PROCEEDINGS OF SCIENCE

B physics from HQET in two-flavour lattice QCD

CERN-PH-TH/2012-289
DESY 12-188
HU-EP-12/35
LPT Orsay/12-107
MS-TP-12-14
SFB/CPP-12-78
TCDMATH 12-09

ALPHA Collaboration

F. Bernardoni[*a], **B. Blossier**[b], **J. Bulava**[c], **M. Della Morte**[d], **P. Fritzsch**[e], **N. Garron**[f], **A. Gérardin**[b], **J. Heitger**[g], **G. von Hippel**[d], **H. Simma**[a], **R. Sommer**[a]

[a] *NIC, DESY, Platanenallee 6, 15738 Zeuthen, Germany*
[b] *Laboratoire de Physique Théorique, CNRS/Université Paris XI, F-91405 Orsay Cedex, France*
[c] *CERN, Physics Department, TH Division, CH-1211 Geneva 23, Switzerland*
[d] *Institut für Kernphysik, University of Mainz, Becher-Weg 45, 55099 Mainz, Germany*
[e] *Institut für Physik, Humboldt-Universität zu Berlin, Newtonstr. 15, 12489 Berlin, Germany*
[f] *School of Mathematics, Trinity College, Dublin 2, Ireland*
[g] *Universität Münster, Institut für Theoretische Physik, Wilhelm-Klemm-Str. 9, 48149 Münster, Germany*
fabio.bernardoni@desy.de

We present our analysis of B physics quantities using non-perturbatively matched Heavy Quark Effective Theory (HQET) in $N_\mathrm{f}=2$ lattice QCD on the CLS ensembles. Using all-to-all propagators, HYP-smeared static quarks, and the Generalized Eigenvalue Problem (GEVP) approach with a conservative plateau selection procedure, we are able to systematically control all sources of error. With significantly increased statistics compared to last year, our preliminary results are $\overline{m}_\mathrm{b}(\overline{m}_\mathrm{b}) = 4.22(10)(4)_z\,\mathrm{GeV}$ for the $\overline{\mathrm{MS}}$ b-quark mass, and $f_\mathrm{B} = 193(9)_\mathrm{stat}(4)_\chi\,\mathrm{MeV}$ and $f_{\mathrm{B_s}} = 219(12)_\mathrm{stat}\,\mathrm{MeV}$ for the B-meson decay constants.

CLS based

The 30th International Symposium on Lattice Field Theory - Lattice 2012,
June 24-29, 2012
Cairns, Australia

*Speaker.

PoS(Lattice 2012)273

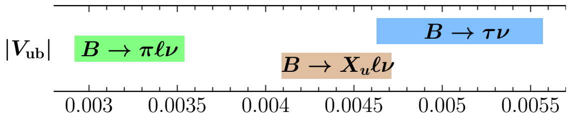

Figure 1: An illustration of the observed tension between the determinations of $|V_{ub}|$ from leptonic, exclusive semileptonic, and inclusive semileptonic channels; $\pm 1\sigma$ bands are shown.

1. Introduction

Weak decays of heavy mesons constrain the CKM matrix encoding the flavour-changing weak interactions. Besides experimental data, lattice QCD results for low-energy hadronic matrix elements decisively contribute to precision tests in the beauty sector. Since the significance of these tests is predominantly limited by theoretical uncertainties, lattice computations with an overall accuracy of a few percent are highly desirable.

A quantity that has attracted much attention in this context recently is the CKM matrix element $|V_{ub}|$. It can be determined in different ways, in particular from inclusive semi-leptonic processes $B \to X_u\ell\nu$ (whose theoretical treatment involves the use of the optical theorem, the heavy quark expansion and perturbation theory), from exclusive semi-leptonic decays $B \to \pi\ell\nu$ involving the hadronic form factor $f_+(q^2)$, and from exclusive leptonic decays $B \to \tau\nu$ involving the hadronic decay constant f_B (where the latter two cases require lattice determinations of $f_+(q^2)$ and f_B in order to extract $|V_{ub}|$). Currently, there is a $\sim 3\sigma$ tension between the two exclusive (semi-leptonic and leptonic) determinations of $|V_{ub}|$, as well as tension with the estimate from inclusive B-meson decays. Precise and reliable lattice calculations with good control of systematic errors are required in order to answer the question whether this tension really hints at New Physics in the B-sector.

