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Exploring walking behavior in SU(3) gauge theory with 4 and 8 HISQ quarks

Yasumichi Aoki[a], **Tatsumi Aoyama**[a], **Masafumi Kurachi**[a], **Toshihide Maskawa**[a], **Kei-ichi Nagai**[*a], **Hiroshi Ohki**[a], **Akihiro Shibata**[b], **Koichi Yamawaki**[a] and **Takeshi Yamazaki**[a]

LatKMI Collaboration

[a]*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya, 464-8602, Japan*
[b]*Computing Research Center, High Energy Accelerator Research Organization (KEK), Tsukuba, 305-0801, Japan*
E-mail: `keiichi.nagai@kmi.nagoya-u.ac.jp`

We present the report of the LatKMI collaboration on the lattice QCD simulation for the cases of 4 and 8 flavors. The Nf=8 in particular is interesting from the model-building point of view: The typical walking technicolor model with the large anomalous dimension is the so-called one-family model (Farhi-Susskind model). Thus we explore the walking behavior in LQCD with 8 HISQ quarks by comparing with the 4-flavor case (in which the chiral symmetry is spontaneously broken). We report preliminary results on the spectrum, analyzed through the chiral perturbation theory and the finite-size hyperscaling, and we discuss the availability of the Nf=8 QCD to the phenomenology.

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*Speaker.

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1. Introduction

The origin of mass is the most urgent issue of the particle physics today. One of the candidates for the theory beyond the Standard Model is the walking technicolor which is the strongly coupled gauge theory having a large anomalous dimension $\gamma_m \simeq 1$ and approximate scale invariance due to the almost non-running (walking) coupling [1, 2]. The walking behavior is in fact realized in the QCD with large number of (massless) flavors N_f which possesses Caswell-Banks-Zaks infrared fixed point (IRFP) [3] in the two-loop beta function. The exact IRFP would be washed out by the dynamical generation of a quark mass m in the very infrared region $\mu < m$ for $N_f < N_f^{cr}$, N_f^{cr} being the critical number. However, for N_f very close to N_f^{cr}, m could be much smaller than the intrinsic scale Λ ($\gg m$), an analogue of $\Lambda_{\rm QCD}$, beyond which the coupling runs as the asymptotically free theory, so that the coupling remains almost walking for the wide infrared region $m < \mu < \Lambda$ as a remnant of the would-be IRFP. The case $N_f > N_f^{cr}$ is called conformal window, although conformality is broken in the ultraviolet asymptotically free region beyond Λ.

Although the results from the two-loop and ladder approximation of Schwinger-Dyson equation analysis [4] are very suggestive, the relevant dynamics is obviously of non-perturbative nature, we would need fully non-perturbative studies. Among others the lattice simulations developed in the lattice QCD would be the most powerful tool for that purpose. The above two-loop and ladder studies suggest that the walking theory if existed would be in between $N_f = 8$ and $N_f = 12$. The $N_f = 8$ in particular is interesting from the model-building point of view: The typical technicolor model [5] is the so-called one-family model (Farhi-Susskind model) which has a one-family of the colored techni-fermions (techni-quarks) and the uncolored one (techni-leptons) corresponding to the each family of the SM quarks and leptons. Thus if the $N_f = 8$ turns out to be a walking theory, it would be a great message for the phenomenology to be tested by the on-going LHC.

Since the pioneering works on the lattice [6, 7] were carried out, a lot of groups have been doing lattice studies nowadays. (See Refs. [8] for a review of recent developments.)

2. Simulation

2.1 Simulation details

In our simulation, we use the tree level Symanzik gauge action and the highly improved staggered quark (HISQ) action [9] without the tadpole improvement and the mass correction in the Naik term. It is expected that the flavor symmetry in the staggered fermion and the behavior towards the continuum limit are improved by HISQ sction. We carried out the simulation by using the standard Hybrid Monte-Carlo (HMC) algorithm. We computed the hadron spectrum as the global survey in the parameter region and we obtained M_π, M_ρ, f_π and $\langle\bar{\psi}\psi\rangle$ as the basic observable.

