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Excited and exotic charmonium spectroscopy from lattice QCD

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We study the charmonium spectrum in full QCD on anisotropic lattices generated by Hadron Spectrum Collaboration. We adopt a large basis of interpolating operators to extract the excited charmonium states using the variational method. A detailed spectrum of excited charmonium mesons in many J^{PC} channels is obtained. Some exotic hybrid states (with $J^{PC} = 0^{+-}$, 1^{-+} , 2^{+-}) are also studied.

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1. Introduction

There has been a resurgence of interest in charmonium spectroscopy over the last decade driven by experimental observations at the B-factories, CLEO-c, the Tevatron and BES of rather narrow states close to or above open-charm thresholds. Some had been predicted but hitherto unobserved whereas many were unexpected, for example the enigmatic $X(3872)$, $Y(4260)$ and other "X,Y,Z"s. Prior to these discoveries, the relatively small number of observed mesons all fitted into the simple pattern predicted by quark models. The new states have spurred theoretical discussion as to their nature with suggestions including conventional quark-antiquark states appearing at unexpected masses, hybrid mesons in which the gluonic field is excited, molecular mesons and tetraquarks. To date, however, there has been no observation of a state in the charmonium region with manifestly exotic J^{PC} (J,P and C are respectively spin, parity and charge-conjugation quantum numbers); an observation of such an exotic would be a smoking gun for physics beyond quark potential models. Experimental interest continues with BESIII, experiments at the LHC, and the planned PANDA experiment at GSI/FAIR. An extensive review of the experimental and theoretical situation is given in Ref. [1].

In this proceeding we present a dynamical lattice calculation of the excited spectrum of charmonium mesons. We will show how we can reliably extract an unprecedented number of states, including the spectrum of exotics below around 4.5 GeV, and identify their continuum quantum numbers. The results of this study have been presented in Ref. [2].

2. The lattice actions

For this study, we make use of the substantial ensembles of anisotropic gauge-field configurations generated by the Hadron Spectrum Collaboration [3, 4]. The gauge action is tree-level Symanzik-improved while in the fermion sector we use the anisotropic Shekholeslami-Wohlert action with tree-level tadpole improvement and three-dimensional stout-link smearing [5] of gauge fields. The fields are drawn from the importance sampling distribution appropriate for studies with two dynamical light quarks and a dynamical strange quark ($N_f = 2+1$); more details are given in Refs. [3, 4].

The charm quark action is the same as that used for the light and strange quarks, except that the parameter describing the weighting of spatial and temporal derivatives in the lattice action is chosen to give a relativistic dispersion relation for low-momentum mesons containing charm quarks. This determination is described in more details in Refs. [3, 2]. The bare mass parameter for the valence charm quarks is determined by ensuring the ratio of the η_c meson and the Ω baryon masses takes its experimental value.

To quote energies and lengths in physical units we follow Ref. [4] and consider the ratio of the mass of the Ω baryon measured on these ensembles, $a_t m_\Omega = 0.2951(22)$ [6], to the experimental mass, $m_\Omega = 1672.45(29)$ MeV [7]. Setting the scale in this way, we find the parameters in the lattice action used in this study correspond to a spatial lattice spacing of $a_s \sim 0.12$ fm and a temporal lattice spacing approximately 3.5 times smaller, $a_t^{-1} \sim 5.7$ GeV. Two volumes are used, $(L/a_s)^3 \times (T/a_t) = 16^3 \times 128$ and $24^3 \times 128$, corresponding to spatial extents of ~ 1.9 fm and ~ 2.9 fm respectively. Full details of the lattice actions and parameters are given in Refs. [3, 4, 2].