In order to describe B physics on the lattice with controlled errors, it is necessary to cover multiple physical scales differing by several orders of magnitude; in principle it would be desirable to have $\Lambda_{IR} = L^{-1} \ll m_\pi, \ldots, m_B \ll a^{-1} = \Lambda_{UV}$. What is achievable in practice is $L \gtrsim 4/m_\pi \approx 6\,\mathrm{fm}$ to suppress finite-size effects for light quarks, and $a \lesssim 1/(2m_D) \approx 0.05\,\mathrm{fm}$ to tame discretization errors in the charm sector and allow the use of relativic c-quarks. However, the b-quark scale $m_b \sim 4m_c$ has to be separated from the others in a theoretically sound way before simulating the theory; in this work, we employ the Heavy Quark Effective Theory (HQET) formulation [1] for the b-quark in heavy-light systems.

In this update on our earlier analysis [2], we are now able to include more statistic, as well as new measurements including a strange valence quark (although not all of the latter are fully analysed yet). The newest analysis also benefits from an improved estimate of the matching scale $L_1 = 0.401(13)\mathrm{fm}$, which previously dominated our eror on m_b, as well as a refinement of the procedure used to select the plateau regions from which we extract our estimates of the large-volume observables.

2. Methods

2.1 Non-perturbative HQET at $O(1/m_b)$

By performing a systematic asymptotic expansion of the QCD Lagrangian in $\Lambda_{QCD}/m_b \ll 1$, and truncating $\mathrm{O}\big(\Lambda_\mathrm{QCD}^2/m_\mathrm{b}^2\big)$ contributions, one arrives at the (continuum) HQET Lagrangian:

$$\mathscr{L}_\mathrm{HQET}(x) = \overline{\psi}_\mathrm{h}(x)\,D_0\,\psi_\mathrm{h}(x) \;-\; \omega_\mathrm{kin}\mathscr{O}_\mathrm{kin}(x) - \omega_\mathrm{spin}\mathscr{O}_\mathrm{spin}(x)\,, \tag{2.1}$$
$$\mathscr{O}_\mathrm{kin}(x) = \overline{\psi}_\mathrm{h}(x)\,\mathbf{D}^2\,\psi_\mathrm{h}(x)\,, \qquad \mathscr{O}_\mathrm{spin}(x) = \overline{\psi}_\mathrm{h}(x)\,\sigma\cdot\mathbf{B}\,\psi_\mathrm{h}(x).$$

Similarly, the time component of the heavy-light axial current A_0 (at $\mathbf{p}=0$) expands as

$$A_{0,\mathrm{R}}^\mathrm{HQET} = Z_\mathrm{A}^\mathrm{HQET}\left[A_0^\mathrm{stat} + c_\mathrm{A}^{(1)}A_0^{(1)}\right]\;,\; A_0^\mathrm{stat} = \overline{\psi}_\mathrm{l}\,\gamma_0\gamma_5\,\psi_\mathrm{h}\;,\; A_0^{(1)} = \overline{\psi}_\mathrm{l}\,\gamma_5\gamma_i\,\tfrac{1}{2}\,(\overrightarrow{\nabla}_i^\mathrm{S} - \overleftarrow{\nabla}_i^\mathrm{S})\,\psi_\mathrm{h}. \tag{2.2}$$

In the HQET approach, the $1/m$–terms appear as local operator insertions in correlation functions, and therefore the renormalizability of the static theory carries over to HQET (at each order in $1/m$). This ensures the existence of the continuum limit, once the HQET parameters $\omega_i \in \big\{m_\mathrm{bare}\,, Z_\mathrm{A}^\mathrm{HQET}, c_\mathrm{A}^{(1)}, \omega_\mathrm{kin}, \omega_\mathrm{spin}\big\}$ have been fixed through *non-perturbative* matching [3] so that no uncancelled power divergences in a^{-1} (which are induced by operator mixing in the effective theory) remain that would spoil taking the continuum limit.

