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Weak Matrix Elements of Beyond the Standard Model $\Delta s = 2$ four-quark operators from $n_f = 2+1$ Domain-Wall fermions

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We report on our computation of the hadronic matrix elements of the four-quark operators needed for the study of $K^0 - \bar{K}^0$ mixing beyond the Standard Model (SM). We consider $n_f = 2+1$ Domain-Wall fermions on Iwasaki gauge action with lightest unitary pion of 290 MeV and a single lattice spacing $a \sim 0.086\,\text{fm}$. The renormalization is performed non-perturbatively through the RI-MOM scheme and our results are converted perturbatively to $\overline{\text{MS}}$. We have estimated the various systematic errors. Our results confirm a previous quenched study, where large ratios of non-SM to SM matrix elements were obtained.

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1. Introduction

Direct and indirect CP violation in kaon systems are being currently investigated by the RBC-UKQCD with a framework of $n_f = 2+1$ dynamical flavours of Domain-Wall. Exciting results concerning direct CP violation obtained though simulations of the decay of a kaon into two pions have been reported in [1, 2, 3]. Important progress has also been achieved concerning indirect CP violation, and in particular our computation of the non-SM contributions to neutral kaon mixing has been recently published in [4]. We summarise here our strategy and the current status of our analysis. We also mention that other collaborations have also recently reported on similar studies at this conference [5, 7, 6]

After performing an operator-product-expansion, neutral kaon mixing can described by a generic $\Delta s = 2$ Hamiltonian of the form

$$H^{\Delta s=2} = \sum_{i=1}^{5} C_i(\mu)\, O_i^{\Delta s=2}(\mu)\,, \tag{1.1}$$

where μ is a renormalization scale. The Wilson coefficients C_i, which encode the short-distance effects, depend on the new physics model under consideration. The long-distance effects are factorised into the matrix elements of the four-quark operators $O_i^{\Delta s=2}$ given here in the so-called SUSY basis[1] [8]

$$\begin{aligned}
O_1^{\Delta s=2} &= (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)\,(\bar{s}_\beta \gamma_\mu (1-\gamma_5) d_\beta)\,,\\
O_2^{\Delta s=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)\,(\bar{s}_\beta (1-\gamma_5) d_\beta),\\
O_3^{\Delta s=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)\,(\bar{s}_\beta (1-\gamma_5) d_\alpha),\\
O_4^{\Delta s=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)\,(\bar{s}_\beta (1+\gamma_5) d_\beta),\\
O_5^{\Delta s=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)\,(\bar{s}_\beta (1+\gamma_5) d_\alpha).
\end{aligned} \tag{1.2}$$

In the SM case ($i = 1$) it is conventional to introduce the kaon bag parameter B_K, which measures the deviation of the SM matrix element from its vacuum saturation approximation (VSA)

$$B_K = \frac{\langle \bar{K}^0 | O_1^{\Delta s=2} | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2}\ . \tag{1.3}$$

Where the normalisation for the decay constant is such that $f_{K^-} = 156.1$MeV. Several normalisations for the BSM operators ($i > 1$) have been proposed in the literature, see for example [9], in this work we follow [10] and define the ratios

$$R_i^{\rm BSM} = \left[\frac{f_K^2}{m_K^2}\right]_{\rm expt} \left[\frac{m_P^2}{f_P^2} \frac{\langle \bar{P} | O_i^{\Delta s=2} | P \rangle}{\langle \bar{P} | O_1 | P \rangle}\right]_{\rm latt}\,, \tag{1.4}$$

where P is a pseudo-scalar particle of mass m_P and decay constant f_P. The term $[\,]_{\rm latt}$ is obtained from our lattice simulations for different values of m_P. For completeness we will also give the BSM bag parametrisation, defined as (where $N_{2,\ldots,5} = \frac{5}{3}, -\frac{1}{3}, -2, -\frac{2}{3}$) [11],

$$B_i \ = \ -\frac{\langle \bar{K}^0 | O_i^{\Delta s=2} | K^0 \rangle}{N_i \langle \bar{K}^0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K^0 \rangle}, \quad i = 2, \ldots, 5. \tag{1.5}$$

[1] We discard the parity odd operators since they are not relevant in the present case.

