PROCEEDINGS OF SCIENCE

Chiral extrapolation of matrix elements of BSM kaon operators

Jon A. Bailey

Department of Physics and Astronomy, Seoul National University, Seoul, 151-747, South Korea

Hyung-Jin Kim

Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA

Weonjong Lee

Department of Physics and Astronomy, Seoul National University, Seoul, 151-747, South Korea

Stephen R. Sharpe*

Physics Department University of Washington Seattle, WA 98195-1560, USA
E-mail: sharpe@phys.washington.edu

Models of new physics induce $K_0 - \overline{K_0}$ mixing through operators having Dirac structures other than the "left-left" form of the Standard Model. To carry out the chiral-continuum extrapolation of results from numerical simulations, one needs to know the quark mass and lattice spacing dependence of the corresponding B-parameters in the partially quenched theory at least at next-to-leading order. For simulations using staggered fermions (such as that we are doing with HYP-smeared valence fermions on the MILC asqtad lattices) one must determine this dependence using staggered chiral perturbation theory (SChPT). We have calculated the required dependence in both SU(3) and SU(2) SChPT, working at next-to-leading order, and we give here an overview of the methodology and results. The SU(3) SChPT result turns out to be much simpler than that for the Standard Model B_K operator, due to the absence of chiral suppression for the new operators. The SU(2) SChPT result turns out to be closely related to that for B_K: the chiral logarithms are identical, up to an operator-dependent sign. Our results are also useful for fermions with chiral symmetry as they provide, in the continuum limit, the partially quenched generalization of existing continuum results.

The 30th International Symposium on Lattice Field Theory
June 24-29, 2012
Cairns, Australia

*Speaker.

PoS(Lattice 2012)106

1. Introduction

Theories of physics beyond the standard model (BSM) are often highly constrained by the observed weakness of flavor-changing neutral processes. One of the strongest constraints comes from $K_0-\overline{K_0}$ mixing, both the CP-conserving and violating parts. For a given model of new physics, integrating out heavy particles leads to a set of local $\Delta S=2$ operators with known Wilson coefficients. In order to determine the constraint placed on the parameters of the model, one must calculate the $K_0-\overline{K_0}$ matrix elements of the $\Delta S=2$ operators. This is a task for which lattice methods are well suited, and several efforts are underway [1, 2, 3]. The results will provide complementary information to the direct searches for new physics at the LHC.[1]

Discretization effects and unphysically large u and d quark masses are important sources of systematic error in these calculations, which fortunately can be largely eliminated by performing chiral and continuum extrapolations with effective field theory as a guide. The need for controlled, model-independent continuum extrapolations is particularly acute for our calculation because we use rooted staggered quarks, and one must extrapolate away the effects of taste-breaking and rooting. To do this, one requires at least a next-to-leading order (NLO) calculation in staggered chiral perturbation theory (SChPT). Here we report on such a calculation, which has been recently published [5]. We have presented results for both SU(3) and SU(2) SChPT, i.e. treating the strange quark as light and heavy, respectively. Our results are also useful for describing data obtained with other fermion discretizations, e.g. domain-wall fermions [1], for we can turn off taste breaking and determine the NLO predictions for a partially quenched (PQ) continuum theory in both SU(3) and SU(2) cases.

Since all the technical details have been provided in Ref. [5], we attempt here a complementary discussion, focusing on the essential features and eschewing most details. We also to correct some minor shortcomings in the original discussion, which, however, lead to no changes in the final results.

BSM theories lead to $\Delta S=2$ operators with Dirac structures which are not constrained to have the "left-left" structure of the standard model (SM) operator. We adopt the basis

$$O_1 = [\bar{s}^a\gamma_\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1-\gamma_5)d^b], \tag{1.1}$$
$$O_2 = [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1-\gamma_5)d^b], \tag{1.2}$$
$$O_3 = [\bar{s}^a\sigma_{\mu\nu}(1-\gamma_5)d^a][\bar{s}^b\sigma_{\mu\nu}(1-\gamma_5)d^b], \tag{1.3}$$
$$O_4 = [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b], \tag{1.4}$$
$$O_5 = [\bar{s}^a\gamma_\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1+\gamma_5)d^b], \tag{1.5}$$

where $\sigma_{\mu\nu}=[\gamma_\mu,\gamma_\nu]/2$, and a and b are color indices. This basis was used in the highest-order continuum calculations of anomalous dimensions [6], and is the most natural choice for our staggered calculation of the matrix elements, in which we match onto continuum operators using 1-loop perturbation theory (PT) [7].

