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Topological susceptibility near T_c in SU(3) gauge theory

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We calculated the topological charge susceptibility χ_t for pure gauge SU(3) theory at finite temperature. The anisotropic lattice and over-improved stout-link smoothing method were used to calculate the topological charge. Near the phase transition point we find a smooth crossover behavior for χ_t with values decreasing from $(189(2)\mathrm{MeV})^4$ to $(67(3)\mathrm{MeV})^4$ as the temperature increased from $0.3T_c$ to $1.9T_c$, which demonstrate that topological excitations exist far above T_c. Our results are consistent with the earlier studies using other methods to calculate the topological charge on smaller lattice with narrower temperature range.

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1. Introduction

Topology in lattice QCD has been widely studied for different purposes, while one of most exciting application is to explore the structure of QCD vacuum, which has not been well understood. The topological charge susceptibility χ_t, which is defined as

$$\chi_t = \frac{1}{V}\left(\langle Q^2\rangle - \langle Q\rangle^2\right). \tag{1.1}$$

where Q is the topological charge, and it has attracted special interest since 1979. The quenched χ_t is related to the $U(1)$ axial anomaly and mass of η' meson as the well-known Witten-Vaneziano relation [1] [2]. On the other hand, lattice studies of QCD at finite temperature and the phase structure of QCD has made great progress recently [3], and the relation between chiral symmetry breaking and confinement, as they happen to be near T_c, are of great interest. χ_t describes the topological fluctuations of the vacuum in quenched situation, and its behavior near T_c are expected to provide a further understanding on the chiral symmetry breaking and confinement.

χ_t in SU(3) pure gauge theory near T_c was studied by Gattringer *et al.* in 2002 [4]. Their simulation find the cross-over behavior of χ_t in the temperature interval of $0.8T_c \sim 1.3T_c$. They use the chiral improved fermion action and the fermionic method based on Atiyah-Singer index theorem to calculate the topological charge. Here we work on the anisotropic lattices, with wider range of temperature, using the bosonic method to explore the topological properties at finite temperature. More recent studies were carried out [5], part of their results is compatible with us.

2. Methods

We executed the calculation on the anisotropic lattices, which have advantages of improving accuracy in lattice QCD for both zero and finite temperatures. The Symanzik and tadpole improvement schemes of the gauge action are found to have better continuum extrapolation behaviors for many physical quantities, that is, the finite lattice spacing effect is well suppressed by these improvements. Considering these facts, we use the following improved gauge action,

$$S_{IA} = \beta\left\{\frac{5}{3}\frac{\Omega_{sp}}{\xi u_s^4} + \frac{4}{3}\frac{\xi\Omega_{tp}}{u_t^2 u_s^2} - \frac{1}{12}\frac{\Omega_{sr}}{\xi u_s^6} - \frac{1}{12}\frac{\xi\Omega_{str}}{u_s^4 u_t^2}\right\} \tag{2.1}$$

where β is related to the bare QCD coupling constant g_0, $\xi = a_s/a_t$ is the aspect ratio for anisotropy (we take $\xi = 5$ in this work), u_s and u_t are the tadpole improvement parameters of spatial and temporal gauge links. $\Omega_C = \sum_C \frac{1}{3}\mathrm{ReTr}(1 - W_C)$, with W_C referring to the path ordered product of link variables along a closed contour C on the lattice. $\Omega_{\rm sp}$ includes the sum over all spatial plaquettes on the lattice, $\Omega_{\rm tp}$ includes the temporal plaquettes, $\Omega_{\rm sr}$ includes the product of link variables about planar 2×1 spatial rectangular loops, and $\Omega_{\rm str}$ refers to the short temporal rectangles (one temporal link, two spatial). Practically, u_t is set to 1, and u_s is defined by the expectation value of the spatial plaquette, $u_s = \left\langle \frac{1}{3}\mathrm{Tr}P_{ss'}\right\rangle^{1/4}$. Besides, it's been proved in [6] that the renormalization of the anisotropy ξ is ignorable as β varies.