The simulation for $N_f = 4$ is carried out at $\beta(= 6/g^2)$= 3.6, 3.7 and 3.8 for various quark masses on $12^3 \times 16$ and $16^3 \times 24$. We took over 1000 trajectories on the small lattice and about 600 trajectories on the large lattice in $N_f = 4$ case. The simulation for $N_f = 8$ is carried out at $\beta(= 6/g^2)$=3.6, 3.7, 3.8, 3.9 and 4.0 for various quark masses on ($12^3 \times 32$,) $18^3 \times 24$, $24^3 \times 32$, $30^3 \times 40$ and $36^3 \times 48$ for various quark masses. We took about 800 trajectories on each size.

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2.2 Analysis methods

In this section, we show preliminary results of the chiral perturbation analysis (ChPT) and the finite-size hyperscaling analysis in 4- and 8-flavor case.

If the system is in the chiral symmetry broken phase (χSB), physical quantities, M_H, are described by the chiral perturbation theory (ChPT); the polinomial behavior. In particular, about the pion decay constant $f_\pi = F + c_1 m_f + c_2 m_f^2 + \cdots$. (Here we don't discuss the existence of the chiral log.) If $F \neq 0$ in the above eq., it is regarded as the χSB.

On the other hand, if the system is in the conformal window, M_H are described by the finite-size hyperscaling relation (FSHS) [10] ; $LM_H = \mathscr{F}(X)$ where $X = Lm_f^{\frac{1}{1+\gamma}}$. The γ in this equation is defined as the anomalous mass-dimension. We carry out the hyperscaling analysis with our data of $M_H = \{M_\pi, f_\pi, M_\rho\}$ by the following fit function; $LM_H = c_0 + c_1 X$.

In the following, we analyse $N_f = 4$ and 8 by these methods.

3. Spectrum

3.1 $N_f = 4$

In this subsection, we analyze $N_f = 4$ system by the ChPT and the finite-size hyperscaling relation. The result of $N_f = 4$ is shown in Fig. 1, in which the pion mass squared, the decay constant and the chiral condensate are plotted on the panel from the left to the right respectively. M_π^2 is proportional to m_f. f_π and $\langle\bar{\psi}\psi\rangle$ have the non zero value in the chiral limit. Thus, the $N_f = 4$ has the property of the χSB phase and this is regarded as the signal of the chiral broken phase in the dynamical case of lattice QCD. Also, if the FSHS test is applied to $N_f = 4$ which is χSB phase