Many authors, including Refs. [1, 2, 5], use an alternate basis in which O_3 and O_5 are replaced by

$$O_3' = [\bar{s}^a(1-\gamma_5)d^b][\bar{s}^b(1-\gamma_5)d^a], \tag{1.6}$$

[1] For a recent review of this complementary information using the first lattice results see Ref. [4].

$$O_5' = [\bar{s}^a(1-\gamma_5)d^b][\bar{s}^b(1+\gamma_5)d^a], \tag{1.7}$$

We do not discuss the pros and cons of the two bases here, since the choice of basis turns out to be irrelevant for the SChPT calculation as *the chiral behavior of the matrix elements of O_3 and O_3' (and of O_5 and O_5') are identical.* Thus the results presented here hold for either basis.

In the SM, only the operator O_1 appears, its matrix element being parametrized by B_K. A large effort by the lattice community has led to an accurate determination of B_K. The matrix elements of the other operators, which we call "BSM operators", are in fact *easier* to calculate on the lattice. This is because they do not vanish in the SU(3) chiral limit, and so have simpler chiral expansions. Thus lattice results for the new matrix elements are following closely on the heels of those for B_K.

As for B_K, calculating ratios of matrix elements is advantageous because many lattice systematics and also some of the chiral logarithms cancel. Standard ratios for the BSM operators are

$$B_j(\mu) = \frac{\langle \overline{K}_0|O_j(\mu)|K_0\rangle}{N_j\langle \overline{K}_0|\bar{s}^a\gamma_5 d^a(\mu)|0\rangle\langle 0|\bar{s}^b\gamma_5 d^b(\mu)|K_0\rangle}, \quad (N_2,N_3,N_4,N_5) = (5/3,4,-2,4/3), \tag{1.8}$$

where μ is the renormalization scale. Note that, unlike for B_K, the denominator does not vanish in the SU(3) chiral limit. Below we present and discuss our results for the chiral-continuum behavior of these B_j.

2. Overview of methodology

Our numerical calculations use a mixed-action set-up with HYP-smeared valence staggered quarks atop $2+1$ flavors of rooted asqtad sea quarks (MILC configurations). We allow the valence d and s masses, labeled m_x and m_y, respectively, to differ from the sea-quark masses, labeled m_u, m_d and m_s. Although our present simulations have $m_u = m_d$, we consider the completely non-degenerate case in the SChPT calculation.[2]

The calculation largely follows the methodology worked out in Ref. [8] for B_K in PQ SChPT and extended to mixed action SChPT in Ref. [9]. In most respects the calculation for the BSM operators is more straightforward than that for B_K because the matrix elements do not vanish in the SU(3) chiral limit. There is, however, an additional complication whose origin is the non-trivial mixing between the BSM operators in PT.

In a continuum ChPT calculation, one would proceed by first mapping the continuum BSM operators into chiral operators in the effective theory, and then calculating one-loop diagrams. The operator mapping is done using transformation properties under the chiral group $SU(N_f)_L \times SU(N_f)_R$. To carry this over to staggered ChPT one must deal with several complications:

- The presence of additional valence tastes implies that the lattice operators are, even in the continuum limit, in a partially quenched theory with an enlarged chiral group. Because Fierz rearrangement differs in the presence of tastes, one must introduce two down and two strange valence fields (D_1, D_2, S_1 and S_2, where uppercase indicates a field with four tastes) in order to have a theory in which Wick contractions can be matched with those of the

[2] All masses are in physical rather than lattice units in this article.

desired continuum theory [8]. One must also choose the taste of the lattice operator—with the standard choice being the Goldstone taste ξ_5.

- Taste-breaking in the staggered action away from the continuum limit introduces additional terms in the chiral Lagrangian which are proportional to a^2, and, in the standard power-counting, these terms appear at leading order (LO) along with terms proportional to m and p^2 [10].

- The sea quarks are rooted and differ from the valence quarks. The rooting can be dealt with in SChPT using the replica method of Ref. [11]. The mixed action introduces further complications [9] that, however, turn out to be unimportant for the BSM operators (though not for B_K).

- Taste-breaking due either to the staggered action or introduced by the use of inexact (i.e. one-loop) matching leads to many new chiral operators. This proliferation of operators is a major problem for B_K [in SU(3) ChPT] but turns out to be a minor issue here as the new operators only contribute to analytic terms at NLO in SChPT (and not to loops).