Computation details

This computation is performed on $32^3 \times 64 \times 16$ Iwasaki gauge configurations with an inverse lattice spacing $a^{-1} = 2.28(3)\,\mathrm{GeV}$, corresponding to $a \sim 0.086\,\mathrm{fm}$ [2]. We have three different values of the light sea quark mass $am^{\rm sea}_{\rm light} = 0.004, 0.006, 0.008$ corresponding to unitary pion masses of approximately $290, 340$ and 390 MeV respectively. The simulated strange sea quark mass is $am^{\rm sea}_{\rm strange} = 0.03$, which is close to its physical value $0.0273(7)$. For the main results of this work, we consider only unitary light quarks, $am^{\rm valence}_{\rm light} = am^{\rm sea}_{\rm light}$, whereas for the physical strange we interpolate between the unitary ($am^{\rm valence}_{\rm strange} = am^{\rm sea}_{strange} = 0.03$) and the partially quenched ($am^{\rm valence}_{\rm strange} = 0.025$) data.

The procedure for the evaluation of the two-point functions and of the three point function is fairly standard (in particular, we have used Coulomb gauge fixed wall sources to obtain very good statistical precision). We define the three point functions $c_i = \langle \bar P(t_f) O_i^{\Delta S=2}(t) \bar P(t_i)\rangle$ and from the asymptotic Euclidean time behaviour of the ratios of three point-functions c_i/c_1 (we fit these ratios to a constant in the interval $t/a = [12, 52]$ [3]) we obtain the bare matrix elements of the four-quark operators normalised by the SM one: $\left[\langle \bar P | O_i^{\Delta S=2} | P\rangle / \langle \bar P | O_1^{\Delta S=2} | P\rangle\right]^{\rm bare}$. In figure 1 (left), we show the corresponding plateaux obtained for our lightest unitary kaon $am^{\rm sea}_{\rm light} = am^{\rm valence}_{\rm light} = 0.004$, $am^{\rm sea}_{\rm strange} = am^{\rm valence}_{\rm strange} = 0.03$.

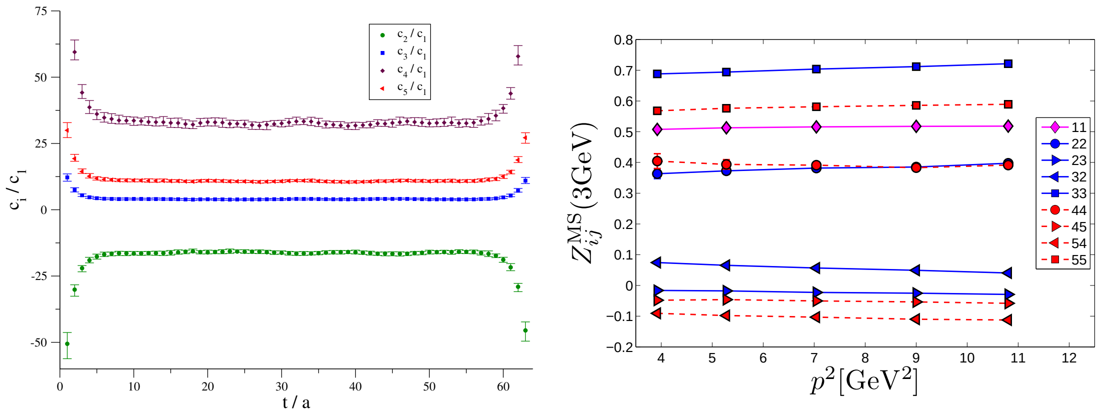

Figure 1: *Left:* ratios of the bare three point functions from which we extract $R_i^{\rm BSM}$. Results are shown for our lightest simulated unitary kaon.
Right: renormalization factors of the four-quark operators. At each value of the simulated momentum p, we run to the scale of 3GeV and convert to $\overline{\rm MS}$. The remaining scale dependence can be imputed to the truncation of the perturbative expansion. We show only the Z-factors allowed by chiral symmetry.

Renormalization

The four-quark operators given in eq. (1.2) mix under renormalization. With Domain-Wall fermions, the renormalization pattern is the same as in the continuum (up to numerically irrelevant

[2] Note that this value was recently updated to $a^{-1} = 2.31(4)\,\mathrm{GeV}$ [12].

[3] From figure 1 we deduce that we have reached the asymptotic region in this range for each operator insertion.

discretisation effects). The SM operator $O_1^{\Delta s=2}$ belongs to a $(27,1)$ irreducible representation of $SU(3)_L \times SU(3)_R$ and renormalizes multiplicatively. The BSM operators fall into two categories: $O_2^{\Delta s=2}$ and $O_3^{\Delta s=2}$ transform like $(6,\bar{6})$ and mix together. Likewise $O_4^{\Delta s=2}$ and $O_5^{\Delta s=2}$ belong to $(8,8)$ and also mix.

