process $e^+e^- \to J/\psi + \eta_c$ by examining the NLO QCD corrections to the full-QCD process. We find that the double logarithms arise from Sudakov or endpoint regions of loop momenta, and we identify the contribution that arises from each region for each NLO Feynman diagram. Our results for the double logarithms agree with those that were obtained in the complete NLO calculations [11, 12]. We find that the Sudakov double logarithms cancel in the sum over diagrams and that power-divergent contributions from the endpoint region vanish. In Sec. 3, we give a general analysis of the Sudakov double logarithms that elucidates the reason for their cancellation in the sum over diagrams. In Sec. 4, we give a general analysis of the endpoint region that establishes the absence of power-divergent contributions. We summarize our results in Sec. 5.

2. Calculation of double logarithms

In this section, we evaluate the double logarithms that appear in the NLO QCD corrections to the amplitude $e^+e^- \to J/\psi + \eta_c$, and we identify the momentum regions that are associated with the logarithms. The process $e^+e^- \to Q\bar{Q}_1(^3S_1) + Q\bar{Q}_1(^1S_0)$ is composed of $e^+e^- \to \gamma^*$, followed by $\gamma^* \to Q\bar{Q}_1(^3S_1) + Q\bar{Q}_1(^1S_0)$. Because the process $e^+e^- \to \gamma^*$ does not receive QCD corrections in relative order $\alpha^0\alpha_s$, we need to consider only the process $\gamma^* \to Q\bar{Q}_1(^3S_1) + Q\bar{Q}_1(^1S_0)$ in the evaluation of the NLO QCD corrections.

2.1 Kinematics, conventions, and nomenclature

Now we describe the kinematics, conventions, and nomenclature that we use in calculating the double logarithms and throughout this paper. We work in the Feynman gauge. We use the light-cone momentum coordinates $k = [k^+, k^-, \boldsymbol{k}_\perp] = [(k^0 + k^3)/\sqrt{2}, (k^0 - k^3)/\sqrt{2}, \boldsymbol{k}_\perp]$ and work in the e^+e^--center-of-momentum frame. Because our calculation is at LO in v, we set the relative momentum of the Q and $\bar{Q}$ in each charmonium equal to zero. Then, the momenta of the Q and $\bar{Q}$ in the J/ψ are both $p = [(\sqrt{P^2 + m_c^2} + P)/\sqrt{2}, (\sqrt{P^2 + m_c^2} - P)/\sqrt{2}, \mathbf{0}_\perp]$, and the momenta of the Q and $\bar{Q}$ in the η_c are both $\bar{p} = [(\sqrt{P^2 + m_c^2} - P)/\sqrt{2}, (\sqrt{P^2 + m_c^2} + P)/\sqrt{2}, \mathbf{0}_\perp]$, where P is the magnitude of the 3-momentum of any of the Qs or $\bar{Q}$s. The momentum of the virtual photon is $Q = 2(p + \bar{p})$, which implies that $Q^2 = 16(P^2 + m_c^2)$. If a momentum k has light-cone components whose orders of magnitude are $P\lambda[1, (\eta^+)^2, \eta^+]$, then we say that k is soft if $\lambda \ll 1$, and we say that k is collinear to plus if $\eta^+ \ll 1$. If k has light-cone components whose orders of magnitude are $P\lambda[(\eta^-)^2, 1, \eta^-]$, then we say that k is soft if $\lambda \ll 1$, and we say that k is collinear to minus if $\eta^- \ll 1$. Hence, p is collinear to plus and $\bar{p}$ is collinear to minus in the limit $m_c^2/P^2 \to 0$.

2.2 Evaluation of the diagrams

Now we calculate the double logarithms that arise from the Feynman diagrams that contribute to the NLO QCD corrections to the amplitude. The amplitudes for each diagram contain spin and color projectors that put the $Q\bar{Q}$ pairs into states of definite spin and color [18]. When the relative momentum of the Q and $\bar{Q}$ in each charmonium is zero, the spin-singlet and spin-triplet projectors are given by

$$\Pi_1(\bar{p}, \bar{p}) = -\frac{1}{2\sqrt{2}m}\gamma^5(\bar{\not{p}} + m_c), \tag{2.1}$$

$$\Pi_3(p, p, \lambda) = -\frac{1}{2\sqrt{2}m}\not{\varepsilon}^*(\lambda)(\not{p} + m_c), \tag{2.2}$$

where $\varepsilon^*(\lambda)$ is the polarization vector for the $Q\bar{Q}$ pair in the spin-triplet state.