2.1 The definition of temperature

The temperature on lattice is defined as follow:

$$T = \frac{1}{N_t a_t}, \tag{2.2}$$

where N_t is the temporal lattice size. T can be changed by varying either N_t or the coupling constant β, which is related directly to the lattice spacing a_t. The critical temperature is determined for a given lattice $N_t = 24$ after the critical coupling β_c has been determined. The order parameter for determining β_c is chosen to be the susceptibility χ_P of the Polyakov line, which is defined as

$$\chi_P = \left\langle \Theta^2 \right\rangle - \left\langle \Theta \right\rangle^2, \tag{2.3}$$

where Θ is the $Z(3)$ rotated Polyakov line. After a rough and refined study the peak position of order parameter gives the critical coupling constant $\beta_c = 2.808$, which corresponds to the critical temperature $T_c \approx 0.724 r_0^{-1} = 296\text{MeV}$ [6]. Here r_0 is the hadronic scale parameter and we take $r_0^{-1} = 410\,(20)\,\text{MeV}$.

Considering both finite volume effects and good resolution of temporal lattice at $T \sim 2T_c$, we set $\beta = 3.2$ in the study of topological susceptibility at finite temperature. The corresponding lattice spacing a_s is set by calculating the static quark potential $V(r)$ on an anisotropic lattice $24^3 \times 128$. The fitting result of string tension σ reads:

$$\frac{a_s}{r_0} = \sqrt{\frac{\sigma a_s^2}{1.6 + e_c}} = 0.1825\,(7)\,, \tag{2.4}$$

taking $r_0^{-1} = 410\,(20)\,\text{MeV}$, we have $a_s = 0.0878\,(4)\,\text{fm}$. Comparing to a_s at $\beta_c = 2.808$ and $N_t = 24$, the N_t of T_c and $2T_c$ at fixed $\beta = 3.2$ can be calculated and the results are $N_t \sim 40$ and $N_t \sim 20$. More parameters details could be found in [7].

In this study we change temperature by varying temporal lattice size N_t. Setting $\beta = 3.2$, we generated a series of lattice $24^3 \times N_t$ with N_t =20, 24, 28, 32, 36, 40, 44, 48, 60, 80 and 128, which cover the range of $T \in [0.3T_c,\ 1.9T_c]$. For each N_t lattice we sampled 1000 configurations, after 10000 sweeps from cold start and with 500 sweeps between each sampling. Each sweep consists of a composite update including 1 pseudo heat-bath and 5 over-relaxing procedures. It should be pointed out that we assume the difference between $T_c\,(\beta = 2.808)$ and $T_c\,(\beta = 3.2)$ is ignorable due to the application of the improved gauge action.

2.2 Over-improved stout-link method

It is well-known that the topological charge Q calculated directly from a typical gauge configuration by the field theoretic definition [8] will not be an integer in general, which is obviously conflicting with the continuum situation. Thus, some cooling or smearing procedure is essentially to be taken on the configuration before calculating Q, aiming to suppress the ultraviolet fluctuations.

All cooling or smearing methods are based on an approximation to the continuum gauge field action:

$$S_g = \frac{1}{2} \int d^4x \,\text{tr}\left[F_{\mu\nu} F_{\mu\nu}\right]. \tag{2.5}$$

and the lattice version of action contains combination of plaquette and larger Wilson loops. But the coefficients of 2×2 and larger loop terms involved quite complicated renormalization problem, while $\mathscr{O}\left(a^4\right)$ errors are not suppressed as expected. Moreover, in these methods the destruction of topological structure is unavoidable. For the purpose of the study on topology susceptibility, we adopt the over-improved stout-link smearing method developed by Moran and Leinweber [9] to filter the lattice, which is proved efficiently preserving instanton-like objects.

Over-improved stout-link smearing method introduces the over-improved parameter ε into stout-link smearing algorithm. Modified link combinations are as follows:

$$\begin{aligned} \mathscr{C}_\mu(x) = \rho_{\text{sm}} \sum \Big\{ & \frac{5-2\varepsilon}{3}\left(1\times 1 \text{ paths touching } U_\mu(x)\right) \\ & -\frac{1-\varepsilon}{12}\big(1\times 2 \\ & +2\times 1 \text{ paths touching } U_\mu(x)\big)\Big\}, \end{aligned} \tag{2.6}$$

There are three free parameters in the over-improved stout-link smearing method, the over-improved parameter ε, the smearing parameter ρ_{sm}, and n_{sm}, the number of smoothing steps taken on each configuration. Moran and Leinweber has practically determined $\varepsilon = -0.25$ to maximize the life of instantons under iterative smearing procedure, so do we set this parameter. We checked this on several configurations at different temperature and confirmed that mostly the topological charge quickly fell into an integer number (near the integer within ± 0.1) as n_{sm} increased and kept quite stable even after more then 1000 steps of smoothing procedure.