To deal with these complications one must follow a rather elaborate series of matching steps. The details are given in Ref. [5]. Here we give only an overview.[3]

We illustrate the required steps by focusing on O_2. In **step 1**, we match from QCD to a PQ continuum theory containing two valence down quarks, $d_{1,2}$, and two valence strange quarks, $s_{1,2}$, as well as their corresponding ghosts.[4] The sea-quarks are as in QCD. O_2 then matches onto

$$O_2^{PQA} = 2\left\{[\bar{s}_1^a(1-\gamma_5)d_1^a][\bar{s}_2^b(1-\gamma_5)d_2^b] + [\bar{s}_1^a(1-\gamma_5)d_2^a][\bar{s}_2^b(1-\gamma_5)d_1^b]\right\}. \tag{2.1}$$

The two terms correspond to the two types of Wick contraction in QCD—note the $[\bar{s}_1 d_2][\bar{s}_2 d_1]$ flavor structure in the second term. The overall factor of 2 compensates for an overall reduction by 2 in the number of Wick contractions. This matching is exact to all orders in PT. The point of this step is to separate the two Wick contractions, without yet having to deal with any staggered complications.

In **step 2** we stay in the same PQ continuum theory, but match onto operators having $[\bar{s}_1 d_1][\bar{s}_2 d_2]$ form. This is in preparation for matching to the staggered theory. The result is a linear combination of four operators (note the different Dirac and color structures):

$$O_2^{PQB} = b_1[\bar{s}_1^a(1-\gamma_5)d_1^a][\bar{s}_2^b(1-\gamma_5)d_2^b] + b_2[\bar{s}_1^a(1-\gamma_5)d_1^b][\bar{s}_2^b(1-\gamma_5)d_2^a] \tag{2.2}$$

$$+\ b_3[\bar{s}_1^a\sigma_{\mu\nu}(1-\gamma_5)d_1^a][\bar{s}_2^b\sigma_{\mu\nu}(1-\gamma_5)d_2^b] + b_4[\bar{s}_1^a\sigma_{\mu\nu}(1-\gamma_5)d_1^b][\bar{s}_2^b\sigma_{\mu\nu}(1-\gamma_5)d_2^a]. \tag{2.3}$$

The coefficients b_j have the form $b_j^{(0)} + \alpha b_j^{(1)} + \ldots$ and the LO and NLO coefficents are known. The $b_j^{(0)}$ can be obtained by a Fierz transform, while the $b_j^{(1)}$ require a 1-loop calculation and one must make a choice of evanescent operators.

[3] This discussion extends and corrects that of Ref. [5], since we have subsequently found that additional matching steps (steps 1 and 2 below) are required. The final results are, however, unaffected. Details will be presented in Ref. [13].

[4] At this stage we can also let the valence and sea quark masses differ.

In **step 3** we match onto the "staggered partially quenched" (SPQ) continuum theory, which contains sea quarks U, D and S and valence quarks $D_{1,2}$ and $S_{1,2}$ (with uppercase again indicating four tastes) as well as sufficient ghosts (25 in all) to cancel both the valence determinant and the contribution of 3 of the 4 tastes for each sea quark. Here, for simplicity, we drop the negative parity parts of the operator, which do not contribute to the matrix element we are ultimately interested in. The operator matching is exemplified by

$$[\bar{s}_1^a(1-\gamma_5)d_1^a][\bar{s}_2^b(1-\gamma_5)d_2^b] \longrightarrow \frac{1}{N_t}\left\{[\bar{S}_1^a(1\otimes\xi_5)D_1^a][\bar{S}_2^b(1\otimes\xi_5)D_2^b] + [\bar{S}_1^a(\gamma_5\otimes\xi_5)D_1^a][\bar{S}_2^b(\gamma_5\otimes\xi_5)D_2^b]\right\}, \tag{2.4}$$

where $N_t = 4$ is the number of tastes. Here we are assuming external kaons of taste ξ_5 created with normalized operators. This matching is exact to all orders in PT. Each operator in the sum (2.3) is matched similarly and the coefficients b_j are unchanged.

We are now ready to match, in **step 4**, to a lattice theory with rooted staggered (asqtad) sea quarks χ_u, χ_d and χ_s and (HYP) valence quarks $\chi_{d1,2}$ and $\chi_{s1,2}$. Our continuum operator from step 3 is in the form that is used in practice in staggered lattice calculations of matrix elements. The matching from the continuum PQ theory to the lattice theory is (at present) done at 1-loop in PT. At this order in α, a large number of lattice operators contribute to the matching, although most have the wrong taste and are dropped in numerical calculations. The form of the lattice operators that are kept is described in Ref. [5]. The errors that are introduced at this stage have the form

$$O_j^{\rm LAT} \cong O_j^{\rm SPQ} + \mathcal{O}(\alpha)[\text{wrong taste ops}] + \mathcal{O}(\alpha^2, a^2)[\text{various taste ops}], \tag{2.5}$$

where $\cong$ indicates that matrix elements (rather than operators) are being matched.