The strategy of non-perturbative matching [4, 5] is as follows: The matching is performed in the Schrödinger Functional scheme in a small volume $L_1 \approx 0.4\,\mathrm{fm}$, where due to $am_\mathrm{b} \ll 1$ simulations with a relativistic b-quark are feasible. The HQET parameters ω_i are fixed by imposing the matching conditions

$$\Phi^\mathrm{HQET}(z,a) = \Phi^\mathrm{QCD}(z,0)\,, \qquad \Phi^\mathrm{QCD}(z,0) = \lim_{a\to 0}\Phi^\mathrm{QCD}(z,a)\,, \tag{2.3}$$

so that ω_i inherit the quark mass dependence from non-perturbatively renormalized QCD via their dependence on $z \equiv L_1 M$, where M is the RGI quark mass [6]. We then use a recursive finite-size scaling procedure to perform the step $L_1 \to L_2 = 2L_1$ and finally to make contact to physically large volumes $L_\infty \gtrsim \max(2\;\mathrm{fm}, 4/m_\pi)$.

As a result of this procedure [5], the $N_\mathrm{f} = 2$ HQET parameters $\omega_i(z,a)$ (which absorb the power divergences of HQET) are non-perturbatively known for a number of z–values around the b mass at the lattice spacings used in our large–volume simulations.

2.2 Large volume computations and techniques

Our large–volume measurements are performed on the $N_\mathrm{f} = 2$ CLS ensembles, which use the plaquette gauge action and non-perturbatively $\mathrm{O}(a)$ improved Wilson quarks and were generated using the DD-HMC [7] and/or the MP-HMC [8] algorithms. The ensembles fulfill the condition $Lm_\pi \gtrsim 4$, and we use a range of pion masses $(190 \lesssim m_\pi \lesssim 440)\,\mathrm{MeV}$ at three lattice spacings $(0.05 \lesssim a \lesssim 0.08)\,\mathrm{fm}$, where the scale has been set through f_K [9].

In the computation of the static-light correlation functions, we use HYP smearing for the static quarks [13] as well as a variant of the stochastic all-to-all propagator method for the relativistic quarks (with multiple noise sources per configuration and full time-dilution) [10, 12] in order to improve statistical precision.

To control excited state contaminations to the HQET energies and matrix elements, we solve the Generalized Eigenvalue Problem (GEVP) [11, 12]

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)\,, \qquad t_0 < t < 2t_0\,, \tag{2.4}$$

β	$a\,[\mathrm{fm}]$	$L^3 \times T$	$m_\pi\,[\mathrm{MeV}]$	#
5.2	0.075	$32^3 \times 64$	380	1000
		$32^3 \times 64$	330	500
5.3	0.065	$32^3 \times 64$	440	1000
		$48^3 \times 96$	310	500
		$48^3 \times 96$	270	600
		$64^3 \times 128$	**190**	**600**
5.5	0.048	$48^3 \times 96$	440	400
		$48^3 \times 96$	**340**	**900**
		$64^3 \times 128$	270	900

Table 1: Overview of the CLS configurations used in this analysis. The ensembles shown in bold are new to the present analysis.