We perform the renormalization of the four-quark operators $O_i^{\Delta s=2}$ non-perturbatively in the RI-MOM scheme [13]. By using both momentum sources [14] and partially twisted boundary conditions, we obtain smooth functions of the external momentum with very good statistical accuracy [4]. Although we have also performed this computation in a non-exceptional intermediate scheme, we quote here the results obtained via the RI-MOM scheme because only in this case are the conversion factors to $\overline{\rm MS}$ (computed in continuum perturbation theory) available for the whole set of operators. We choose to impose the renormalization conditions at $\mu = 3\,{\rm GeV}$, the conversion between 3 and 2 GeV can be found in the appendix.

We observe that the effects of chiral symmetry breaking are not completely negligible, even at 3 GeV [15]. Therefore we must assess a systematic error to the mixing of operators of different chirality (see next section). We have checked that in a non-exceptional scheme this small chirally forbidden mixing is strongly reduced and becomes numerically irrelevant at 3 GeV [15, 18]. Thus we conclude that this effect is a physical manifestation of the infrared behaviour of the exceptional intermediate scheme. The results for the chirally allowed renormalization factors $Z_{ij}^{\overline{\rm MS}}(3{\rm GeV})$ are shown in figure 1 (right). They relate the bare four-quark operators to the renormalized ones through the usual relation (Z_q is the renormalization factor of the quark wave function and cancels in the ratios)

$$O_i^{\Delta s=2,\overline{\rm MS}}(3{\rm GeV}) = \frac{Z_{ij}}{Z_q^2}^{\overline{\rm MS}}(3{\rm GeV})\, O_j^{\Delta s=2,{\rm bare}} \,. \tag{1.6}$$

Physical results and error estimation

Once the bare ratios have been renormalized, we extrapolate them to the physical kaon mass. The chiral functional form of the BSM operators are discussed for example in [19, 20, 5]. Since we find that the $R^{\rm BSM}$'s exhibit a very mild quark mass dependence (see figure 2), we take the results obtained by a simple Taylor expansion as our central values.

Our final results are the $R_i^{\rm BSM}$ quoted in $\overline{\rm MS}$ at 3 GeV given in table 1. For completeness, we also convert these to the BSM bag parameters, using eq. (1.5). We also note that, using the same framework, the SM contribution is found to be $B_1 = B_K = 0.517(4)_{\rm stat}$ in the $\overline{\rm MS}$ scheme at 3 GeV, whereas a continuum value of $0.529(5)_{\rm stat}(19)_{\rm syst}$ was quoted in [21]. The difference comes from the fact that a different intermediate scheme was used in [21] (such a difference is accounted for in our estimation of the systematic errors). From the same reference, the discretisation effects for B_K on this lattice are seen to be of the order of 1.5%. Since we have only one lattice spacing for the BSM ratios, we make the assumption that the discretisation errors are of the same size as those affecting B_K, and estimate a 1.5% error to all the different operators. This might appear like a crude estimate, but this effect is expected to be sub-dominant compared to other sources of systematic errors. The next systematic error (called "extr.") represents the spread of the results obtained from

[4] More details about the computation of the renormalization factors can be found in [15, 16, 17]

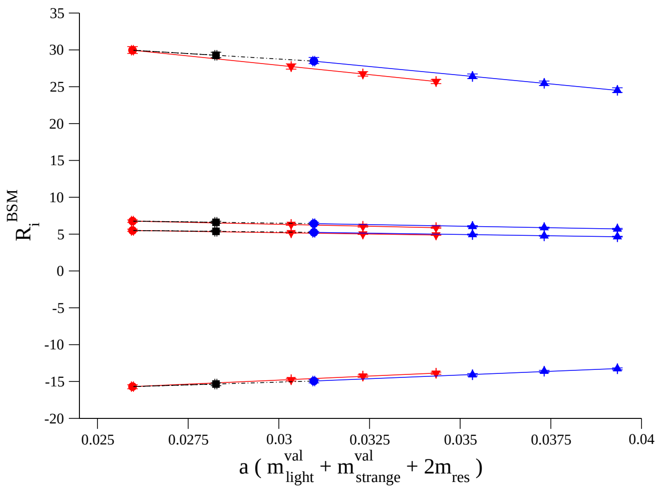

Figure 2: Renormalized BSM ratios $R_i^{\rm BSM}$ in function of the bare valence quark mass. We show the three unitary light quarks for both the unitary strange (upward blue triangles) and the partially quenched strange (downward blue triangles), together with their extrapolations in the light sector (blue and red circles) and their interpolation to the physical kaon mass (black squares).

