

Recent progress in calculation of B_K using staggered fermions

Weonjong Lee*, Yong-Chull Jang, Hwancheol Jeong, Jangho Kim, Kwangwoo Kim, Seonghee Kim, Jaehoon Leem, Boram Yoon

Lattice Gauge Theory Research Center, CTP, and FPRD,

Department of Physics and Astronomy, Seoul National University, Seoul, 151-747, South Korea

E-mail: wlee@snu.ac.kr

Taegil Bae

Korea Institute of Science and Technology Information, Daejeon, 305-806, South Korea

E-mail: esrevinu@gmail.com

Chulwoo Jung, Hyung-Jin Kim

Physics Department, Brookhaven National Laboratory, Upton, NY11973, USA

E-mail: chulwoo@bnl.gov

Jongjeong Kim

Physics Department, University of Arizona, Tucson, AZ 85721, USA

E-mail: rvanguard@gmail.com

Stephen R. Sharpe

Physics Department, University of Washington, Seattle, WA 98195-1560, USA

E-mail: sharp@phys.washington.edu

SWME Collaboration

We report on recent progress in the calculation of B_K using HYP-smearred improved staggered fermions on the MILC asqtad lattices. We have added measurements on fine ($a \sim 0.09$ fm) and superfine ($a \sim 0.06$ fm) ensembles at different values of the light sea quark mass (am_ℓ), as well as increased the statistics on some other ensembles. We find that the results on the fine lattices show a significantly stronger am_ℓ dependence than those on the superfine and coarse ($a \sim 0.12$ fm) lattices. We discuss different methods for accounting for these new results when doing the am_ℓ and continuum extrapolations.

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1. Introduction

The calculation of the kaon mixing matrix element B_K is one of the successes of lattice QCD. Results with all errors controlled are available with several fermion discretizations [1, 2]. The value for B_K is a key input into the standard model prediction for ε_K . At present there is an $\approx 3\sigma$ tension between this prediction and the experimental value (if one uses the “exclusive” V_{cb} obtained using lattice QCD) [5, 6, 7]. Thus it is important to continue to further improve the lattice calculations.

In this proceedings we provide an update on our results for B_K . These are obtained using improved staggered fermions, specifically HYP-smearred valence quarks on asqtad sea-quarks. We describe here results obtained using chiral extrapolations based on SU(2) staggered chiral perturbation theory (SChPT), which is our most reliable procedure [3, 4]. Details of the fitting functions and the analysis method are given in Ref. [3, 4], and are not repeated here.

Table 1 lists all the ensembles on which we have calculated B_K , and notes which results have changed in the last year. In particular, since Lattice 2011 we have accumulated much higher statistics on the F2 and S1 ensembles (in addition to a small increase on the U1 ensemble) and added new measurements on ensembles F3, S2 and S3. Of these, the most important updates are those on ensembles F2, F3, S2, and S3, since they give information on the light sea quark mass dependence which was previously lacking on the fine and superfine ensembles. We focus on this dependence here.

a (fm)	am_l/am_s	geometry	ID	ens \times meas	status
0.12	0.03/0.05	$20^3 \times 64$	C1	564×9	old
0.12	0.02/0.05	$20^3 \times 64$	C2	486×9	old
0.12	0.01/0.05	$20^3 \times 64$	C3	671×9	old
0.12	0.01/0.05	$28^3 \times 64$	C3-2	275×8	old
0.12	0.007/0.05	$20^3 \times 64$	C4	651×10	old
0.12	0.005/0.05	$24^3 \times 64$	C5	509×9	old
0.09	0.0093/0.031	$28^3 \times 96$	F3	949×9	new
0.09	0.0062/0.031	$28^3 \times 96$	F1	995×9	old
0.09	0.0031/0.031	$40^3 \times 96$	F2	959×9	update
0.06	0.0072/0.018	$48^3 \times 144$	S3	593×9	new
0.06	0.0036/0.018	$48^3 \times 144$	S1	749×9	update
0.06	0.0025/0.018	$56^3 \times 144$	S2	799×9	new
0.045	0.0028/0.014	$64^3 \times 192$	U1	747×1	update

Table 1: MILC asqtad ensembles used to calculate B_K . am_l and am_s are the masses, in lattice units, of the light and strange sea quarks, respectively. “ens” indicates the number of configurations on which “meas” measurements are made. Note that the numbering of the ID tags on the fine and superfine lattices do not follow the ordering of am_l .

