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Progress Towards $\Delta I = 1/2$ $K \to \pi\pi$ decays with G-parity Boundary Conditions

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Christopher Kelly*

Columbia University,
905 Pupin Hall,
116th St & Broadway,
New York, NY 10027, USA.
E-mail: `ckelly@phys.columbia.edu`

For the RBC and UKQCD collaborations

We discuss the RBC&UKQCD collaboration's efforts to calculate the $K \to \pi\pi$ decay amplitude with an Isospin-zero final state on the lattice using domain wall fermions. We focus upon the use of G-parity boundary conditions for obtaining physical kinematics, presenting our progress in their implementation and some preliminary results.

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*Speaker.

1. Introduction

In the Standard Model, CP-violation occurs due to a non-zero phase δ in the CKM matrix that parameterises the mixing between quark flavours through the weak interaction. As a result the low-energy effective Hamiltonian of the weak interactions and CP become non-commuting operators. This manifests indirectly as a mixing of CP-eigenstates, e.g. in the $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ systems, and also directly in decays, as was first observed in the decay of CP-odd kaon states into the CP-even two-pion state.

$K \to \pi\pi$ decays can occur via the $\Delta I = 3/2$ channel to an isospin-2 final state,e.g. $K^\pm \to \pi^\pm\pi^0$, and also via the $\Delta I = 1/2$ channel to an isospin-0 final state, e.g. $K^0 \to (\pi^+\pi^-)_{I=0}$ and $K^0 \to (\pi^0\pi^0)_{I=0}$ (we include a subscript because $\pi^+\pi^-$ and $\pi^0\pi^0$ can also form $I = 2$ states). The decay amplitudes are labelled A_2 and A_0 respectively. Direct CP-violation manifests here as a difference in the complex phases of the two amplitudes:

$$\varepsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}} \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right), \tag{1.1}$$

where $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$ and δ_i are the strong-rescattering phase shifts. The quantity ε' parameterises the amount of direct CP-violation, and is highly sensitive to beyond the Standard Model sources of CP-violation. As a result there is a strong motivation for obtaining a precise first-principles determination of this quantity within the Standard Model.

Non-perturbative strong interactions play an important role in these decays, for example they give rise to a strong preference for the kaon to decay into the isospin-0 state over the isospin-2 state – the so-called $\Delta I = 1/2$ rule – the mechanism for which is not yet fully understood. Lattice QCD is currently the only method by which the strong dynamics can be studied from first principles, however measuring such hadronic decays on the lattice has only recently become possible and has required significant theoretical and technological advances. The RBC&UKQCD collaboration have recently published [5, 6] the first realistic ab initio calculation of A_2 from the $K^+ \to \pi^+\pi^0$ decay channel. Unfortunately the calculation of A_0 poses further challenges that must be overcome before we can provide a complete first-principles calculation of ε'.

Below we discuss the difficulties in the measurement of A_0 and present G-parity boundary conditions as a solution to one such problem. We then detail their implementation on the lattice and present some of the more unusual aspects of calculations involving them. We present some preliminary results showing that the boundary conditions have the desired effect, and discuss further difficulties that must be overcome before the technique can be applied to the $K \to \pi\pi$ decays.

2. Challenges in Measuring A_0 and G-parity Boundary Conditions

One difficulty in measuring the $\Delta I = 1/2$ amplitude is the presence of disconnected diagrams in the $\pi\pi$ propagator that appear due to the fact that the I=0 two-pion state, unlike the I=2 state, has vacuum quantum numbers and can therefore be absorbed into the vacuum and recreated at a later time. The measurement of disconnected diagrams on the lattice is usually very noisy, requiring large statistics and many source positions for the quark propagators to resolve. Fortunately, using advanced propagator measurement techniques such as all-mode-averaging or all-to-all propagators, such calculations are now feasible. This is demonstrated in ref. [4], in which the all-to-all technique was used in calculating the $\Delta I = 1/2$ $K \to \pi\pi$ decay at zero momentum transfer on a $16^3 \times 32$ domain wall lattice with unphysical quark masses. Of course the larger-lattice, physical calculations we outline in the conclusions of this document will require considerably more powerful machines such as the IBM BlueGene/Q machines now available to RBC&UKQCD.