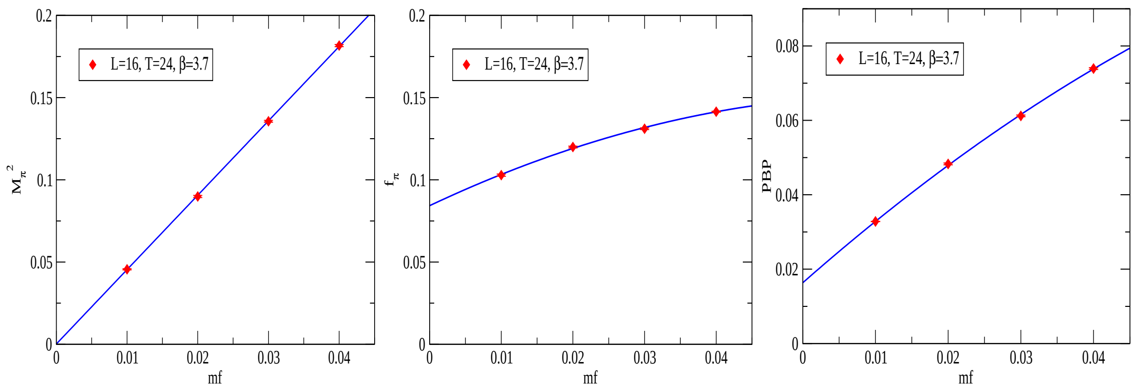

Figure 1: In $N_f = 4$ $SU(3)$ gauge theory on $16^3 \times 24$ at $\beta = 3.7$; **Left:** M_π^2 as functions of m_f, **Center:** f_π as functions of m_f, **Right:** $\langle\bar{\psi}\psi\rangle$ as functions of m_f. The blue-solid line in the left panel is the linear fit and the lines in other panels are the quadratic fit.

(the ordinary QCD), what happens? The result of this attempt for f_π is shown in Fig. 2. From these figs., there is no data alignment in the region $0 \le \gamma \le 2$. This is the property of QCD when the finite size hyperscaling is applied to.

These properties (ChPT and FSHS) in $N_f = 4$ may hint whether $N_f = 8$ is χSB or the conformal/walking.

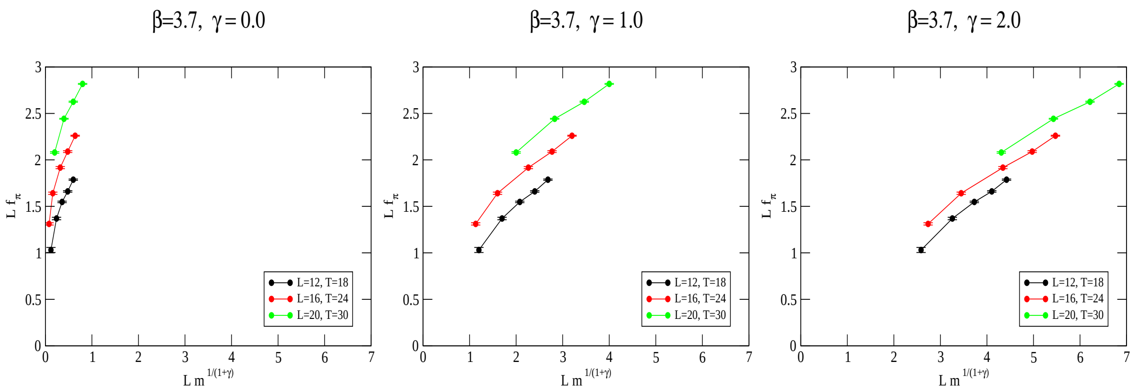

Figure 2: Finite size hyperscaling test of f_π in $N_f = 4$ $SU(3)$ gauge theory; **Left:** at $\gamma = 0.0$, **Center:** at $\gamma = 1.0$, **Right:** at $\gamma = 2.0$.

3.2 $N_f = 8$

In this subsection, we analyze $N_f = 8$ system by the ChPT and the finite-size hyperscaling test. The panels in Fig. 3 are M_π, f_π and M_ρ at $\beta = 3.8$ in partucular as a function of the quark mass m_f, and the polynomial fit (quadratic fit) and the power fit are plotted.

In M_π and M_ρ, the plateau appears in $m_f \lesssim 0.06$ on small lattice ($12^3 \times 32$) and in $m_f \lesssim 0.02$ on large lattice ($24^3 \times 32$) at $\beta = 3.8$. In the corresponding region of the plateau, f_π behaves like the linear toward the zero. Since these might be the effect of the finite size effect or might be in a different vacuum, these data are not included in the following analyses.