for an $N \times N$ correlator matrix $C(t)$ with $N = 3$, and derive the energies and matrix elements from the eigenvalues and eigenvectors λ_n, v_n, such that the corrections to the energies behave like $\propto \exp\{-(E_{N+1} - E_1)t\}$ and the corrections to the matrix elements like $\propto \exp\{-(E_{N+1} - E_1)t_0\} \times \exp\{-(E_2 - E_1)(t - t_0)\}$; for details see [11, 2]. We minimize our systematic errors by a conservative choice of plateau ranges: for fixed t_max we vary t_min such that the errors $\sigma(t_\mathrm{min})$ of the plateau average $A(t_\mathrm{min})$ fulfil $\sigma_\mathrm{stat} \gtrsim 3\sigma_\mathrm{sys}$. Specifically, for each value of t_min we compute

$$r(t_\mathrm{min}) = \frac{|A(t_\mathrm{min}) - A(t_\mathrm{min} - \delta)|}{\sqrt{\sigma^2(t_\mathrm{min}) + \sigma^2(t_\mathrm{min} - \delta)}}\,, \tag{2.5}$$

where $\delta = \frac{2}{3} r_0 \approx 2/(E_{N+1} - E_1)$ is chosen such that we expect the influence of excited-state contributions to have decayed by a factor $\sim \mathrm{e}^2$, and take the first value of t_min satisfying $r(t_\mathrm{min}) \leq 3$.

3. Results

Combining the HQET parameters with the GEVP results for matrix elements and energies, we obtain observables depending on the pseudoscalar (sea) mass m_π and lattice spacing a, as well as (through the HQET parameters) on the heavy quark mass parameter z.

3.1 Mass of the b-quark

We fix m_b by imposing $m_\mathrm{B}(z_\mathrm{b}, m_\pi^\mathrm{exp}, a = 0) \equiv m_\mathrm{B}^\mathrm{exp} = 5279.5\mathrm{MeV}$ through the fit ansatz

$$m_\mathrm{B}\,(z, m_\pi, a, \mathrm{HYPn}) = B(z) + Cm_\pi^2 - \frac{3\widehat{g}^2}{16\pi f_\pi^2} m_\pi^3 + D_\mathrm{HYPn} a^2, \qquad \widehat{g} = 0.51(2)\,[16]. \tag{3.1}$$

Using the NLO mass definition of HQET, $m_\mathrm{B} = m_\mathrm{bare} + E^\mathrm{stat} + \omega_\mathrm{kin} E^\mathrm{kin} + \omega_\mathrm{spin} E^\mathrm{spin}$, we find

$$z_\mathrm{b} = 13.34(33)(13)_z\,, \qquad \text{or equivalently} \qquad \overline{m}_\mathrm{b}(\overline{m}_\mathrm{b}) = 4.22(10)(4)_z\,\mathrm{GeV}\,. \tag{3.2}$$

Having fixed the physical mass of the b-quark, we interpolate the HQET parameters to $z \equiv z_\mathrm{b}$.