A more difficult challenge is that of conserving energy in the decay, which requires the two pions to be moving away from each other in the kaon rest frame. As the momentum is discretised by the finite lattice volume, careful tuning of the lattice size is necessary. For the calculation of A_2 we investigated several approaches, finding that the strongest signal was obtained using a stationary kaon in the lattice reference frame. The required non-zero momentum $\pi\pi$ state is an excited state of the $\pi\pi$ spectrum, hence the measurement of the physical $K \to \pi\pi$ decay requires the subtraction of the much larger contribution from the unphysical decay to the zero momentum ground state. This can be avoided using antiperiodic boundary conditions (APBC) on the down quark propagators such that their lowest momentum is π/L, where L is the lattice spatial box length. Any charged pions formed from these therefore also have a minimum momentum of π/L. This introduces two further difficulties: firstly the quark momentum cancels between the two down quarks in the neutral pion state, hence we are restricted to decay channels containing only charged pions; and secondly applying APBC only to the down quarks breaks the isospin symmetry, allowing states of different isospin to mix. For the A_2 calculation, both of these difficulties were avoided by using the Wigner-Eckart theorem to relate the physical $\Delta I_z = 1/2$, $K^+ \to \pi^+\pi^0$ decay to the unphysical $\Delta I_z = 3/2$, $K^+ \to \pi^+\pi^+$ decay, which contains only charged pions and is protected from mixing with other isospin states by virtue of charge conservation due to it being the only charge-2 state.

For the calculation of A_0 we must consider both the $K^0 \to (\pi^+\pi^-)_{I=0}$ and $K^0 \to (\pi^0\pi^0)_{I=0}$ channels. Here there are no alternate $I = 0$ states to which these can be related in order to avoid the effects of the isospin breaking introduced by imposing APBC on the down quarks. In order to proceed we could attempt to pick out the excited state contribution in the decay, however, as we expect the measurement to be very noisy due to the presence of disconnected diagrams, it is highly unlikely that we would be able to find a signal for the excited state contribution. Another possibility have a non-zero total momentum in the lattice reference frame; the lowest energy configuration has a moving kaon in the initial state, along with one stationary pion and one moving pion (carrying the kaon momentum) in the final state. For a typical lattice this configuration contains large momentum amplitudes that are typically noisy and hard to compute [3]. The alternative is to find a method of imposing momentum on both the charged and neutral pions without breaking isospin. One possibility is to use G-parity boundary conditions [1, 2] (GPBC).

G-parity is the combined action of the charge conjugation operation and an isospin rotation by π radians about the y-axis: $\hat{G} = \hat{C}e^{i\pi\hat{I}(y)}$. Both the charged and neutral pions are G-parity odd eigenstates, hence applying this operation on the lattice boundary is equivalent to imposing APBC on the pion states, giving them a minimum momentum of π/L. As G-parity commutes with isospin we can avoid any mixing of the final states with those of different isospin. At the quark level,

$$\hat{G}\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} -C\bar{d}^T \\ C\bar{u}^T \end{pmatrix}, \tag{2.1}$$

where $C = \gamma^2\gamma^4$ in Euclidean space. If we define $\psi^{(0)} = d$ and $\psi^{(1)} = C\bar{u}^T$ ($\bar{\psi}^{(1)} = -u^T C^\dagger$), the GPBC (here in the x-direction) take on the simple form:

$$\psi^{(0)}(s + L\hat{x}) = \psi^{(1)}(s), \qquad \psi^{(1)}(s + L\hat{x}) = -\psi^{(0)}(s), \tag{2.2}$$

where s is a general lattice coordinate. This implies that the fields $\psi^{(i)}$ are antiperiodic in $2L$ and therefore that the quarks have a minimum momentum of $\pi/2L$. Due to the presence of disconnected diagrams, these boundary conditions must be applied in both the valence and sea sectors, necessitating the generation of a new set of ensembles.