The NLO diagrams that contain the double logarithms in Q^2/m_c^2 are shown in Fig. 1. Because the process $e^+e^- \to J/\psi + \eta_c$ does not satisfy quark-helicity conservation, its amplitude must contain a numerator factor m_c, which produces a helicity flip. Hence, the amplitude is suppressed by a factor of m_c/Q relative to a helicity-conserving amplitude. The factor m_c can come from the numerators of the quark propagators or from the numerators of the spin projectors. We must also retain a nonzero quark mass in denominators because logarithmic collinear and endpoint divergences that appear in the calculation are sensitive to that mass.

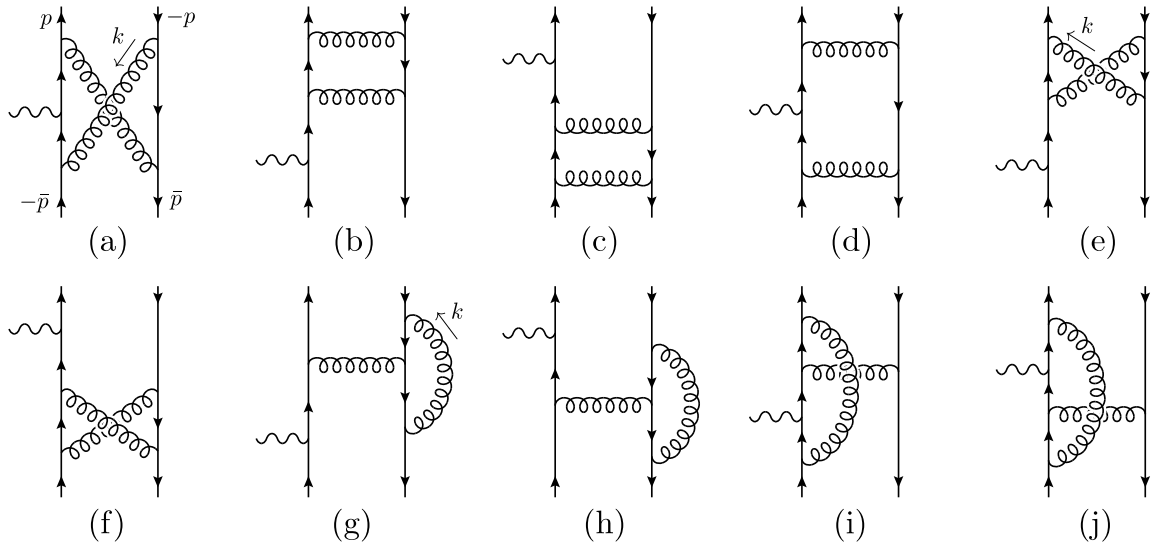

Figure 1: One-loop diagrams that produce double logarithms of Q^2/m_c^2. The upper $Q\bar{Q}$ pair corresponds to the J/ψ, and the lower $Q\bar{Q}$ pair corresponds to the η_c.

A straightforward calculation of the diagram in Fig. 1(a) gives

$$i\mathscr{A}_{\rm LO} \times \frac{-i\alpha_s\pi Q^2}{2}\left(C_F - \frac{1}{2}C_A\right) \times \left\{ \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2+i\varepsilon)[(k+p)^2-m_c^2+i\varepsilon][(k-\bar{p})^2-m_c^2+i\varepsilon]} + \int \frac{d^dk}{(2\pi)^d} \frac{1}{[(k+p+\bar{p})^2+i\varepsilon][(k+2p+\bar{p})^2-m_c^2+i\varepsilon][(k+p)^2-m_c^2+i\varepsilon]} + \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2+i\varepsilon)[(k+p)^2-m_c^2+i\varepsilon][(k+p+\bar{p})^2+i\varepsilon]} + \cdots \right\}, \tag{2.3}$$