2. Chiral fits

In our numerical study, our lattice kaons are composed of valence (anti)quarks with masses

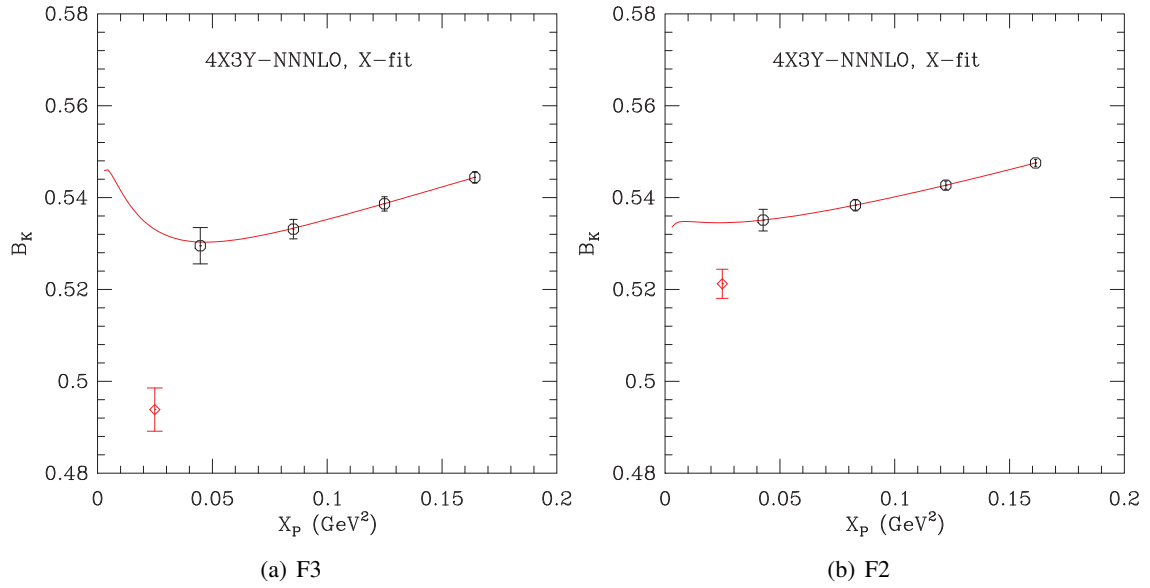


Figure 1: X-fits to $B_K(1/a)$ for the F3 and F2 ensembles. The red diamond is explained in the text.

m_x and m_y . These are, respectively, the masses of the valence d and s quarks. On each MILC ensemble, we use 10 valence masses:

$$am_x, am_y = am_s \times n/10 \quad \text{with} \quad n = 1, 2, 3, \dots, 10, \quad (2.1)$$

where am_s is the nominal strange sea quark mass. In our standard fits we extrapolate to am_d^{phys} using the lowest 4 values for am_x (the ‘‘X-fit’’—done at fixed am_y), and then extrapolate to m_s^{phys} using the highest 3 values of am_y (‘‘Y-fit’’). As described in Ref. [3], these choices keep us in the regime where we expect next-to-leading order (NLO) SU(2) ChPT to be reasonably accurate. The X-fits described here are done to the form predicted by NLO partially quenched SChPT (which is given in Refs. [3, 8]), augmented by analytic and generic non-analytic terms of next-to-next-to-leading order (NNLO) and a single analytic term of next-to-next-to-next-to-leading order (NNNLO). We use the Bayesian method to constrain the higher order coefficients, as described in Refs. [3, 4, 9]. The am_y dependence (which is not controlled by ChPT) is very close to linear and we use a linear fit for our central values. We dub this entire fitting procedure the ‘‘4X3Y-NNNLO fit’’, and use it for our central values.