different extrapolation strategies to the physical point. The systematic error associated with the non-perturbative renormalization (NPR) has been estimated from the breaking of chiral symmetry. The mixing of the $(6,\bar{6})$ with the $(8,8)$ operators is forbidden by chiral symmetry, but is likely to be enhanced by the exchange of light pseudo-scalar particles. As the matrix element of $O_4^{\Delta s=2}$ is numerically large, this non-physical mixing has an effect on $O_2^{\Delta s=2}$ and $O_3^{\Delta s=2}$ of the order of $8-9\%$. This unwanted infrared effect is absent if a non-exceptional scheme is used. The last error we quote ("PT") arises from the matching between the intermediate RI-MOM scheme and $\overline{\rm MS}$, which is performed at one-loop order in perturbation theory [22, 23] in the three-flavour theory. The associated error is obtained by taking half the difference between the leading order and the next to leading order result [5]. We note that this error is one of the dominant ones in our budget, and we expect this error to be reduced by an important factor if a non-exceptional scheme were used, since the latter are known to converge faster in perturbation theory. We neglect the finite volume effects which have been found to be small in [21], as one can expect from the value of $m_\pi L \sim 4$ for our lightest pion mass $m_\pi \sim 290\,$MeV.

Conclusions

We have computed the electroweak matrix elements of the $\Delta s = 2$ four-quark operators which contribute to neutral kaon mixing beyond the SM. Our work improves on other studies by using $n_f = 2+1$ flavours of dynamical chiral fermions. We confirm the effect seen in a previous quenched computation [10], where large enhancements of the non-standard matrix elements were observed.

[5] To obtain α_s at 3 GeV in the three-flavour theory, we start from $\alpha_s(M_Z) = 0.1184$ [24], we use the four-loop running [25, 26] to compute the scale evolution down to the charm mass, while changing the number of flavours when crossing a threshold, and then run up to 3 GeV in the three-flavour theory.

i	R_i^{BSM}	B_i	stat.	discr.	extr.	NPR	PT	total
2	$-15.3(1.7)$	0.43 (5)	1.3	1.5	4.0	9.4	4.7	11.3
3	5.4(0.6)	0.75 (9)	2.0	1.5	3.9	7.8	7.6	12.0
4	29.3(2.9)	0.69 (7)	1.3	1.5	4.1	3.0	8.2	9.8
5	6.6(0.9)	0.47 (6)	2.1	1.5	3.8	3.2	12.6	13.8

Table 1: Final results of this work: the first two columns show the ratios R_i^{BSM} and the corresponding bag parameters B_i in $\overline{\mathrm{MS}}$ at 3 GeV, together with their total error, combining systematics and statistics. In the remaining columns, we give our error budget for the R^{BSM}, detailing the contributions in percentage of the different sources of systematics (see text for more details).

The errors quoted in this work are of the order of 10%. We note that the main limitation of this study comes from the lack of matching factors between non-exceptional renormalization schemes (such as SMOM) and $\overline{\mathrm{MS}}$. Once these factors are available, we expect to reach a precision better than 5%. We also plan to utilise another lattice spacing in order to have a better handle on the discretisation effects.

Appendix

We have computed the non-perturbative scale evolution of the R^{BSM}'s between 3 and 2 GeV, and then converted the results to $\overline{\mathrm{MS}}$ using one-loop perturbation theory [22, 23]:

$$U^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}, 3\,\mathrm{GeV}) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.87 & 0.02 & 0 & 0 \\ 0 & 0.09 & 1.09 & 0 & 0 \\ 0 & 0 & 0 & 0.86 & -0.01 \\ 0 & 0 & 0 & -0.03 & 0.98 \end{pmatrix}. \tag{1.7}$$

Our conventions are such that

$$R^{\mathrm{BSM}}(2\,\mathrm{GeV}) = U^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}, 3\,\mathrm{GeV}) R^{\mathrm{BSM}}(3\,\mathrm{GeV}). \tag{1.8}$$

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