3. Operator construction and correlator analysis

In lattice calculations, meson masses are extracted from two-point correlation functions,

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle \;, \tag{3.1}$$

where the interpolating operators $\mathcal{O}_j^\dagger(0)$ create the state of interest at $t = 0$ and $\mathcal{O}_i(t)$ annihilate the state at a later Euclidean time t. Inserting a complete set of eigenstates of the Hamiltonian we obtain the spectral decomposition,

$$C_{ij}(t) = \sum_{\mathfrak{n}} \frac{Z_i^{\mathfrak{n}*} Z_j^{\mathfrak{n}}}{2E_{\mathfrak{n}}} e^{-E_{\mathfrak{n}}t} \;, \tag{3.2}$$

where the vacuum-state matrix elements, $Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n}|\mathcal{O}_i^\dagger|0\rangle$, are referred to as *overlaps*, and the sum is over a discrete set of states because the calculation is performed in a finite volume. It is essential that the operators overlap well with the states under consideration. A well-established way to achieve this is to apply a smearing to the quark fields in the operators to emphasise the relevant low-energy modes. We use the *distillation* technique described in Ref. [8].

In order to reliably and precisely extract the energies of excited states, we require a large basis of operators in each quantum number channel. We use the derivative-based construction for operators described in Refs. [9, 10]: gauge-covariant spatial derivatives are combined with a gamma matrix within a fermion bilinear. The operator is of the general form

$$\bar{\psi}\Gamma \overleftrightarrow{D}_i \overleftrightarrow{D}_j \cdots \psi \;, \tag{3.3}$$

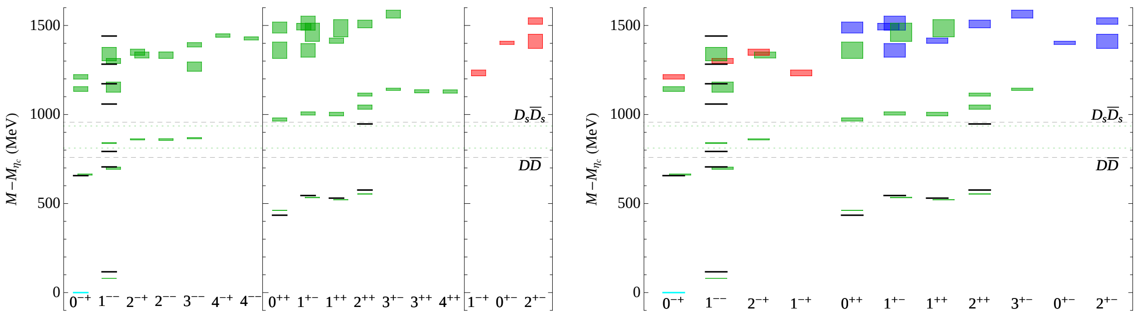
where $\overleftrightarrow{D} \equiv \overleftarrow{D} - \overrightarrow{D}$ is a lattice-discretised gauge-covariant derivative. As discussed in Ref. [10], one can combine any number of derivatives with gamma matrices to construct operators with the desired quantum numbers, $\mathcal{O}^{J,M}$, where M is the J_z component. The choice of Γ and the way in which the derivatives have been coupled determine the parity, P, and charge conjugation, C, quantum numbers. In this work we use the operators up to three derivatives.

In lattice QCD calculations the full three-dimensional rotational symmetry of an infinite volume continuum is reduced to the symmetry group of a cube. Instead of the infinite number of irreducible representations labelled by spin, $J \geq 0$, we instead have a finite number of lattice irreducible representations (*irreps*); the five single-cover irreps for states at rest are A_1, T_1, T_2, E and A_2. Because a state with $J > 1$ is split across many lattice irreps and each lattice irrep contains an infinite tower of J, it is not straightforward to identify the J of states extracted in lattice computations. Refs. [9, 10] presented a method to address this problem and we follow this method. For the details of the spin identification, please see Ref. [2].

We use the variational method [11, 12] to extract spectral information from the two-point correlation functions with the particular implementation described in detail in Ref. [10]. In brief, for each lattice irrep we form a matrix of correlators, $C_{ij}(t)$, where i and j label operators in our basis for that irrep. The best extraction of the energies and overlaps then follows from solving a generalised eigenvalue problem,

$$C_{ij}(t)v_j^{\mathfrak{n}} = \lambda^{\mathfrak{n}}(t,t_0)C_{ij}(t_0)v_j^{\mathfrak{n}} \;, \tag{3.4}$$

where an appropriate reference time-slice t_0 is chosen as described in Refs. [10, 13], the energies are determined by fitting the dependence of the eigenvalues, $\lambda^{\mathfrak{n}}$, on $t - t_0$.