The X-fits on the new fine ensembles are shown in Fig. 1, while those on the new superfine ensembles are shown in Fig. 2. In each panel, a fit to the SU(2) SChPT form is shown, with X_p the squared-mass of the Goldstone-taste valence $\bar{x}x$ pion, whose mass is very close to linear to am_x . Also shown (as the red diamond) is the result obtained after (a) extrapolating $am_x \rightarrow am_d^{\text{phys}}$, (b) removing the known taste-breaking in the pion masses appearing in the chiral logarithms, and (c) setting the light sea-quark mass to its physical value in the chiral logarithmic terms. See Ref. [3] for details of this procedure. We have incorporated finite volume corrections, as predicted by NLO SChPT, into the fitting function [10].

A notable feature of both figures is that the difference between the extrapolated/corrected result and the fit curve is larger on the ensembles with larger am_ℓ (F3 and S3). This is due almost entirely

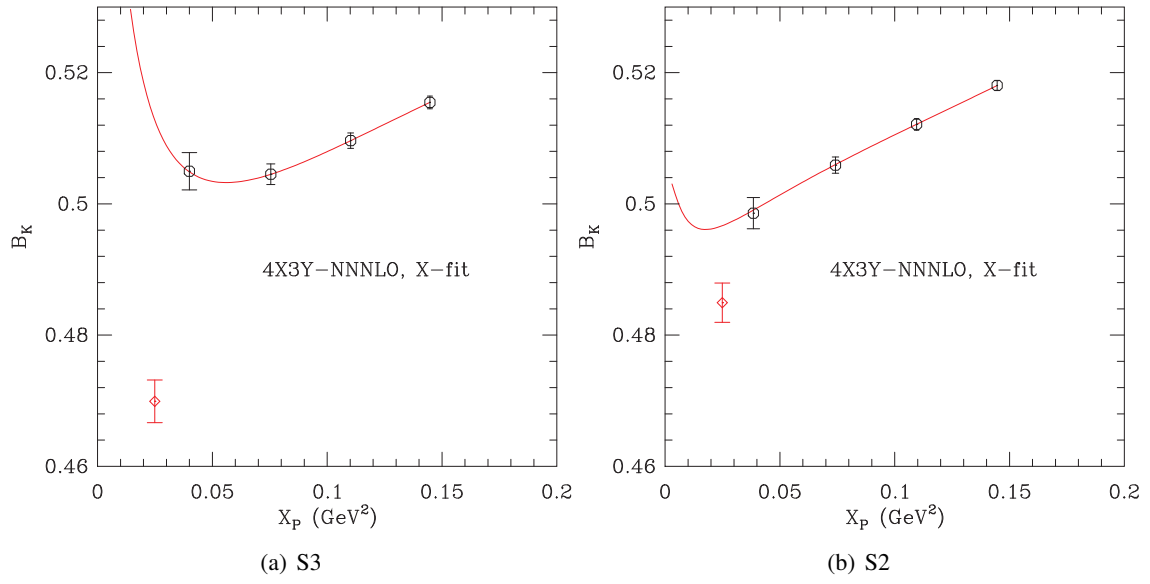


Figure 2: X-fits to $B_K(1/a)$ for the S3 and S2 ensembles. Notation as in Fig. 1.

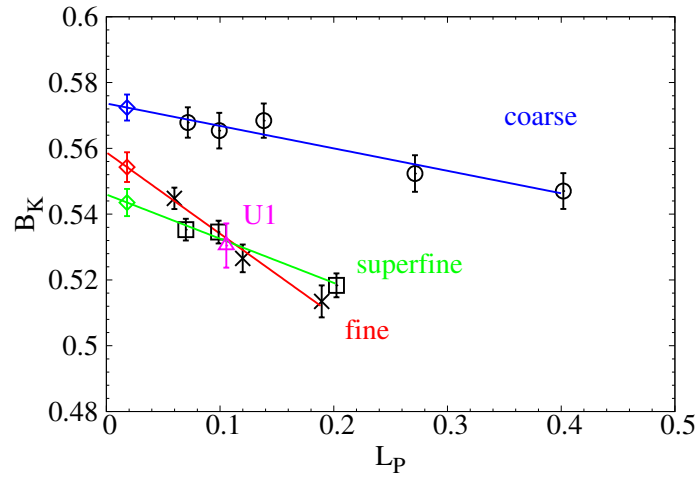


Figure 3: $B_K(\mu = 2 \text{ GeV})$ vs. $L_P (\text{GeV}^2)$, where L_P is a squared mass of the Goldstone-taste pion composed of two light sea quarks ($\bar{\ell}\ell$).

to the larger shift required to bring the sea-quark pion mass to its physical value on these lattices (correction (c) above). The other aspects of the extrapolation/correction are similar for the pairs of lattices. We return to this point below.