In Fig. 3, to take the infinite volume limit is difficult. Then we take the data on the largest volume at each m_f for the fitting. The fit range is $0.0 \le m_f \le 0.1$. The fit result of M_π^2 is obtained as follows: $M_\pi^2 = 2.31(2)m_f + 12.5(1)m_f^2, (\chi^2/\mathrm{dof} = 17.9)$ and $M_\pi^2 = 5.43(4)m_f^{1.197(3)}, (\chi^2/\mathrm{dof} = 34.0)$. For f_π, $\chi^2(f_\pi)/\mathrm{dof} = 14.7$ in the power fit and $\chi^2(f_\pi)/\mathrm{dof} = 6.1$ in the quadratic fit, then $f_\pi = 0.0295(3)$ in the limit $m_f \to 0$ as the quadratic fit result. For M_ρ, $\chi^2(M_\rho)/\mathrm{dof} = 6.5$ in the power fit and $\chi^2(M_\rho)/\mathrm{dof} = 1.3$ in the quadratic fit, then $M_\rho = 0.191(8)$ in the limit $m_f \to 0$ as the the quadratic fit result. In all cases, since the χ^2/dof in the quadratic fit is better than that in the power fit, the chiral limit by the quadratic fit gives the non-zero value of f_π and M_ρ. Thus, it seems that the $N_f = 8$ is in the χSB phase.

Here we discuss the validity of the ChPT fit. We used the expansion parameter defined as $\chi = N_f \left(\frac{M_\pi(m_f)}{4\pi F_\pi(m_f=0)}\right)^2$. In our simulation, $\chi \simeq 1.2 - 2.5 \simeq O(1)$ at the minimum value of $M_\pi \simeq 0.2$ in our simulation. Therefore, our result in $N_f = 8$ is consistent with ChPT.

Next we consider the chiral condensate by the direct calculation, $\langle \bar{\Psi}\Psi \rangle = \mathrm{Tr}[D_{HISQ}^{-1}(x,x)]$, and GMOR relation, $\Sigma = f_\pi^2 M_\pi^2 / m_f$. In the result of the direct calculation, $\langle \bar{\Psi}\Psi \rangle \sim m_f$ and then the chiral limit is very small in the lattice unit. In GMOR relation, the solid line is the combination of the quadratic fit results of M_π^2 and f_π. This chiral limit is is $\Sigma \simeq O(0.001)$. Thus the chiral limit of the chiral condensate is very small in the lattice unit. This means the chiral condensate of the techni-quark, $\langle \bar{Q}Q \rangle$, is tiny value and then the χSB is not strong to give the mass in the SM quark sector. Therefore, in the following, we attempt to find the tail (the remnant) of the conformal.

If the system is in the conformal window, the data is in the good agreement with the FSHS having the universal value of γ Although $N_f = 8$ is consistent with ChPT (χSB), because of the

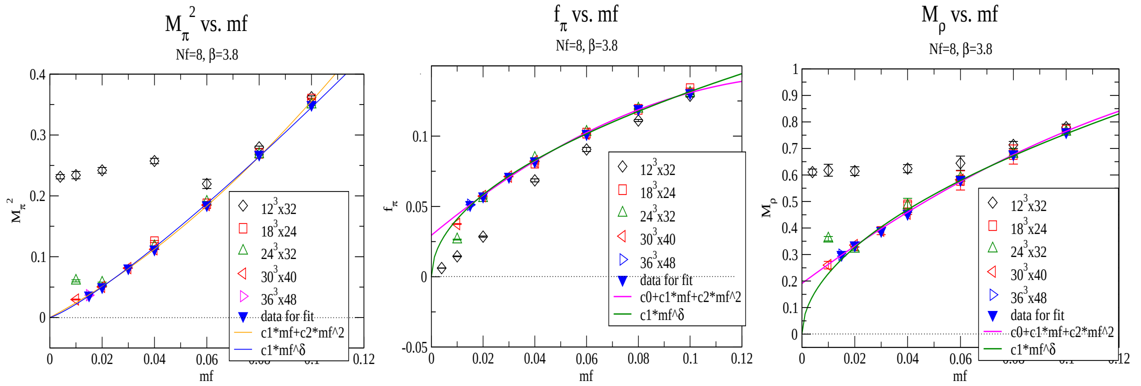

Figure 3: ChPT (the quadratic fit) in f_π in $N_f = 8$ $SU(3)$ gauge theory; **Left:** M_π^2 as a function of m_f, **Center:** f_π, **Right:** M_ρ. The solid lines are the quadratic fit and the power fit.