where we retain only the terms that contain the double logarithms in Q^2/m_c^2. Here, $C_F = (N_c^2-1)/(2N_c)$, $C_A = N_c$, $d = 4-2\varepsilon$, and $i\mathscr{A}_{\rm LO}$ is the LO (order-α_s) contribution to the amplitude $i\mathscr{A}[\gamma^* \to Q\bar{Q}_1(^3S_1) + Q\bar{Q}_1(^1S_0)]$:

$$i\mathscr{A}_{\rm LO} = \frac{-i256\pi\alpha_s C_F}{m_c Q^4}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_\nu^*(\lambda)p_\alpha\bar{p}_\beta, \tag{2.4}$$

where μ is the polarization of the virtual photon. We use the nonrelativistic normalization for the spinors. $\varepsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor in 4 dimensions, for which we use the convention $\varepsilon_{0123} = 1$. We evaluate the first integral in Eq. (2.3) using dimensional regularization to control the soft divergence. We find that

$$\begin{aligned}\mathscr{S} &\equiv \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2+i\varepsilon)[(k+p)^2-m_c^2+i\varepsilon][(k-\bar{p})^2-m_c^2+i\varepsilon]} \\ &= \frac{i}{4\pi^2Q^2}\left\{ \left[\frac{1}{\varepsilon_{\rm IR}} - \log(m_c^2/\mu^2)\right]\log(m_c^2/Q^2) + \frac{1}{2}\log^2(m_c^2/Q^2) + \cdots \right\},\end{aligned} \tag{2.5}$$

where we retain only the terms that are singular in ε and the double logarithms.

We have carried out a detailed analysis of the first integral in Eq. (2.3) that makes use contour integration for the integration over k_0. That analysis shows that double logarithm in this integral comes from the region in which the gluon with momentum k is simultaneously soft and collinear. That is, it is a Sudakov double logarithm. The second integral in Eq. (2.3) is identical to the first integral if we change the loop momentum to $k+p+\bar{p}$. Hence, the second integral gives a Sudakov double logarithm that comes from the region in which the gluon with the momentum $k+p+\bar{p}$ is simultaneously soft and collinear. The last integral in Eq. (2.3) yields

$$\mathscr{E} \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2+i\varepsilon)[(k+p)^2-m_c^2+i\varepsilon][(k+p+\bar{p})^2+i\varepsilon]} = \frac{i}{8\pi^2 Q^2}\left[\log^2(m_c^2/Q^2)+\cdots\right], \tag{2.6}$$

where again we show only the double-logarithmic contributions. (In this integral, there is no singularity in ε.) A detailed analysis, which makes use of contour integration for the k_0 integration, shows that the double logarithm in this integral comes from the momentum region in which the gluons carry almost all of the collinear-to-plus and collinear-to-minus momenta from the spectator-quark line to the active-quark line. We call a contribution from this momentum region an endpoint contribution. An important observation is that $\mathscr{E}$ can be made to look similar to the integral that gives the Sudakov double logarithm by changing the loop momentum to the spectator-quark momentum $\ell = -k-p$. Then, we have

$$\mathscr{E} = \int \frac{d^4\ell}{(2\pi)^4} \frac{1}{(\ell^2-m_c^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon]}. \tag{2.7}$$

It follows that the endpoint double logarithm arises from the region in which the momentum of the spectator quark, ℓ, is soft and collinear. From Eq. (2.7), we see that the soft and collinear divergences are regulated by m_c. An analysis of Eq. (2.7) also shows that single logarithms of Q^2/m_c^2 can arise from the region in which the momentum of the spectator quark is soft.

Carrying out similar analyses of the remaining diagrams that contribute to the amplitude $\gamma^* \to Q\bar{Q}_1(^3S_1) + Q\bar{Q}_1(^1S_0)$ at order α_s^2, we find that the double logarithms in each diagram are accounted for by Sudakov double logarithms and endpoint double logarithms. The Sudakov and endpoint double logarithms that arise from each diagram are summarized in Table. 1. Our result for the sum of double logarithms in all of the NLO diagrams agrees with the results in Refs. [12, 16]. We also find that our results in the Feynman gauge for the double logarithm in each diagram agree with the results that were obtained in carrying out the calculation of Ref. [16].[1]

Our detailed calculations are consistent with two general properties of the Sudakov and endpoint singular regions: (1) the Sudakov double logarithms cancel in the sum over diagrams; (2) the endpoint region produces only logarithmic singularities, not power singularities. In the following sections, we show how these general properties arise from soft-collinear approximations that are valid in the Sudakov and endpoint regions.