3. Implementation of G-parity Boundary Conditions

3.1 Gauge Field Boundary Conditions

Imposing GPBC on the quarks requires a modification of the boundary conditions and gauge transformations of the links. To see this consider the bilinear operator $\psi^{(0)}(L-1)U_x(L-1)\psi^{(0)}(L)$ at the lattice boundary in the x-direction, where we have suppressed all but the x-coordinate in the brackets, and impose GPBC on the quarks: $\bar{\psi}^{(0)}(L-1)U_x(L-1)\psi^{(1)}(0)$. Under a gauge transformation $V(x)$ this transforms as

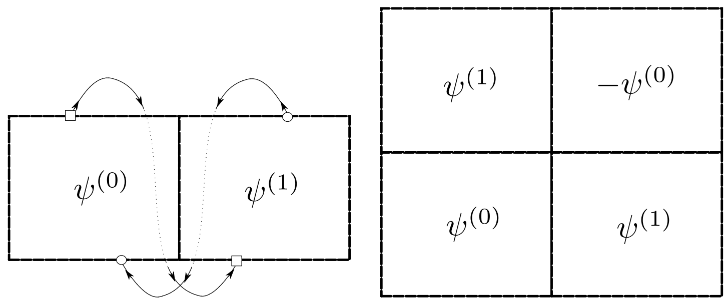

Figure 1: Two strategies for imposing GPBC in a second direction in the one-flavour setup. In the first approach (left) we modify the communications such that the left and right fields join on the upper and lower lattice boundaries as well as the left and right boundaries. This requires non-local communication for a standard 4d toroidal computer. In the second approach (right) we double the lattice again in the y-direction. The fields in the four quadrants must be set up and maintained as shown in the figure. Here the minus sign on the upper-right quadrant appears due to the double application of the GPBC when moving from the lower-left to upper-right quadrants.

$$\overline{\psi}^{(0)}(L-1)U_x(L-1)\psi^{(1)}(0) \longrightarrow \overline{\psi}^{(0)}(L-1)V^\dagger(L-1)U_x(L-1)V^*(0)\psi^{(1)}(0)\,. \tag{3.1}$$

We see that in order to maintain gauge invariance, the link $U_x(L-1)$ crossing the G-parity boundary must transform as $U_x(L-1) \to V(L-1)U_x(L-1)V^T(0)$. Building a staple across the boundary and applying a gauge transformation to the links crossing the boundary,

$$\begin{aligned} &U_x(L-1,y)U_y(L,y)U_x^\dagger(L-1,y+1) \\ &\quad \to V(L-1,y)U_x(L-1,y)V^T(L,y)U_y(L,y)V^*(L,y+1)U_x^\dagger(L-1,y+1)V^\dagger(L-1,y+1)\,, \end{aligned} \tag{3.2}$$

we see that $U_y(L,y) \to V^*(L,y)U_y(L,y)V^T(L,y+1)$, from which we can identify $U_y(L,y) = U_y^*(0,y)$, i.e. the links obey charge conjugation boundary conditions in the G-parity direction.

3.2 Implementation Strategies

Conceptually the simplest implementation is to maintain on each lattice site two quark fields, $\psi^{(0)}$ and $\psi^{(1)}$, that mix at the G-parity boundary under the parallel transport according to eqn. 2.2. The Dirac operator for the $\psi^{(1)}$ field takes the same form as that for $\psi^{(0)}$ only with complex-conjugated gauge links (in practise it is convenient to store both U and U^* fields in memory). We refer to this as the 'two-flavour' method. In order to implement this strategy in a standard lattice library (CPS, QDP++/Chroma, etc.) extensive changes are required to accomodate two fields that are joined at the boundary as well as the unusual boundary conditions on the gauge links.