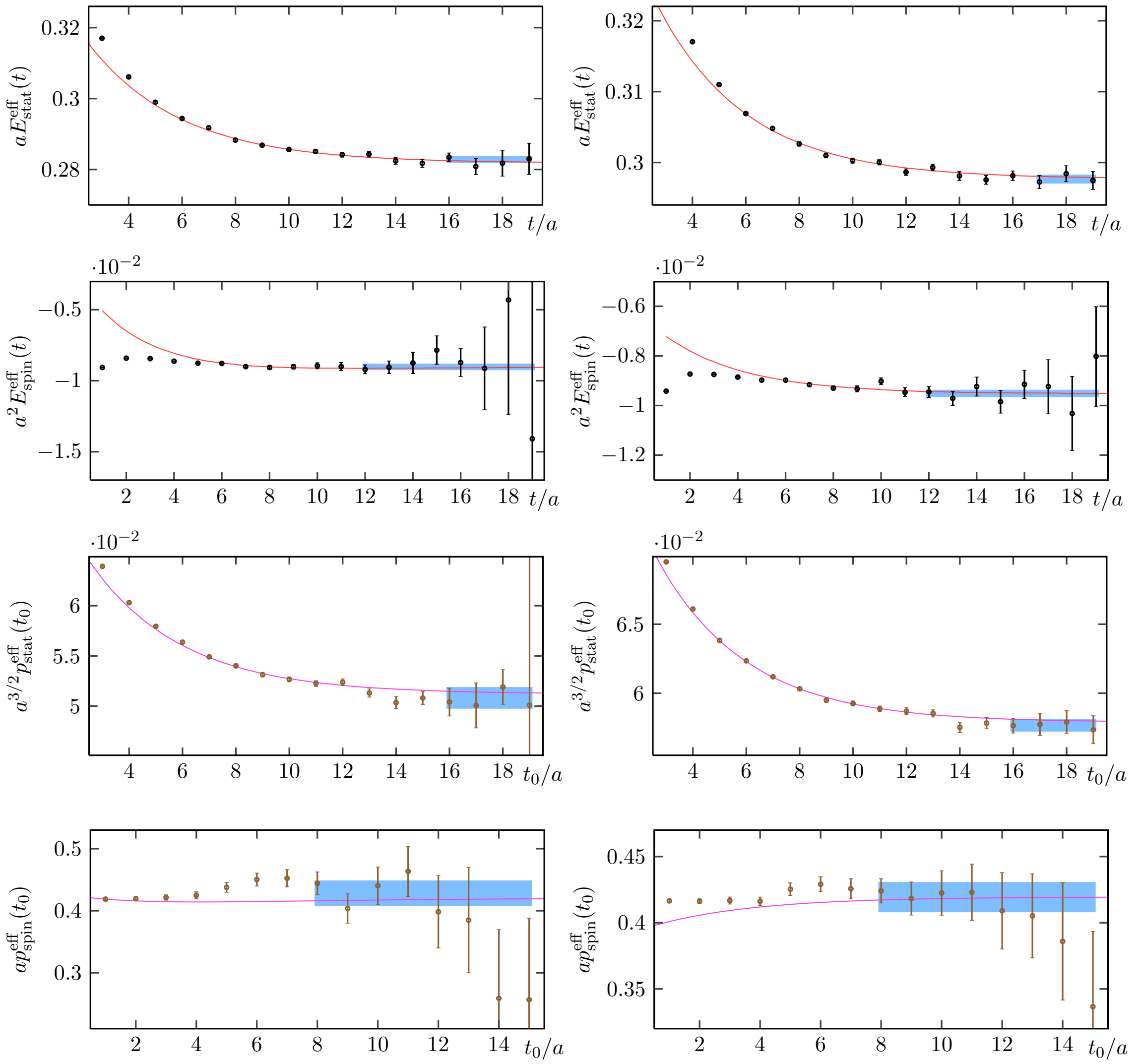

Figure 2: Examples of preliminary results from an ensemble with $a = 0.048\,\mathrm{fm}$, $m_\pi \simeq 340\,\mathrm{MeV}$ and $L^3 \times T = 48^3 \times 96$; *left:* HYP2 static-light; *right:* HYP2 static-strange. Shown are the effective masses or matrix elements as a function of $t = t_0 + a$, together with a band indicating the extracted plateaux. The result of a global fit to the data at $t_0/a \geq 6$, which also includes a large number of data points not shown here (including results from $t > t_0 + a$, and from $N = 4, 5$), is also displayed as a solid curve, showing good agreement between the fit and the more conservative analysis ultimately employed.

3.2 B-meson decay constants at NLO of HQET

We determine the $\mathrm{B_r}$-meson decay constant for light ($\mathrm{r} = d$) and strange ($\mathrm{r} = s$) quarks through

$$\ln(a^{3/2} f_{\mathrm{B_r}} \sqrt{m_{\mathrm{B_r}}/2}) = \ln(Z_\mathrm{A}^\mathrm{HQET}) + \ln(a^{3/2} p_\mathrm{r}^\mathrm{stat}) + b_\mathrm{A}^\mathrm{stat} a m_{\mathrm{q,r}} + \omega_\mathrm{kin} p_\mathrm{r}^\mathrm{kin} + \omega_\mathrm{spin} p_\mathrm{r}^\mathrm{spin} + c_\mathrm{A}^{(1)} p_\mathrm{r}^{\mathrm{A}^{(1)}} \,. \tag{3.3}$$