3. General analysis of the Sudakov double logarithms

As we have mentioned, Sudakov double logarithms arise from a region in which the momentum of a gluon is simultaneously soft and collinear. We would like to apply a collinear approxima-

[1] We thank Yu Jia and Xiu-Ting Yang for providing us with the logarithmic contributions of the individual NLO Feynman diagrams.

Diagram	Endpoint double logarithm	Sudakov double logarithm
(a)	$(C_F - \frac{1}{2}C_A)\mathscr{E}$	$2(C_F - \frac{1}{2}C_A)\mathscr{S}$
(b)	$C_F\mathscr{E}$	0
(c)	$2\,C_F\mathscr{E}$	0
(d)	$\frac{1}{2}\,C_F\mathscr{E}$	0
(e)	$(C_F - \frac{1}{2}C_A)\mathscr{E}$	$(C_F - \frac{1}{2}C_A)\mathscr{S}$
(f)	0	$(C_F - \frac{1}{2}C_A)\mathscr{S}$
(g)	0	$-(C_F - \frac{1}{2}C_A)\mathscr{S}$
(h)	0	$-(C_F - \frac{1}{2}C_A)\mathscr{S}$
(i)	0	$-(C_F - \frac{1}{2}C_A)\mathscr{S}$
(j)	0	$-(C_F - \frac{1}{2}C_A)\mathscr{S}$

Table 1: Endpoint and Sudakov double logarithms in each diagram, in units of $i\mathscr{A}_{\rm LO} \times (-i\alpha_s\pi Q^2)/2$.

tion to such gluons. Consider, for example, Figs. 1(e) and (g), in which a gluon with momentum k that is collinear to minus is emitted from a quark line with momentum $\bar{p}$. Then, the quark-gluon vertex and the two propagator numerators surrounding it can be written as

$$(\bar{p}\!\!\!/ + m_c)\gamma^\mu(\bar{p}\!\!\!/ + k\!\!\!/ + m_c) = 2(\bar{p}^\mu + k^\mu)(\bar{p}\!\!\!/ + m_c) - k\!\!\!/\gamma^\mu m_c, \tag{3.1}$$

where μ is the polarization index of the gluon, we have used $k\!\!\!/\bar{p}\!\!\!/ \propto \bar{p}\!\!\!/\bar{p}\!\!\!/$ and $\bar{p}\!\!\!/\bar{p}\!\!\!/ = m_c^2$, and we have dropped terms of order m_c^2. In the collinear-to-minus approximation, one retains only the first of the two terms on the right side of Eq. (3.1). In the case of nonzero quark masses, this approximation is not valid in general. However, if k is soft in comparison to $\bar{p}$, as well as collinear, then we can drop the second term on the right side of Eq. (3.1), and the standard collinear approximation holds. [We can also drop k in the first term on the right side of Eq. (3.1).] Since the current in Eq. (3.1) now lies in the minus light-cone direction, up to terms of order m_c^2, we can make a collinear-to-minus approximation in the gluon propagator [19, 20, 21], by making the replacement in the polarization tensor

$$g_{\mu\nu} \to \frac{k_\mu p_\nu}{k\cdot p - i\varepsilon}, \tag{3.2}$$

where the index ν corresponds to the attachment of the gluon to the quark line that is collinear to minus and the sign of $i\varepsilon$ is fixed by the sign in the original Feynman diagram. This approximation is valid unless the μ attachment of the gluon is to a line that is also collinear to minus. Hence, the approximation always holds for the diagrams that produce Sudakov logarithms because the invariant Q^2 in the logarithm can appear only if the soft-collinear gluon connects a line carrying momentum $\bar{p}$ with a line carrying momentum p. The replacement (3.2) can also be regarded as a soft approximation [22, 23] to the μ attachment of the gluon. However, the collinear approximation can be more useful in applications other than the present one because its form is independent of the direction of the momentum p, while the form of the soft approximation is not.