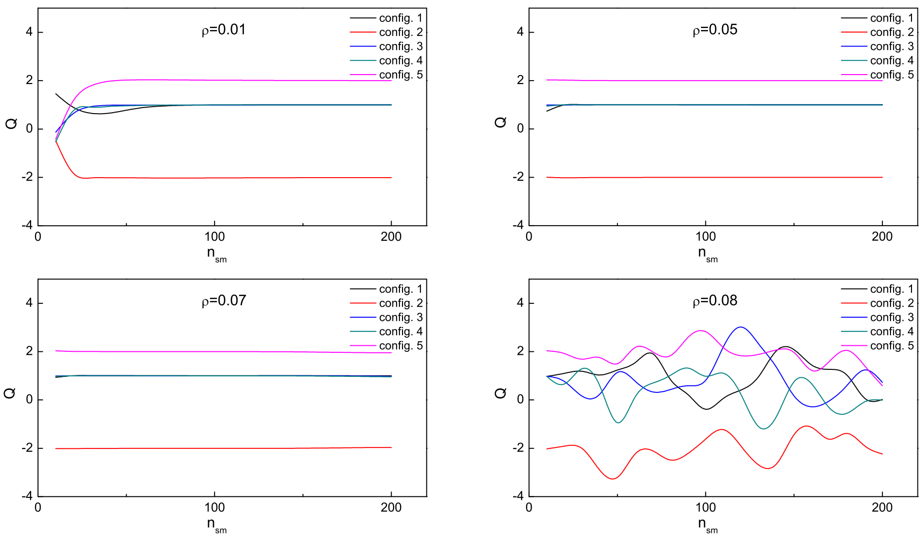

Figure 1: Test for different ρ at $N_t = 40$

For the other two parameters ρ_{sm} and n_{sm}, we take some test to find suitable values for them. 5 configurations were randomly selected for each N_t and were smoothed 200 times for $\rho_{\text{sm}} = 0.01 \sim 0.08$, and the results of topological charge for some ρ_{sm} values are shown in Figure 1. ρ_{sm} is the strength to suppress the fluctuation, but in practice we found that when ρ_{sm} is larger than

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0.08, it will leads to completely unstable topological charge, as shown in Figure 1. Finally we set $n_{\rm sm} = 40$ and $\rho_{\rm sm} = 0.05$ for all lattices (except for $N_t = 128$ we set $\rho_{\rm sm} = 0.07$, keeping $n_{\rm sm} = 40$). Moreover, the $F^{ab}_{\mu\nu}(x)$ used in computing the topological charge Q is calculated by highly improved operators [10], including 1×1, 2×2 and 3×3 loops.

3. Results and discussion

We get 1000 Q charge data for each N_t lattice, which are calculated after 40 times of smoothing procedures. Most of them are integers, as shown in Figure 2 that points biased from the integers