For GPBC in a single direction, a far more straightforward 'one-flavour' approach [3] is to consider the superscript i on the fields $\psi^{(i)}$ as an index labelling the two halves of a doubled lattice. Essentially we unroll the two fields onto a single lattice of doubled size upon which the field $\psi^{(0)}$ and the gauge links lie on the first half, and $\psi^{(1)}$ and the complex-conjugated links lie on the second half. As the fields are antiperiodic in $2L$, the GPBC are equivalent to regular antiperiodic BCs on this doubled lattice. Although this approach requires only minor code changes, it does not scale well when GPBC are required in more than one direction (as was required for obtaining physical kinematics in the A_2 calculation). The difficulty is that in traversing a G-parity boundary from a site on $\psi^{(0)}$, one must always reach the neigbouring site but on $\psi^{(1)}$. The doubling of the lattice accomplishes this in one direction, but in any other G-parity direction we are forced either to communicate non-locally (assuming a standard four-dimensional toroidal layout of compute nodes) or to double the lattice again, taking pains to ensure that the fields on the upper quadrant remain identical to those on the lower quadrant. These two scenarios are sketched in figure 1. The second approach is highly inefficient due to the necessity of doubling the lattice size and maintaining the redundant degrees of freedom on the upper half. The efficiency of the first approach is dependent on how the machine handles non-local data communication and would require substantial code modifications of a similar scale to the two-flavour approach. On the other hand, the two flavour approach suffers no performance hits when applying GPBC in multiple directions. As a result we chose to implement this approach for our A_0 calculation. The one-flavour strategy (double/quad lattice), however, provides an extremely useful cross-check of any code modifications, therefore in practise we implemented both strategies.

3.3 G-parity Propagators

Due to the mixing of the quark flavours at the boundary, GBPC allow for unusual contractions:

$$\begin{aligned} \psi^{(1)}(x)\overline{\psi}^{(0)}(y) &= \mathscr{G}^{(1,0)}(x,y) = C\overline{u}^T(x)\overline{d}(y)\,, \\ \psi^{(0)}(x)\overline{\psi}^{(1)}(y) &= \mathscr{G}^{(0,1)}(x,y) = -d(x)u^T(y)C^{\dagger}\,, \end{aligned} \tag{3.3}$$

where $\mathscr{G}^{(i,j)}(x,y)$ is a quark propagator between positions y and x and flavour indices j and i. In the first line of the above, a down quark propagates through the boundary where it is annihilated as an anti-up quark. In the second line an anti-up quark propagates over the boundary and is annihilated as a down quark. We may interpret this as the boundary destroying and creating quark flavour respectively, thus violating baryon number conservation.

The flavour mixing at the boundary can dramatically increase the number of Wick contractions associate with a given amplitude. Fortunately the underlying symmetries provide at least two relations that can be used to simplify a calculation. We have the usual γ^5-hermiticity, which allows us to relate propagators crossing the G-parity boundary as follows:

$$\left[\gamma^5\mathscr{G}^{(0,1)}(x,y)\gamma^5\right]^{\dagger} = \mathscr{G}^{(1,0)}(y,x)\,. \tag{3.4}$$

We can also exploit the fact that $\psi^{(0)}$ and $\psi^{(1)}$ interact with the same gauge fields (albeit complex-conjugated for the latter). If we write $U^{(0)}_\mu(x) = U_\mu(x)$ and $U^{(1)}_\mu(x) = U^*_\mu(x)$, the Dirac matrix has the following dependence on the gauge links:

$$\not{D}^{(i,j)}(x,y) \sim \sum_\mu \left(P^\mu_-\delta(x,y+\hat{\mu})\beta^{(i,j)}U^{(j)\dagger}_\mu(y) + P^\mu_+\delta(x,y-\hat{\mu})\beta^{(j,i)}U^{(i)}_\mu(y-\hat{\mu})\right) + \ldots\,, \tag{3.5}$$