In order to estimate a systematic error in our combined chiral and continuum extrapolation, we use both a fit motivated by Heavy Meson Chiral Perturbation Theory (HMχPT) [14, 15] and a linear

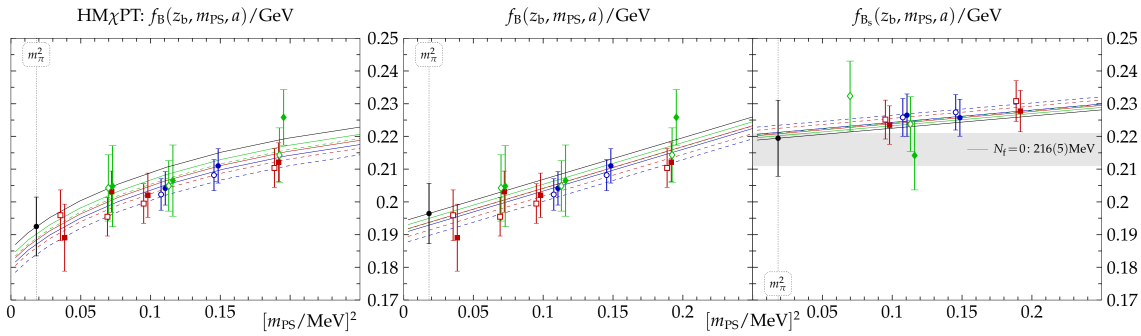

Figure 3: *Left:* HMχPT extrapolation of f_B; *centre/right:* linear extrapolation of f_B and f_{B_s}. The blue, red and green points correspond to ensembles at $a = 0.075$ fm, 0.065 fm and 0.048 fm, respectively. Filled symbols denote HYP2, empty symbols HYP1; the joint continuum and chiral extrapolation is shown in black, with the fit formulae evaluated at each given lattice spacing shown in colour (solid for HYP2, dashed for HYP1).

fit in m_π^2:

$$f_{B_r}(m_\pi, a, \text{HYPn}) = b_r + c_r m_\pi^2 + d_{r,\text{HYPn}} a^2 , \qquad \text{(linear)} \tag{3.4}$$

$$f_B(m_\pi, a, \text{HYPn}) = b' \left[1 - \frac{3}{4} \frac{1+3\hat{g}^2}{(4\pi f_\pi)^2} m_\pi^2 \ln(m_\pi^2) \right] + c' m_\pi^2 + d'_{\text{HYPn}} a^2 . \qquad (\text{HM}\chi\text{PT}) \tag{3.5}$$

Both of these fit formulae are based on a simultaneous expansion in a and $1/m_b$, where $\mathrm{O}(a)$ discretization effects are dropped, since they are also $\mathrm{O}(1/m_b)$; for f_B, only the non-analytic terms coming from the lowest order in $1/m_b$ are retained.

Our analysis for Lattice 2012 (where for f_{B_s} not all ensembles are analysed yet) gives

$$f_B = 193(9)_{\text{stat}}(4)_\chi \,\text{MeV}, \qquad f_{B_s} = 219(12)_{\text{stat}} \,\text{MeV}.$$

The quenched value was $f_{B_s} = 216(5)$ (using $r_0 = 0.5$fm), indicating small quenching effects. Note that our present $N_f = 2$ estimate of f_B is about one standard deviation larger than the previous value from [2], due both to increased statistics and the inclusion of additional sea quark masses. A further improvement is an improved control of systematic errors from excited-state contributions (which enlarges the statistical errors).

4. Conclusions

We have presented results with a significant increase in statistics and improved control of systematic errors. We obtain a value for f_B that is similar to those obtained by other collaborations [17, 18, 19, 20]. In the case of f_{B_s}, we find quenching effects to be undetectable, while for f_B such a comparison is not meaningful due to the pathological chiral behaviour of the quenched theory.

We are now finalizing our analysis, and more detailed publications are forthcoming. We are also investigating further spectral quantities within our approach, such as the B-meson spin splittings, and are preparing for the determination of $B \to$ light semileptonic form factors [21].