In **step 5** we match onto the Symanzik effective continuum theory describing the long distance degrees of freedom of the lattice theory. Lattice symmetries are used to determine corrections to the action and operators, and factors of a^2 are now explicit.

Finally, in **step 6** we match onto the chiral effective theory (SChPT), using the chiral transformation properties of the action and operators. This process is straightforward, though tedious. A crucial step in this matching (which we do not have space to enter into) is that certain symmetry properties of the original operator percolate through the multiple steps and constrain the low-energy constants (LECs) that multiply the chiral operators, reducing the number of independent constants. The details of this are slightly modified from the discussion in Ref. [5] by the addition of steps 1 and 2, but the final conclusion is the same.

With the chiral form of the operators in hand, the requisite one-loop calculation is now straightforward. We have done this in SU(3) SChPT, and obtained the SU(2) result using the method explained and justified in Refs. [9, 5].

3. Results and conclusions

The general form of the NLO result in SChPT is

$$B_j = B_j^{\rm LO}\left[1 + \delta B_j^{\rm anal} + \delta B_j^{\rm log}\right]. \tag{3.1}$$

The analytic terms involve five unknown LECs at NLO:

$$\delta B_j^{\rm anal} = c_{j1}(m_x + m_y) + c_{j2}(m_u + m_d + m_s) + c_{j3}a^2 + c_{j4}a_\alpha^2 + c_{j5}\alpha^2. \tag{3.2}$$

Here we have shown the SU(3) result; for SU(2) the m_y and m_s terms are absent (and the coefficients are different). The first two terms are present in the continuum, while the remaining three represent lattice artifacts. The a^2 term comes from standard discretization errors (which are not suppressed by α since our valence fermions are not fully improved). The $a^2_\alpha = a^2\alpha^2$ term comes from taste-breaking discretization errors, which are numerically enhanced such that they are comparable to standard $\mathcal{O}(a^2)$ errors. Finally, the α^2 term arises because we use 1-loop operator matching—it would be absent if we used non-perturbative renormalization.

These analytic terms are similar to those for B_K and one can deal with them in a similar fashion. In a chiral extrapolation at fixed a the last three terms are constants and can be absorbed into $B_j^{\rm LO}$. These terms then enter into a (fairly complicated) continuum extrapolation.

The non-analytic terms are completely predicted in terms of the pion decay constant in the appropriate chiral limit (f_3 or f_2) and the masses of the valence and sea pions. They lead to curvature in the predicted chiral behavior. We show explicitly only the SU(2) result for $N_f = 2+1$ (which is the most useful in practice):

$$\delta B_j^{\rm log} = \pm \frac{1}{(4\pi f_2)^2}\left[-\frac{1}{16}\sum_B \ell(M^2_{xx;B}) + \frac{1}{2}\left\{\ell(M^2_{xx;I} + (M^2_{\pi;I} - M^2_{xx;I})\tilde{\ell}(M^2_{xx;I})\right\}\right]. \tag{3.3}$$

Here $M_{xx;B}$ is the LO mass of the valence $\bar{x}x$ pion of taste B and $M_{\pi;I}$ is the LO mass of the taste-singlet sea-quark pion. The chiral logarithms are $\ell(X) = X\ln(X/\mu_\chi^2)$ and $\tilde{\ell}(X) = -d\ell(X)/dX$. The $+$ sign holds for $j = 2$ and 3, while the $-$ sign applies for $j = 4$ and 5.

Key features of this result are:

- The chiral logarithms for $j = 2$ and 3 (and for $j = 4$ and 5) are identical because the chiral transformation properties of these pairs of operators are the same [12]. This holds generally—i.e. also for SU(3) SChPT, and irrespective of degeneracies in sea-quark masses.
- The $j = 2,3$ and $j = 4,5$ cases differ only in sign. This relationship does *not* hold for the SU(3) result.
- The chiral log for $j = 2,3$ is identical to that for B_K. This identity does *not* hold for the SU(3) result.
- The result is unaffected by the use of a mixed action—this only impacts some coefficients in $\delta B_j^{\rm anal}$ as well as the values of the pion masses and decay constant. This conclusion holds also for the SU(3) result (but not for B_K).