where we only show the links in the G-parity direction(s) $\{\mu\}$. Here $i,j \in \{0,1\}$ are flavour indices, and $\beta^{(0,0)} = \beta^{(1,1)} = \beta^{(1,0)} = -\beta^{(0,1)} = 1$ provides the minus-sign on the G-parity boundary between flavours 1 and 0 (i.e. the global lattice boundary in the one-flavour model). $P^\mu_\pm$ are spin operators: for naïve fermions $P^\mu_\pm = \pm\frac{1}{2}\gamma^\mu$ and for Wilson/domain wall fermions $P^\mu_\pm = \frac{1}{2}(1\pm\gamma^\mu)$.

Taking the complex conjugate and exploiting $\gamma^5 C(\gamma^\mu)^* C^\dagger\gamma^5 = -\gamma^5(\gamma^\mu)^\dagger\gamma^5 = \gamma^\mu$ (the γ-matrices are Hermitian in Euclidean space), we find:

$$\begin{aligned} &\gamma^5 C\left[\sum_\mu \left(P^\mu_-\delta(x,y+\hat{\mu})\beta^{(i,j)}U^{(j)\dagger}_\mu(y) + P^\mu_+\delta(x,y-\hat{\mu})\beta^{(j,i)}U^{(i)}_\mu(y-\hat{\mu})\right) + \ldots\right]^* C^\dagger\gamma^5 \\ &\qquad = \sum_\mu \left(P^\mu_-\delta(x,y+\hat{\mu})\beta^{(i,j)}U^{(1-j)\dagger}_\mu(y) + P^\mu_+\delta(x,y-\hat{\mu})\beta^{(j,i)}U^{(1-i)}_\mu(y-\hat{\mu})\right) + \ldots\,. \end{aligned} \tag{3.6}$$

Using the definition $\sum_{y,j}\not{D}^{(i,j)}(x,y)\mathscr{G}^{(j,k)}(y,z) = \delta^{(i,k)}\delta(x,z)$ and applying the above relationship:

$$\begin{aligned} &\sum_{y,j}\left[\sum_\mu \left(P^\mu_-\delta(z-\hat{\mu},y)\beta^{(k,j)}U^{(1-j)\dagger}_\mu(z-\hat{\mu}) + P^\mu_+\delta(z+\hat{\mu},y)\beta^{(j,k)}U^{(1-k)}_\mu(z)\right) + \ldots\right] \times \\ &\qquad \gamma^5 C[\mathscr{G}^{(j,k)}(y,z)]^* C^\dagger\gamma^5 = 1\,. \end{aligned} \tag{3.7}$$

Writing $\beta^{(j,k)} = (-1)^{|j-k|}\beta^{(1-j,1-k)}$, this can be restated as

$$\sum_{y,j}\not{D}^{(1-k,1-j)}(x,y)(-1)^{|j-k|}\gamma^5 C[\mathscr{G}^{(j,k)}(y,z)]^* C^\dagger\gamma^5 = 1\,, \tag{3.8}$$

from which we can identify

$$\mathscr{G}^{(1-j,1-k)}(y,z) = (-1)^{|j-k|}\gamma^5 C[\mathscr{G}^{(j,k)}(y,z)]^* C^\dagger\gamma^5\,. \tag{3.9}$$

3.4 The Strange Quark

The ultimate goal is to calculate the $K \to \pi\pi$ amplitude A_0, for which, as discussed previously, we desire to have the neutral kaon stationary in the lattice rest frame. Unfortunately the K^0 state, $\frac{1}{\sqrt{2}}(\overline{s}d + \overline{d}s)$, is not a G-parity eigenstate, therefore not only will the GPBC prevent this from forming a stationary particle (recall the minimum light quark momentum is $\pi/2L$), but the state will also mix with other unphysical states with non-zero baryon number.