(a) (b)

Figure 1: Summary of the charmonium spectrum up to around 4.5 GeV labelled by J^{PC}. In the panel (a), the red and green boxes are the masses calculated on the 24^3 volume; black lines are experimental values from the PDG [7]. We show the calculated (experimental) masses with the calculated (experimental) η_c mass subtracted. The vertical size of the boxes represents the one sigma statistical uncertainty on either side of the mean. The dashed lines indicate the lowest non-interacting $D\bar{D}$ and $D_s\bar{D}_s$ levels using the D and D_s masses measured on the 16^3 ensemble (fine green dashing) and using the experimental masses (coarse grey dashing). The panel (b) shows the spectrum for the subset of J^{PC} channels in which we identify candidates for hybrid mesons. Red (dark blue) boxes are states suggested to be members of the lightest (first excited) hybrid supermultiplet as described in the text and green boxes are other states.

4. Results

Our final results, the well-determined states on the 24^3 volume, are shown in Fig. 1(a) along with experimental masses taken from PDG summary tables [7]. The $X(3872)$ is not shown because its J^{PC} (1^{++} or 2^{-+}) has not been determined experimentally. In the plot, the calculated (experimental) mass of the η_c has been subtracted from the calculated (experimental) masses in order to reduce the systematic error from the tuning of the bare charm quark mass. The dashed lines indicate the lowest non-interacting $D\bar{D}$ and $D_s\bar{D}_s$ levels using the D and D_s masses measured on the 16^3 ensemble (fine green dashing) and using the experimental masses (coarse grey dashing). We note that higher up in the spectrum the states become less well determined. This is in part because, although we have a relatively large number of operators in each channel, the basis is still restricted in size. In order to better determine these states we would need to include operators with different radial structures and, in order to determine states with $J \geq 5$, different angular structures, for example higher numbers of derivatives.

4.1 Interpretation of the results

Many of the states with non-exotic J^{PC} in Fig. 1(a) appear to follow the $n^{2S+1}L_J$ pattern predicted by quark potential models, where S is the spin of the quark-antiquark pair, L is the relative orbital angular momentum, J is the total spin of the meson and n is the radial quantum number.

In the left panel of Fig. 1(a) we show the negative parity states. We observe the ground-state S-wave pair $[0^{-+}, 1^{--}]$ and at $M - M_{\eta_c} \sim 700$ MeV the first excitation; a second excitation is observed at $M - M_{\eta_c} \sim 1150$ MeV. There is a complete D-wave set $[(1,2,3)^{--}, 2^{-+}]$ just above the $D\bar{D}$ threshold. In the region above 1200 MeV there is a complete set of D-wave excitations and parts of a G-wave indicated by the presence of spin-4 states. In addition, there are three states at $M - M_{\eta_c} \sim 1200$ to 1300 MeV with $J^{PC} = (0,2)^{-+}, 1^{--}$ which do not appear to fit with the pattern expected by quark models. We note that these three states all have a relatively large overlap onto operators proportional to the gluonic field-strength tensor, something not observed for the states that fit into quark-antiquark supermultiplets. Following Ref. [10] we suggest that these 'excess' states can be identified as non-exotic hybrid mesons and we return to this in Section 4.2.

In the middle panel of Fig. 1(a) we show the positive parity states. Below $D\bar{D}$ threshold there is a clear P-wave set $[(0,1,2)^{++}, 1^{+-}]$. In the region of $M - M_{\eta_c} \sim 1000$ MeV there is a P-wave excitation and, slightly higher, an F-wave $[(2,3,4)^{++}, 3^{+-}]$. The band of states around $M - M_{\eta_c} \sim$ 1400 MeV probably contains part of the second excitation of the P-wave set and several non-exotic hybrids which lie above the lightest negative-parity hybrids; we will discuss these hybrid candidates in Section. 4.2.

The states with exotic J^{PC} are presented in the right panel of Fig. 1(a). These cannot consist solely of a quark-antiquark pair – more degrees of freedom are necessary. In general, there are several possible interpretations such as hybrid mesons where the gluonic field is excited, molecular mesons or tetraquarks. The exotic states in our spectrum have significant overlap onto operators with non-trivial gluonic structure and this suggests that we can identify them as hybrid mesons. In Section 4.2 we discuss the hybrid meson phenomenology suggested by our spectra.