In Ref. [5] we present also the SU(2) result for non-degenerate sea quarks, and the SU(3) result both for $N_f = 1+1+1$ and $N_f = 2+1$. We only note here that the SU(3) chiral logarithms for the BSM operators are much simpler than those for B_K. The latter involve 13 additional LECs, whereas the former involve none. This simplicity can be traced back to the fact that the matrix elements of the BSM operators do not vanish in the SU(3) chiral limit.

Some time ago, Becirevic and Villadoro (BV) calculated the chiral logarithms for the BSM operators in continuum, unquenched SU(3) ChPT [12] (results which we use as a check on ours in the appropriate limit). They noted that the chiral logarithmss cancel in the ratios B_2/B_3 and B_4/B_5. They dubbed these ratios "golden" and advocated their use in minimizing errors in chiral

extrapolations. Our results allow us to extend the set of golden combinations. First (and this is almost trivial) the two golden ratios of BV remain golden in the PQ theory, and in SU(2) ChPT. Second, the ratio B_2/B_K becomes golden in SU(2) ChPT. Finally, the product B_2B_4 is also golden in SU(2) ChPT. Neither of these new combinations is golden in SU(3) ChPT, although there is a partial cancellation of chiral logarithms (and so in BV's parlance they are "silver" combinations).

These golden combinations could be particularly important for a calculation using staggered fermions. This potential importance arises because chiral logarithms are responsible for a significant fraction of the taste breaking in the B_j (since the pion masses entering the logarithms are taste dependent). Thus the chiral behavior of the golden combinations may be smoother and easier to fit. One need only fit one of the five B-parameters including chiral logarithms, determining the other four using golden ratios or products. We are presently testing this approach in our companion numerical calculation [3].

References

[1] P. A. Boyle, N. Garron and R. J. Hudspith, *Neutral kaon mixing beyond the standard model with $n_f = 2+1$ chiral fermions,* Phys. Rev. D **86**, 054028 (2012) [arXiv:1206.5737 [hep-lat]].

[2] V. Bertone *et al., Kaon Mixing Beyond the SM from Nf=2 tmQCD and model independent constraints from the UTA,* arXiv:1207.1287 [hep-lat].

[3] H. -J. Kim *et al.,* these proceedings.

[4] F. Mescia and J. Virto, *Natural SUSY and Kaon Mixing in view of recent results from Lattice QCD,* arXiv:1208.0534 [hep-ph].

[5] J. A. Bailey, H. -J. Kim, W. Lee and S. R. Sharpe, *Kaon mixing matrix elements from beyond-the-Standard-Model operators in staggered chiral perturbation theory* Phys. Rev. D **85**, 074507 (2012) [arXiv:1202.1570 [hep-lat]].

[6] A. J. Buras, M. Misiak and J. Urban, *Two loop QCD anomalous dimensions of flavor changing four quark operators within and beyond the standard model,* Nucl. Phys. B **586**, 397 (2000) [hep-ph/0005183].

[7] J. Kim, W. Lee and S. R. Sharpe, *One-loop matching of improved four-fermion staggered operators with an improved gluon action,* Phys. Rev. D **83**, 094503 (2011) [arXiv:1102.1774 [hep-lat]].

[8] R. S. Van de Water and S. R. Sharpe, *B_K in staggered chiral perturbation theory,* Phys. Rev. D **73**, 014003 (2006) [hep-lat/0507012].

[9] T. Bae, *et al., B_K using HYP-smeared staggered fermions in $N_f = 2+1$ unquenched QCD,* Phys. Rev. D **82**, 114509 (2010) [arXiv:1008.5179 [hep-lat]].

[10] W. -J. Lee and S. R. Sharpe, *Partial flavor symmetry restoration for chiral staggered fermions,* Phys. Rev. D **60**, 114503 (1999) [hep-lat/9905023].

[11] C. Aubin and C. Bernard, *Pion and kaon masses in staggered chiral perturbation theory,* Phys. Rev. D **68**, 034014 (2003) [hep-lat/0304014].

[12] D. Becirevic and G. Villadoro, *Remarks on the hadronic matrix elements relevant to the SUSY $K_0 - \overline{K_0}$ mixing amplitude,* Phys. Rev. D **70**, 094036 (2004) [hep-lat/0408029].

[13] J.A. Bailey, H.-J. Kim, W. Lee and S.R. Sharpe, in progress.