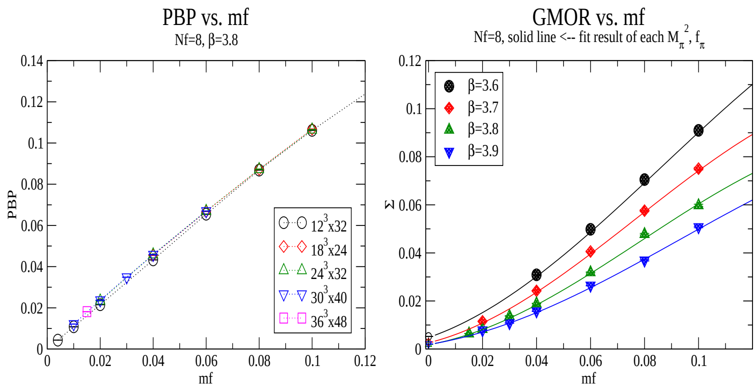

Figure 4: Ciral condensate in $N_f = 8$ $SU(3)$ gauge theory; **Left:** $\langle\bar{\Psi}\Psi\rangle$, **Right:** GMOR relation and the line obtained from the quadratic fit results of M_π^2 and f_π.

tiny value of the chiral condensate, we apply the FSHS to $N_f = 8$ system in order to catch the tail of the confromal if there is. Fig. 5 is the FSHS test of f_π for the various γ. The data is aligned (collapsing) at around $\gamma = 1$.

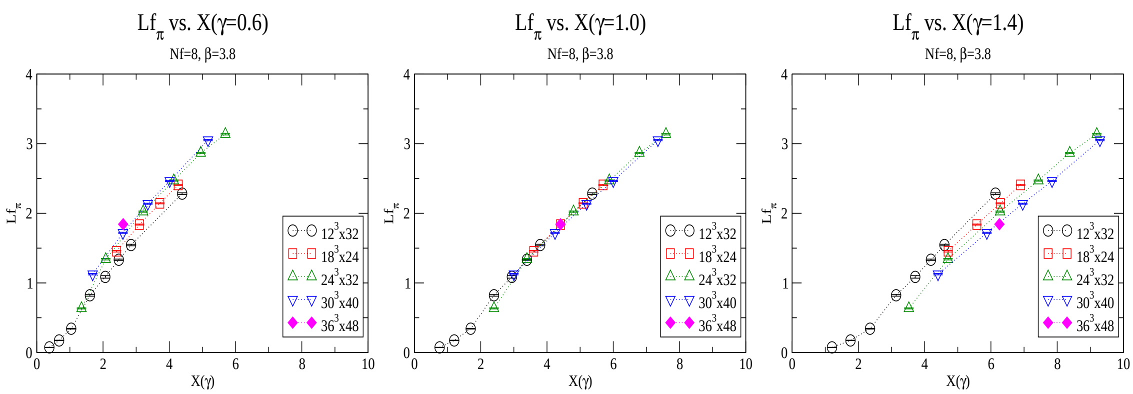

Figure 5: Finite size hyperscaling test of f_π in $N_f = 8$ $SU(3)$ gauge theory; **Left:** at $\gamma = 0.6$, **Center:** at $\gamma = 1.0$, **Right:** at $\gamma = 1.4$.

Therefore, to quantify this alignment, we attempt the linear fit as the leading approximation of FSHS. Panels in Fig. 6 are the linear fit result of FSHS for M_π, f_π and M_ρ from the left to the right. Although χ^2/dof is not small, the linearity for each observable is not worse and it seems that the remnant of conformal exists. The result, $\gamma(M_\pi) \neq \gamma(M_\rho) \neq \gamma(f_\pi) \sim 1.0$, indicates the remnant of the conformal. This situation is very interesting to construct the walking model.