One possible solution is to place the strange quark in a fictional isospin doublet with a degenerate partner that we label s'. If we also impose GPBC on the strange doublet we can write down a 'neutral kaon' analogue $K^{0\prime} = \frac{1}{2}(\bar{s}d + \bar{d}s + \bar{s}'u + \bar{u}s')$ that is a G-parity even eigenstate, and hence can form a stationary state. The only required modification to the A_0 calculation would be to apply an extra factor of $\frac{1}{2}$ as only half of the components will couple to the $\pi\pi$ state.

This strategy has one pitfall: although there are no disconnected strange-quark loops, the charge-conjugation symmetry of the gauge field forces us to also impose GPBC on the strange sea quarks. Here, the introduction of the fictional strange quark partner results in there being one too many flavours in our simulation. This can be rectified by taking the square-root of the s'/s determinant in the gauge evolution using the rational approximation, however in doing so the action becomes non-local. Note that the non-locality appears due to the minus sign in the boundary conditions; if we change $s \to C\bar{s}'^T \to -s$ to $s \to C\bar{s}'^T \to +s$, the s'/s determinant becomes the square of the Pfaffian of the one-flavour determinant [3]. The non-locality is therefore only a boundary effect and should remain small at sufficiently large volumes. However this remains an issue that must be examined more closely in the future, possibly by observing the effects of switching the sign on the boundary, or through the use of staggered chiral perturbation theory.

3.5 Correlation Functions

The amplitude of the propagation of a π^+ meson from y to x has the following form:

$$\langle \bar{d}_x\gamma^5 u_x \bar{u}_y\gamma^5 d_y\rangle = \langle \bar{\psi}^{(0)}_x[\gamma^5 C]\bar{\psi}^{(1)\,T}_x \psi^{(1)\,T}_y[C\gamma^5]\psi^{(0)}_y\rangle\,. \tag{3.10}$$

Due to flavour mixing this has two contractions as opposed to the usual one. Applying the relationships derived in the previous section, these reduce to

$$\mathrm{tr}\left\{\mathscr{G}^{(0,0)\dagger}(x,y)\mathscr{G}^{(0,0)}(x,y)\right\} - \mathrm{tr}\left\{\mathscr{G}^{(1,0)\dagger}(x,y)\mathscr{G}^{(1,0)}(x,y)\right\}, \tag{3.11}$$

which have a form similar to the usual pion contraction; indeed, taking $L\to\infty$, the second component involving propagation across the boundary must vanish leaving only the regular contraction. Although this may appear to require only a single propagator inversion from a source of flavour zero, the necessity of applying a phase at the source location to project onto the correct momentum component along with the fact that this contains both regular and hermitian-conjugate propagators, means that, unless one is using a point source or a source that is invariant under $p\to -p$ (for example a cosine source), two inversions are required.

It is interesting to observe the K^+ propagator in the setup described in the previous section. The analogue-K^+ creation operator is $K^{+\prime} = \frac{1}{\sqrt{2}}\left(\bar{u}\gamma^5 s - \bar{s}'\gamma^5 d\right)$, which is a G-parity even eigenstate. Labelling $\psi^{(2)} = s$ and $\psi^{(3)} = C\bar{s}'^T$, the contractions are:

$$\begin{aligned}&\tfrac{1}{2}\mathrm{tr}\left\{\mathscr{G}^{(2,2)\dagger}(x,y)\mathscr{G}^{(0,0)}(x,y)\right\} + \tfrac{1}{2}\mathrm{tr}\left\{\mathscr{G}^{(2,2)}(x,y)\mathscr{G}^{(0,0)\dagger}(x,y)\right\}\\ &\quad+\tfrac{1}{2}\mathrm{tr}\left\{\mathscr{G}^{(3,2)\dagger}(x,y)\mathscr{G}^{(1,0)}(x,y)\right\} + \tfrac{1}{2}\mathrm{tr}\left\{\mathscr{G}^{(3,2)}(x,y)\mathscr{G}^{(1,0)\dagger}(x,y)\right\}\,.\end{aligned} \tag{3.12}$$

In the case of degenerate quarks $m_s = m_{s'} = m_d = m_u$ this reduces to the same form as eqn. 3.11 only with the opposite sign between the two contractions. This suggests that the non-zero momentum of G-parity odd eigenstates arises from the relative phases of the Wick contractions.

