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Direct determination of strange and light quark condensates from full lattice QCD

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We determine the strange and light quark condensates in full lattice QCD for the first time. This is done by direct calculation of the expectation value of the trace of the quark propagator followed by subtraction of the appropriate perturbative contribution to convert to a value for the condensate in the $\overline{MS}$ scheme at 2 GeV. We use lattice QCD configurations including u, d, s and c quarks in the sea with u/d quark masses going down to the physical value. We find the ratio of the strange to the light quark condensate to be 1.08(16).

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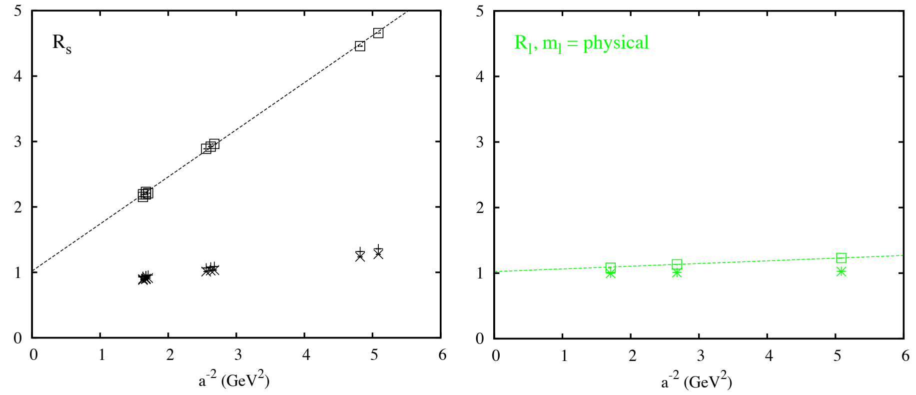

Figure 1: Results for R_q, defined in Eq. 2.1, as a function of the square of the inverse lattice spacing. The plot on the left is for s quarks and on the right for light quarks at the physical light quark mass ($m_l = (m_u + m_d)/2$). Open squares give raw results and pluses have tree-level perturbative corrections. Crosses have in addition one-loop perturbative corrections, using $\alpha_V^{(n_f=4)}(2/a)$. Dashed lines give a simple linear fit to the raw results.

1. Introduction

The condensation of quark-antiquark pairs in the vacuum signalling the breakdown of chiral symmetry is an important feature of low energy QCD. The value of the chiral condensate (the quark condensate for zero quark mass) is given by the Gell-Mann, Oakes, Renner relation [1]:

$$\frac{f_\pi^2 M_\pi^2}{4} = -\frac{m_u + m_d}{2} \frac{\langle 0|\overline{u}u + \overline{d}d|0\rangle}{2}. \tag{1.1}$$

A value for the chiral condensate can be extracted accurately but indirectly from chiral extrapolation of lattice QCD results for meson masses and decay constants. A harder question to answer is that of the value of the light and strange quark condensates at their physical non-zero quark masses. This is because the condensate must be carefully defined, taking into account the quark mass dependent mixing of $m\overline{\psi}\psi$ (not normal-ordered) with the unit operator. On the lattice, in a direct calculation of the condensate, such mixing gives rise to terms which diverge as m/a^2. We have calculated these terms, through $\mathscr{O}(\alpha_s)$ in lattice QCD perturbation theory and used these to convert the lattice QCD results to a well-defined condensate in the $\overline{MS}$ scheme at a fixed scale (2 GeV). Since we have results at multiple values of the lattice spacing we can fit for remaining m/a^2 pieces at higher order in α_s. We can also extrapolate our results to the continuum limit at $a = 0$. Since the conference we have finalised a paper on this work [2] and refer the reader there for all details.

2. Results

We use gluon field configurations generated by the MILC collaboration including u, d ($m_u = m_d$ here), s and c quarks in the sea at 3 different values of the lattice spacing and having u/d quark masses down to the physical value [3]. The quark formalism used is the Highly Improved Staggered Quark (HISQ) action [4] that has good chiral properties and small discretisation errors.

Figure 1 shows our raw and corrected lattice results for s quarks and for light quarks of physical mass. The quantity plotted is the ratio R_q given by:

$$R_q = -\frac{4m_q\langle\overline{\psi}\psi_q\rangle}{f^2_{\eta_q}M^2_{\eta_q}}. \tag{2.1}$$

Here η_q is the pseudoscalar meson made from quarks of mass m_q. This is the π for light quarks and a fictitious particle called the η_s for s quarks. The η_s can be kept distinct from the η and η' on the lattice by not allowing it to decay and it then becomes a very useful particle in the determination of the lattice spacing [5]. Its mass and decay constant in the continuum and chiral limits can be determined accurately in terms of those of the K and the π [6]. Interestingly, values very close to those expected in leading order chiral perturbation theory are found. R_q is a good quantity to use in this analysis because it has reduced finite volume and quark mass dependence over the condensate alone and is 1 for quarks of zero mass, as given by the GMOR relation.