4.2 Exotic mesons and hybrid phenomenology

In Fig. 1(b) we show the charmonium spectrum for the subset of J^{PC} channels in which, by considering operator-state overlaps, we identify candidate hybrid mesons. A state is suggested to be dominantly hybrid in character if it has a relatively large overlap onto an operator proportional to the commutator of two covariant derivatives, the field-strength tensor. We note that within QCD non-exotic hybrids can mix with conventional charmonia. We find that the lightest exotic meson has $J^{PC} = 1^{-+}$ and is nearly degenerate with the three states observed in the negative parity sector suggested to be non-exotic hybrids, $(0,2)^{-+}, 1^{--}$. Higher in mass there is a pair of states, $(0,2)^{+-}$, and a second 2^{+-} state slightly above this. Not shown on the figures, an excited 1^{-+} appears at around 4.6 GeV, there is an exotic 3^{-+} state at around 4.8 GeV and the lightest 0^{--} exotic does not appear until above 5 GeV.

The observation that there are four hybrid candidates nearly degenerate with $J^{PC} = (0,1,2)^{-+}, 1^{--}$, coloured red in Fig. 1(b), is interesting. This is the pattern of states predicted to form the lightest hybrid supermultiplet in the bag model [14, 15] and the P-wave quasiparticle gluon approach [16], or more generally where a quark-antiquark pair in S-wave is coupled to a 1^{+-} chromomagnetic gluonic excitation. This is not the pattern expected in the flux-tube model [17] or with an S-wave quasigluon. In addition, the observation of two 2^{+-} states, with one only slightly heavier than the other, appears to rule out the flux-tube model which does not predict two such states so close in mass. The pattern of J^{PC} of the lightest hybrids is the same as that observed in light meson sector [10, 18]. They appear at a mass scale of $1.2 - 1.3$ GeV above the lightest conventional charmonia. This suggests that the energy difference between the first gluonic excitation and the ground state in charmonium is comparable to that in the light meson [18] and baryon [19] sectors.

A heavier hybrid supermultiplet composed of a P-wave colour-octet quark-antiquark pair coupled to a gluonic field with $J_g^{P_g C_g} = 1^{+-}$ would contain states with $J^{PC} = 0^{+-}$, $(1^{+-})^3$, $(2^{+-})^2$, 3^{+-}, 0^{++}, 1^{++}, 2^{++}. The exotic 0^{+-} state and two 2^{+-} states are observed in our spectrum with small mass splittings and these have relatively large overlap onto operators with the structure of a P-wave $q\bar{q}$ coupled to a gluonic 1^{+-}. In the non-exotic positive-parity sector, we have evidence from similar operator overlaps that in the region around $M - M_{\eta_c} \sim 1400$ to 1500 MeV there are 0^{++}, 1^{++}, 2^{++} and 3^{+-} hybrids as well as candidates for three 1^{+-} hybrids. We therefore observe candidates for all expected members of this excited hybrid supermultiplet and these are coloured dark blue in Fig. 1(b). To pin these states down more precisely we would need to add operators with more derivatives and spatial structures to our basis; this is beyond the scope of the current work.

5. Summary

Using distillation and the variational method with a large basis of carefully constructed operators, we have successfully computed an extensive spectrum of charmonium mesons with high statistical precision and reliably identified the continuum J^{PC} of the extracted states. The large number of extracted states up to 4.5 GeV, across all PC combinations, goes beyond any other dynamical calculation. For the first time, we have computed the dynamical spectrum of charmonia with exotic quantum numbers (0^{+-}, 1^{-+}, 2^{+-}). A determination of these exotic states is particularly interesting because it points to the presence of explicit gluonic degrees of freedom. The spectrum of non-exotic states has many features in common with the $n^{2S+1}L_J$ pattern expected by quark models along with some states which do not appear to fit into such a classification. We have suggested that these extra states can be interpreted as non-exotic hybrids and have identified the lightest hybrid supermultiplet consisting of states with $J^{PC} = (0,1,2)^{-+}, 1^{--}$, as well as an excited hybrid supermultiplet.

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