3. Dependence On Light Sea Quark Mass

In Fig. 3, we show the dependence of B_K on the light sea quark mass for all lattice spacings, including the U1 point for completeness. We use L_P as a proxy for am_ℓ , since these quantities are proportional to very good accuracy. We have run B_K to a common renormalization scale $\mu = 2 \text{ GeV}$

(in the $\overline{\text{MS}}$ scheme), so that the results are directly comparable. We expect, at NLO, only a linear behavior on L_P .

As can be seen from the plot, a linear fit works well for each of the three lattice spacings for which we have multiple values of am_ℓ . The results on the coarse lattices, which were presented in Ref. [4], show a mild dependence on L_P . The slope, $-1/(2.9 \text{ GeV})^2$, is consistent with naive dimensional analysis, which predicts a magnitude of $\mathcal{O}(1/\Lambda_\chi^2)$ with $\Lambda_\chi \approx 1 \text{ GeV}$. The slope on the superfine lattices, which is a new result, has a slightly larger magnitude than that on the coarse lattices but is comparable. What is striking, however, is the slope on the fine lattices, whose magnitude is about twice that on the coarse lattices (slope $\approx -1/(2 \text{ GeV})^2$). Its value per se is reasonable—what is unexpected is that it should differ so much from that at the other lattice spacings. Since we are in the regime where corrections are small ($\leq 10\%$) in both chiral and continuum extrapolations, one would expect that the slope would depend only weakly on a^2 , and any dependence should be linear. Instead, our results appear to require a contribution to B_K proportional to $am_\ell(a^2)^2$, since only with a quadratic dependence on a^2 can the magnitude of the slope increase and then decrease as a^2 is raised.

This peculiar behavior has only recently come to light and we are in the process of investigating it. One avenue we are following is to do a global continuum-chiral extrapolation to all results. This would incorporate the constraint that fit parameters in the chiral fits should depend smoothly on a^2 . Another approach is to remove the step (c) described above from our correction procedure and leave this to be dealt with by the am_ℓ extrapolation. This is possible because, at NLO in SChPT, L_P does not appear in the argument of the chiral logarithms, but only in the prefactor. Pending a more complete understanding, we have considered the continuum extrapolation of results obtained after the extrapolation to physical am_ℓ .

4. Continuum Extrapolation

In Fig. 4, we show the results obtained on the coarse, fine and superfine lattices after the linear extrapolation to am_ℓ^{phys} plotted vs. a^2 . We also include the U1 result which, however, is not extrapolated to physical am_ℓ , since there is only one value of am_ℓ available for the ultrafine lattices. Thus it cannot be consistently used in the continuum extrapolation.

Previously, when the high-statistics data from ensembles F2, F3, S2 and S3 were not available, we made the continuum extrapolation using ensembles C3, F1, S1 and U1, all of which have close to the same sea-quark masses. We then included systematic errors accounting for the use of unphysical sea-quark masses (the errors being 1.5% and 1.3% for the light and strange sea-quark masses, respectively [4]). We found a continuum behavior which was far from linear in a^2 (as can be seen from Fig. 3—the fit corresponds to $L_P \approx 0.1 \text{ GeV}^2$), and we could not obtain reasonable fits including the coarse ensemble. Dropping this, and trying various fits (along the lines described below) we estimated an extrapolation error of 1.9%.