For the diagram of Fig. 1(e) we can apply the soft-collinear approximation (3.2), where the index μ corresponds to the connection of the collinear-to-minus gluon to the active-quark line that

carries momentum p:

$$\bar{u}(p)\gamma^\nu \frac{1}{\not{p}-\not{k}-m_c+i\varepsilon} \to \bar{u}(p)\frac{\not{k}p^\nu}{k\cdot p-i\varepsilon}\frac{1}{\not{p}-\not{k}-m_c+i\varepsilon} = -\bar{u}(p)\frac{p^\nu}{k\cdot p-i\varepsilon}, \tag{3.3}$$

where, in the last equality, we have applied the graphical Ward identity (Feynman identity). Similarly, for the diagram of Fig. 1(g) we can apply the soft-collinear approximation (3.2), where the index μ corresponds to the connection of the collinear-to-minus gluon to the spectator-quark line that carries momentum p:

$$\frac{1}{-\not{p}+\not{k}-m_c+i\varepsilon}\gamma^\nu v(p) \;\to\; \frac{1}{-\not{p}+\not{k}-m_c+i\varepsilon}\frac{\not{k}p^\nu}{k\cdot p-i\varepsilon}v(p) = +\frac{p^\nu}{k\cdot p-i\varepsilon}v(p). \tag{3.4}$$

Because the outgoing $Q\bar{Q}$ pairs are in color-singlet states, these two contributions have the same color factor and cancel. This type of cancellation extends to all diagrams involving a gluon with soft-collinear-to-minus momentum. Similar cancellations occur for diagrams involving a gluon with soft-collinear-to-plus momentum. Therefore, in the sum of all diagrams, Sudakov double logarithms cancel.

4. General analysis of the endpoint region

As we have mentioned, the endpoint double logarithms of Q^2/m_c^2 arise from the region of loop integration in which the momentum ℓ of the internal spectator-quark line is simultaneously soft and collinear. Hence, we need to consider only diagrams that can produce such a momentum configuration. These diagrams are shown in Figs. 1(a)–(f). (A diagram and its charge conjugate give equal contributions to the amplitude. The charge-conjugate diagrams are not shown in Fig. 1.) The endpoint double logarithms arise from contributions in which there is a numerator factor m_c, which produces the helicity flip, and in which integrals diverge logarithmically in the limit $m_c \to 0$. In general, integrals can also diverge as inverse powers of m_c in the limit $m_c \to 0$, but, as we shall see, such contributions vanish when the numerator trace is taken.

In diagrams (a), (e), and (f), the momenta of the propagators on the active-quark lines contain both p and $\bar{p}$. Since $p\cdot\bar{p} \sim P^2 \sim Q^2$, we can ignore ℓ in the denominators of those propagators. In the limit $m_c \to 0$, the two gluon-propagator denominators and spectator-quark-propagator denominator produce factors $1/(\ell^2+2p\cdot\ell)$, $1/(\ell^2-2\bar{p}\cdot\ell)$, and $1/\ell^2$, respectively, where we have dropped the m_c^2 terms in the propagator denominators. Hence, in order to obtain a logarithmically divergent soft power count (λ^{-4}) and logarithmically divergent collinear power counts [$(\eta^\pm)^{-4}$], we must drop all numerator factors of ℓ. This implies that the helicity flip comes from the factor m_c in the numerator of the spectator-quark propagator.

In diagram (b), the momentum of the outermost active-quark propagator contains the momentum p, but not the momentum $\bar{p}$. Hence, in the limit $m_c \to 0$, the denominator of this propagator produces a factor $1/(\ell^2+4p\cdot\ell)$. Then, all of the propagator denominators taken together produce a linearly divergent soft power count and a linearly divergent collinear-to-plus power count. However, it is easy to see that numerator factors reduce both of these power counts to logarithmic ones. First, we rewrite the numerator factors that are associated with the outermost gluon and the spin projector for the J/ψ $Q\bar{Q}$ pair as $\gamma_\mu(\not{p}-m_c)\not{\varepsilon}^*\gamma^\mu = 2m_c\not{\varepsilon}^*$, where we have used the fact that

$p \cdot \varepsilon^* = 0$. Now, because the numerator power of m_c is in this factor, the numerator of the spectator-quark propagator must contribute a factor $\not{\ell}$. Furthermore, the numerator vanishes, up to terms of order m_c^2, if ℓ is proportional to p because there are then two factors of $\not{p}$ that are separated by γ matrices with which they anticommute. Hence, in the trace over the γ matrices, ℓ must appear in the combination $\ell \cdot p$. This numerator factor reduces both the soft and collinear power counts to logarithmic ones.