In Figure 1 the open squares give the raw results from the lattice, determining the condensate from the trace of the quark propagator. The x-axis is the inverse square of the lattice spacing and it is quite clear from these plots that there is a quark-mass dependent $1/a^2$ divergence. The pluses give the results after subtraction of the difference of the tree-level lattice QCD (for HISQ quarks) and $\overline{MS}$ perturbative expressions. The divergence is largely, but not entirely, removed. The crosses continue this process by removing the perturbative contribution through $\mathcal{O}(\alpha_s)$. This makes little difference because the coefficient of the $\mathcal{O}(\alpha_s)$ divergence is very small. It is then clear that the remaining divergence evident in the lattice calculations must come from higher orders in perturbation theory, and we take account of this in our fits. We also use the fact that the form of the divergence is strongly constrained and fit simultaneously results at multiple values of the quark mass and lattice spacing.

Figure 2 on the left shows the one-loop subtracted results from Figure 1 along with the final results from our fit. The x-axis is now the square of the lattice spacing and the grey and green bands show the continuum and chiral limit values of the strange and light quark condensates respectively, in the $\overline{MS}$ scheme at 2 GeV. Our fit form allows for discretisation errors and quark mass dependence of the physical condensate as well as the divergent pieces discussed above. Fitting the remaining divergence is the key issue in determining the final error, however, and the reason why the error on the strange quark condensate is 15% and on the light quark condensate 0.5%. Full details, along with an error budget, are given in [2].

Our results are:

$$\begin{aligned}
\langle\bar{s}s\rangle^{\overline{MS}}(2\,\mathrm{GeV}) &= -0.0245(37)(3)\,\mathrm{GeV}^3 = -(290(15)\,\mathrm{MeV})^3 \\
\langle\bar{l}l\rangle^{\overline{MS}}(2\,\mathrm{GeV}) &= -0.0227(1)(4)\,\mathrm{GeV}^3 = -(283(2)\,\mathrm{MeV})^3 \\
\frac{\langle\bar{s}s\rangle^{\overline{MS}}(2\,\mathrm{GeV})}{\langle\bar{l}l\rangle^{\overline{MS}}(2\,\mathrm{GeV})} &= 1.08(16)(1),
\end{aligned} \tag{2.2}$$

where the first error comes from our fitted result for R_q and the second error in each case comes from the error in the quark masses, taken from lattice QCD results [7].

Figure 2 shows on the right a comparison of our result for the ratio of the strange to light quark condensate to previous values from QCD sum rule calculations [8, 9, 10]. Our result has the

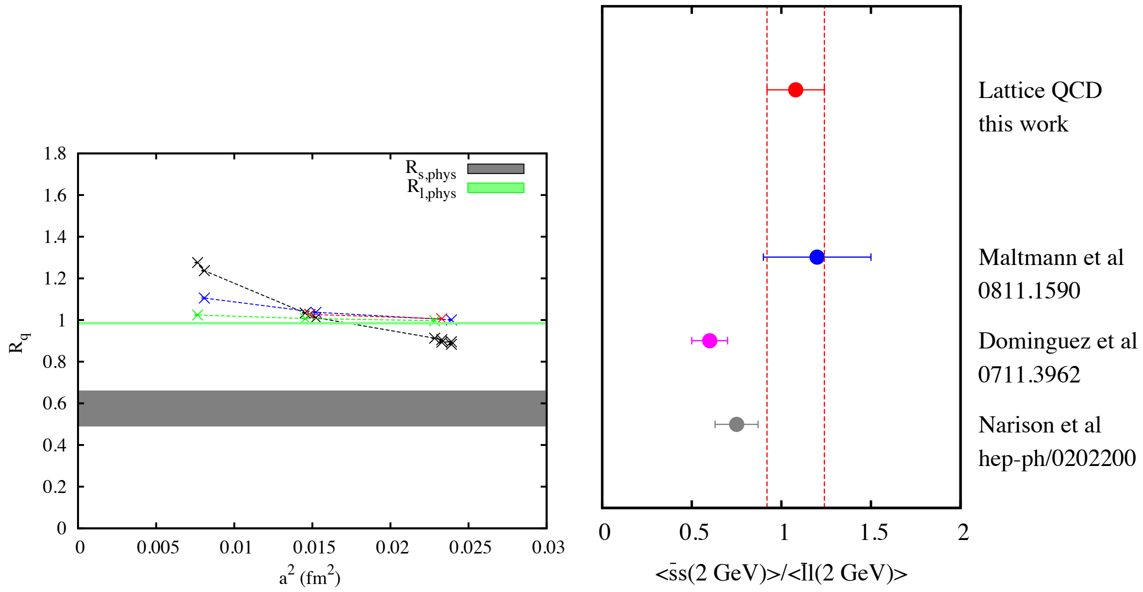

Figure 2: The figure on the left shows the results from fitting the ratio R_q given by the crosses after subtracting the perturbative contribution through $\mathcal{O}(\alpha_s)$. 4 different quark masses are used: m_s (black), $m_s/5$ (blue), $m_s/10$ (red) and the physical light quark mass (green). The green and black bands show the physical result from the fit with $\pm 1\sigma$ errors. Black is for the strange quark condensate and green for the light quark condensate. The figure on the right shows the comparison of our result for the ratio of strange to light quark condensate to those of previous sum rule analyses.

advantage of being a direct determination with a full error budget. Our value for the strange quark condensate agrees well with independent results on gluon field configurations including u, d and s quarks in the sea and covering a wider range of values of a [2].

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