Using the data extrapolated to physical am_ℓ leads to a much smoother continuum extrapolation, as can be seen in Fig. 4. We can now obtain good fits including the coarse lattice result. As noted above, we can consistently fit only to the larger three values of a^2 . Following Ref. [4], we fit both to a linear dependence and to a five-parameter form containing dependence that we know enters due to incomplete operator matching and higher-order contributions. The explicit forms are given

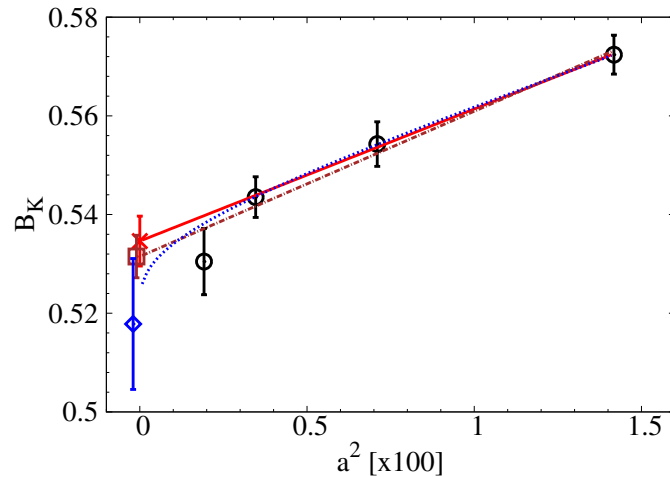


Figure 4: $B_K(\mu = 2 \text{ GeV})$ vs. $a^2 (100 \times \text{fm}^2)$. See text for description of data points. Note that the U1 point (smallest a^2) has not been extrapolated to physical am_ℓ . Red, brown and blue curves and points show the `lin3`, `lin4` and `g4-a2g2-a4` fits, respectively.

in Table 2, For the five-parameter fit we use Bayesian constraints for $c_2 - c_5$, with central values set to zero and spreads taken as $\sigma_2 = \sigma_3 = \sigma_5 = 2\Lambda^2$ and $\sigma_4 = 2$, with $\Lambda = 300 \text{ MeV}$. These fits are shown in Fig. 4; values for $\chi^2/\text{d.o.f.}$ and extrapolated B_K are listed in the Table. Also shown, for completeness, is a linear fit including the U1 point, labeled `lin4`.

fit type	# data	fit function	$\chi^2/\text{d.o.f.}$	$B_K(2 \text{ GeV})$
<code>lin3</code>	3	$c_1 + c_2 a^2$	0.0307	0.5346(51)
<code>g4-a2g2-a4</code>	3	$c_1 + c_2 a^2 + c_3 a^2 \alpha_s(a) + c_4 \alpha_s^2(a) + c_5 a^4$	0.0298	0.5178(133)
<code>lin4</code>	4	$c_1 + c_2 a^2$	0.703	0.5314(43)

Table 2: Details of continuum fits. For the `g4-a2g2-a4` fit with Bayesian constraints, the augmented $\chi^2/\text{d.o.f.}$ is shown. Only statistical errors are shown for B_K .

At this stage we are not ready to quote an updated result for B_K , since we clearly need to improve our understanding of the am_ℓ dependence. Nevertheless, we note that, taking the linear continuum extrapolation (fit `lin3`) the final value for \widehat{B}_K is 0.732 with a $\sim 1\%$ statistical error.¹ This is, in fact, consistent with our previous result of 0.727 ± 0.038 [4] (where the error is dominated by systematics).² Setting aside the matching error of 4.4%, we note that the combined am_ℓ and continuum extrapolation systematics were estimated at 2.4%. This is certainly large enough to accommodate any small shift we might find from our new analysis. Thus we do not expect a significant change in our final result. We do hope, however, that the am_ℓ extrapolation error will

¹We have also taken the opportunity to update the value of r_1 to 0.3117 fm, as given in Ref. [11].

²The smallness of the shift in the central value can be understood from Fig. 3. Extrapolating at physical L_P there is a larger a^2 dependence than at $L_P \approx 0.1 \text{ GeV}^2$ (ignoring the coarse lattice point in the latter case). This difference approximately cancels the increase in the values one obtains when extrapolating to physical L_P .

come down. The total error will, at present, remain dominated by the matching error (due to our use of 1-loop perturbative matching).

In summary, results at different light sea-quark masses on the fine and superfine MILC asqtad ensembles have uncovered an am_ℓ dependence that is hard to understand. An optimistic interpretation of the new results is that the difficulties we had previously when extrapolating at unphysical am_ℓ were due to artifacts introduced by our X-fitting and extrapolation procedure, while the new continuum fits (shown in Fig. 4) do not contain these artifacts. Analysis is underway to test this interpretation, and final results should be available soon.

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