Acknowledgments

This work is supported in part by the SFB/TR 9 and grant HE 4517/2-1 (P.F. and J.H.) of the Deutsche Forschungsgemeinschaft and by the European Community through EU Contract MRTN-CT-2006-035482, "FLAVIAnet". P.F. thanks the ECT* in Trento for support during the workshop "Beautiful Mesons and Baryons on the Lattice". We thank our colleagues in the CLS effort for the joint production and use of gauge configurations. We gratefully acknowledge the computer resources provided within the Distributed European Computing Initiative by the PRACE-2IP, with funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement RI-283493, by the Grand Équipement National de Calcul Intensif at CINES in Montpellier, and by the John von Neumann Institute for Computing at FZ Jülich, at the HLRN in Berlin, and at DESY, Zeuthen.

References

[1] E. Eichten and B. Hill, Phys. Lett. B234 (1990) 511; E. Eichten and B. Hill, Phys. Lett. B 243 (1990) 427.

[2] B. Blossier *et al.*, PoS LATTICE2011 (2011) 280, arXiv:1112.6175.

[3] J. Heitger and R. Sommer, JHEP 0402 (2004) 022, hep-lat/0310035.

[4] B. Blossier *et al.*, JHEP 1006 (2010) 002, arXiv:1001.4783.

[5] B. Blossier *et al.*, JHEP 1209 (2012) 132, arXiv:1203.6516.

[6] P. Fritzsch, J. Heitger and N. Tantalo, JHEP 1008 (2010) 074, arXiv:1004.3978.

[7] M. Lüscher, Comput. Phys. Commun. 156 (2004) 209, hep-lat/0310048; Comput. Phys. Commun. 165 (2005) 199, hep-lat/0409106; JHEP 0712 (2007) 011, arXiv:0710.5417.

[8] M. Hasenbusch, Phys. Lett. B 519 (2001) 177, hep-lat/0107019; M. Marinkovic and S. Schaefer, PoS LATTICE2010 (2010) 031, arXiv:1011.0911.

[9] P. Fritzsch *et al.*, Nucl.Phys. B865 (2012) 397, arXiv:1205.5380.

[10] J. Foley *et al.*, Comput. Phys. Commun. 172 (2005) 145, hep-lat/0505023.

[11] B. Blossier *et al.*, JHEP 0904 (2009) 094, arXiv:0902.1265.

[12] B. Blossier *et al.*, JHEP 1005 (2010) 074, arXiv:1004.2661; B. Blossier *et al.*, JHEP 1012 (2010) 039, arXiv:1006.5816.

[13] A. Hasenfratz and F. Knechtli, Phys. Rev. D 64 (2001) 034504, hep-lat/0103029; M. Della Morte *et al.*, Phys. Lett. B 581 (2004) 93, hep-lat/0307021; M. Della Morte, A. Shindler and R. Sommer, JHEP 0508 (2005) 051, hep-lat/0506008.

[14] J.L. Goity, Phys. Rev. D46 (1992) 3929, hep-ph/9206230.

[15] S.R. Sharpe and Y. Zhang, Phys. Rev. D53 (1996) 5125, hep-lat/9510037.

[16] J. Bulava, M.A. Donnellan and R. Sommer, PoS LATTICE2010 (2010) 303, arXiv:1011.4393.

[17] H. Na *et al.* [HPQCD Collaboration], Phys. Rev. D **86** (2012) 034506, arXiv:1202.4914.

[18] A. Bazavov *et al.* [Fermilab Lattice and MILC Collaboration], Phys. Rev. D **85** (2012) 114506, arXiv:1112.3051.

[19] C. McNeile *et al.* [HPQCD collaboration], Phys. Rev. D **85** (2012) 031503, arXiv:1110.4510.

[20] P. Dimopoulos *et al.* [ETM Collaboration], JHEP **1201** (2012) 046, arXiv:1107.1441.

[21] F. Bernardoni *et al.* [ALPHA Collaboration], in these proceedings, arXiv:1210.3478.