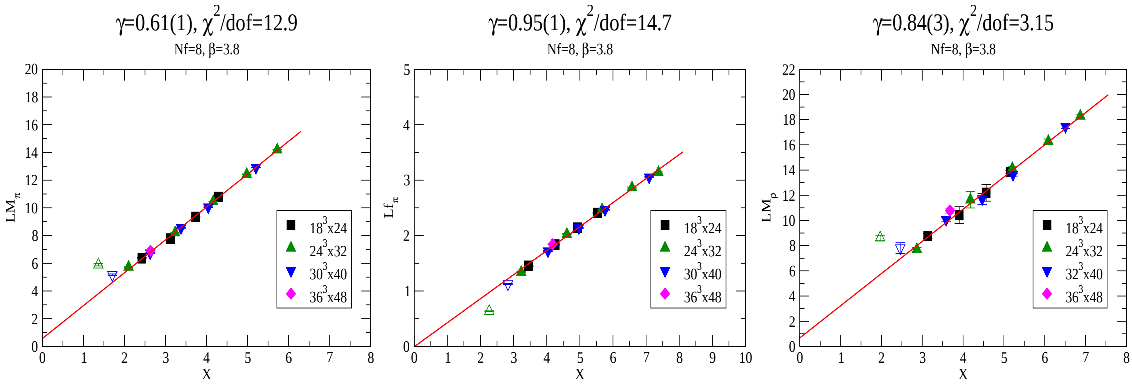

Figure 6: Finite size hyperscaling fit by the linear ansatz of f_π in $N_f = 8$ $SU(3)$ gauge theory; **Left:** M_π, **Center:** f_π, **Right:** M_ρ. The open symbol is not included in the fit data.

4. Discussion and Summary

We have made simulations of lattice QCD with 4 and 8 flavors by using the HISQ action. We obtained the following *preliminary* result; The $N_f = 4$ QCD is in good agreement with the chiral broken phase and the f_π data is not aligned in $0 \leq \gamma \leq 2$, which is the characteristics of QCD applied to the FSHS test. The $N_f = 8$ is consistent with ChPT (χ-parameter) and is not inconsistent with FSHS. We extracted the γ-value from the FSHS test; non-universal γ and $\gamma(f_\pi) \sim 1.0$. We show the table for various βs as the very preliminary result. To understand the behavior of $N_f = 8$,

	$\beta = 3.6$	$\beta = 3.7$	$\beta = 3.8$	$\beta = 3.9$	$\beta = 4.0$
γ in M_π	0.64(1)	0.63(1)	0.61(1)	0.56(1)	0.56(1)
γ in f_π	0.98(2)	0.99(1)	0.95(1)	0.92(1)	0.91(1)
γ in M_ρ	1.02(2)	0.91(4)	0.84(3)	0.79(4)	0.77(6)

Table 1: Preliminary. The statistical error only

We compare with our simulation of $N_f = 12$ which is consistent with the conformal [11] and the Schwinger-Dyson equation analysis on the finite size and mass [12]. According to SD-eq. analysis, $\gamma \simeq 1.0$ for the near conformal in χSB phase. This might indicate that the system with $\gamma \sim 1.0$ is near conformal/walking.

Therefore, from these analyses (ChPT, FSHS and the comparison with $N_f = 4$ and 12 simulations and with SD-eq. analysis), $N_f = 8$ is the candidate of the walking theory.

We should mention, however, that there are several possible systematic uncertainties not considered in this report; As pointed out in Ref. [12], there exists the mass correction in the hyperscaling relation for the heavy quark region. Then the linear ansatz adapted in Fig. 6 may not be sufficient to fit our data. To improve the situation for better understanding, we will accumulate more data for various fermion masses and βs on larger lattices, and carry out detailed analysis using those data.

Acknowledgments

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