In the case of diagram (c), the denominator of the outermost active-quark propagator contributes a factor $1/(\ell^2 - 4\bar{p}\cdot\ell)$. Hence, the soft and collinear-to-minus power counts from the propagator denominators are both linearly divergent. Now, we rewrite the numerator factors that are associated with the outermost gluon and the spin projector for the η_c $Q\bar{Q}$ pair as $\gamma_\mu(-\not{\bar{p}} - m_c)\gamma_5\gamma^\mu = (-2\not{\bar{p}} + 4m_c)\gamma_5$. If the numerator factor m_c comes from the spectator-quark propagator, then there must be a factor $\not{\ell}$ from the outermost active-quark propagator, or else there are two adjacent factors of $\not{\bar{p}}$ and the numerator vanishes, up to terms of order m_c^2. If the numerator factor m_c does not come from the spectator-quark propagator, then that propagator must yield a numerator factor $\not{\ell}$. In both cases, the numerator factor vanishes, up to terms of order m_c^2, if ℓ is proportional to $\bar{p}$ because, in that case, there are two factors of $\not{\bar{p}}$ that are either adjacent or are separated by γ matrices with which they anticommute. Therefore, we conclude that the trace contains a factor $\ell \cdot \bar{p}$, which reduces both the soft and collinear-to-minus power counts to logarithmic ones.

In the case of diagram (d), the denominators of the active-quark propagators produce factors $1/(\ell^2 + 4p\cdot\ell)$ and $1/(\ell^2 - 4\bar{p}\cdot\ell)$. Hence, the propagator denominators, taken together, produce a quadratically divergent soft power count and linearly divergent collinear-to-plus and collinear-to-minus power counts. One can apply the arguments that were used for the numerators of diagrams (b) and (c) separately to each of the gluons in diagram (d), with the conclusion that the numerator contains two factors of $\not{\ell}$ and that the numerator vanishes, up to terms of order m_c^2, if ℓ is proportional to p or to $\bar{p}$. Hence, the trace contains a factor $\ell \cdot p\, \ell \cdot \bar{p}$ or a factor ℓ^2. In either case, the numerator factor reduces the soft and collinear power counts to logarithmic ones.

In the case of one-loop corrections to helicity-conserving charmonium-production processes, in which there is no numerator factor m_c, the previous arguments show that the contribution from the region in which the spectator quark carries a soft-collinear momentum vanishes, implying that there are no endpoint double logarithms. This general conclusion is confirmed in explicit calculations [16, 24, 25].

5. Summary

In this work we have investigated the origin of the double logarithms of Q^2/m_c^2 that appear in the NLO QCD corrections to the process $e^+e^- \to J/\psi + \eta_c$. We find that the double logarithms in each diagram are accounted for by Sudakov double logarithms and endpoint double logarithms. The Sudakov double logarithms cancel in the sum of all diagrams. We have given a general argument for this cancellation that is based on the soft-collinear approximation and graphical Ward identities. We have reinterpreted the region of a loop integration that gives rise to an endpoint double logarithm as a leading region in which the momentum of the spectator-quark line is both soft and collinear. Under this reinterpretation, we would also expect single logarithms of Q^2/m_c^2 to arise from a leading region in which the momentum of the spectator-quark line is soft. This

reinterpretation may be useful in establishing an all-orders factorization theorem for helicity-flip quarkonium production. Such a factorization theorem might allow one to resum logarithms of Q^2/m_c^2 to all orders in α_s for helicity-flip processes. We have also given a power-counting analysis of the one-loop endpoint contributions, which shows that they can arise only in the presence of a helicity flip and that the loop integration can produce logarithms, but not inverse powers, of the heavy-quark mass m_c.

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