4. Preliminary Results

We have implemented both the one-flavour and two-flavour approaches in the CPS++ codebase. As the intention is to run on our IBM BlueGene/Q supercomputers, we have also implemented these approaches in the Bagel/Bfm library (called from within CPS++), which contains assembly routines specifically optimised for this hardware.

As a preliminary test we generated using HMC a series of $8^4\times 32\times 10$ quenched ensembles with the Iwasaki gauge action and domain wall valence quarks. For GPBC in 0,1,2 and 3 directions, we generated 150 configurations separated by 20 MD time units. We measured the π^+ and

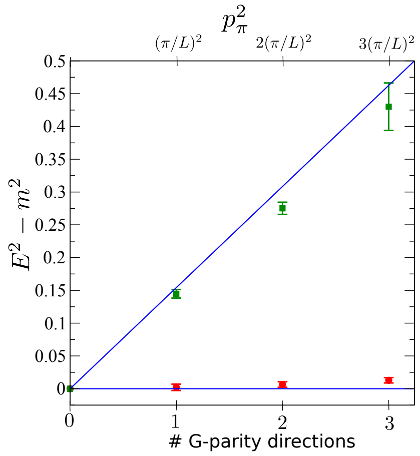

Figure 2: The value of $E^2 - m^2$ for the pion (green squares) and kaon (red circles) plotted against the number of G-parity directions n, where E is the measured energy and m is the rest-mass as measured on a the regular periodic lattice. For the pion we also plot the expected momentum, $p_\pi^2 = n(\pi/L)^2$, along the x-axis with the same scale as the y-axis such that the continuum dispersion relation $E_\pi^2 - m_\pi^2 = p_\pi^2$ (blue line) lies along the diagonal. Although we expect the kaon to have zero momentum we plot the points at the same x-axis positions for comparison, and also plot the expected (flat) dispersion relation in blue.

degenerate $K^{+\prime}$ correlation functions given in the previous section using Coulomb gauge-fixed wall source propagators with the appropriate source phases. In figure 2 we plot the dispersion relations of the pion and kaon against the continuum dispersion relations, seeing good agreement. Note that at higher momenta we should not expect perfect agreement in any case as the continuum dispersion relation becomes modified on the lattice.

5. Conclusions and Outlook

In these proceedings we have discussed several of the challenges involved in measuring the $\Delta I = 1/2$ $K \to \pi\pi$ amplitude on the lattice. This is a very important quantity to measure as, when combined with our existing measurement of the $\Delta I = 3/2$ amplitude, a first-principles calculation of the amount of direct CP-violation in the Standard Model can be performed.

We introduced G-parity as a solution to the difficulty of imposing momentum on the final state pions, such that energy is conserved in the decay while retaining the isospin symmetry. We discussed two strategies for implementation and also some of the unusual aspects of calculations involving these boundary conditions. We noted that taking the square-root of the fictional strange-quark doublet leads to a non-local effective action; although this is only a boundary effect that we expect will be small, this remains an issue which requires further study. In the final section we generated several quenched ensembles with domain wall fermions and G-parity boundary conditions in multiple directions, demonstrating that the pion does indeed have a non-stationary ground state with the expected momentum and that stationary kaon-analogue states can be produced.

At the time of writing we have already started generating $16^3 \times 32$ fully dynamical 2+1f domain wall fermion ensembles for testing purposes using our IBM BlueGene/Q resources, and we intend to start a $32^3 \times 64$ domain wall ensemble with physical quark masses and G-parity BCs in the near future for the purpose of measuring the $\Delta I = 1/2$ $K \to \pi\pi$ amplitude.

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