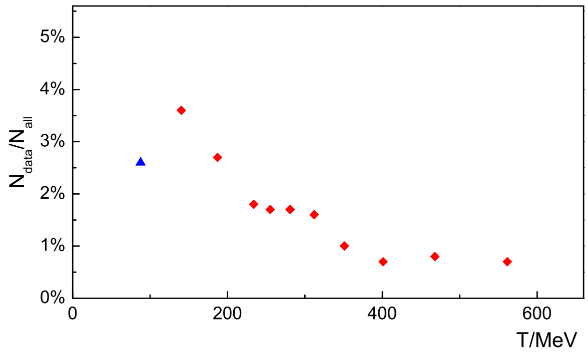

Figure 2: Numbers of Q that bias from integer > 0.1

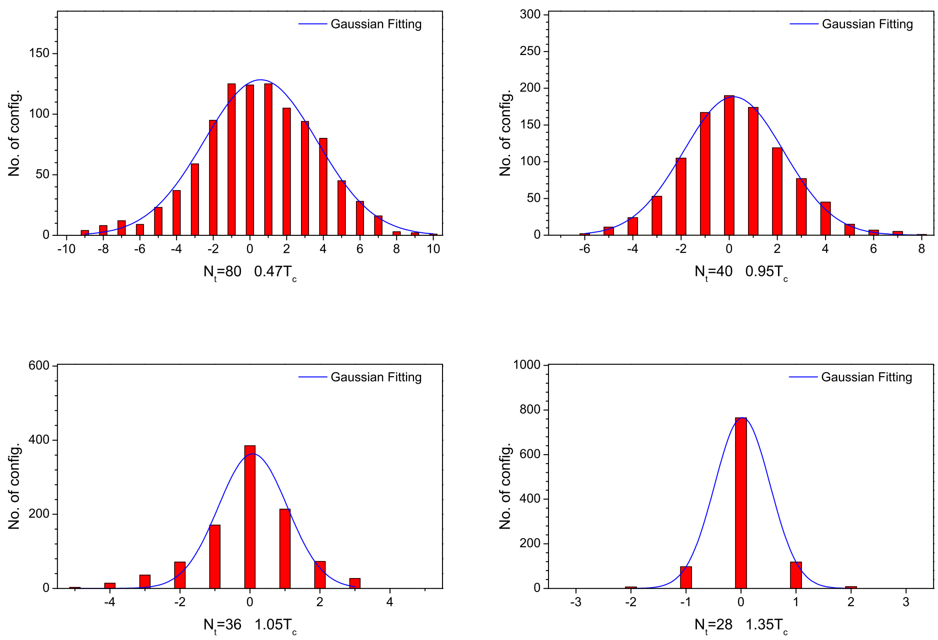

Figure 3: Distribution of topological charge

are less than 4%. Noting that for $N_t = 128$ we set $\rho_{\rm sm} = 0.07$ to ensure that 40 steps of smooth are practically enough.

Firstly, we display the histogram of the topological charge Q for different temperature in Figure 3. As the temperature increase, the Q has been suppressed as expected. We fit the distribution of Q with Gaussian function, and the results are good, especially near T_c.

Next, susceptibility and its standard derivation could be worked out for each N_t, or say for different temperature, the results are shown in Table 1 and Figure 4.

N_t	T/MeV	$\chi^{1/4}$/MeV
20	561.5	67.1 ± 2.8
24	467.9	81.8 ± 1.9
28	401.1	97.5 ± 1.7
32	350.9	125.4 ± 2.0
36	311.9	148.0 ± 2.1
40	280.8	181.7 ± 2.2
44	255.2	186.5 ± 2.0
48	234.0	187.0 ± 2.0
60	187.2	189.1 ± 2.1
80	140.4	183.4 ± 2.1
128	87.7	187.9 ± 2.2

Table 1: χ results

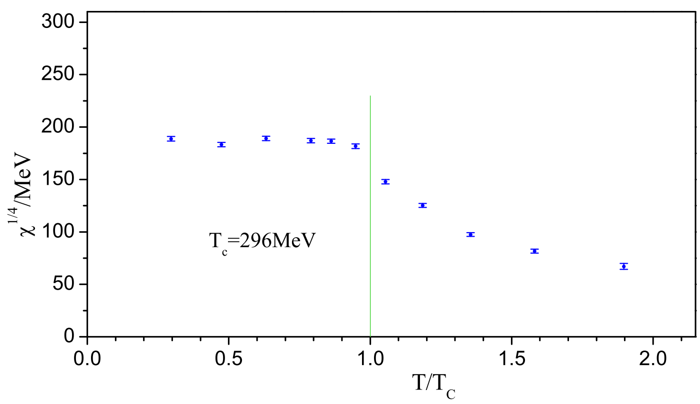

Figure 4: χ results

The susceptibility below T_c results as $\chi_t = 185.95$MeV, which is consistent with former work. It could be seen clearly that topological excitation is observed non-zero even at $T = 1.9T_c$. That means instanton-like structure may exist above to 2 times of the "critical" temperature, supporting the conclusion of Gattringer. As the Figure 4 shows, a smooth crossover behavior for χ_t decreasing from $(188(2)\text{MeV})^4$ to $(67(3)\text{MeV})^4$ is found.

Another purpose of this work is checking the over-improved stout-link smearing method, which is an effective and cheaper (than the fermionic method) method to reveal the cloaking nature of QCD vacuum while preserving topological structure as well as suppressing the ultraviolet fluctuations on the lattice.

Acknowledgments

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