

QCD with colour-sextet quarks

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We study QCD with 2 colour-sextet quarks as a model for walking Technicolor, using lattice gauge theory simulations (RHMC) at finite temperature. Our goal is to determine if the massless theory is QCD-like (confining, with spontaneously-broken chiral symmetry) with a slowly varying coupling (walks) or if it is a conformal field theory. We do this by simulating the theory at finite temperature and observing how the coupling at the chiral-symmetry restoration temperature depends on the temporal extent N_t of the lattice (in lattice units). If the theory is QCD-like, this coupling should approach zero in the large N_t limit in the manner predicted by asymptotic freedom. If it is conformal, this coupling should approach a finite value in this limit, i.e. the transition would be a bulk transition. We discuss new results at $N_t = 6, 8$ and 12 . These preliminary results indicate that the coupling does decrease with increasing N_t , but it is unclear if this is consistent with asymptotic freedom.

We also present new results at $N_t = 8$ for QCD with 3 colour-sextet quarks. This theory is believed to be conformal, and is studied for comparison with the 2-colour case. Preliminary results suggest that $N_t = 8$ is still too small to access the weak coupling limit.

The 30th International Symposium on Lattice Field Theory

June 24-29, 2012

Cairns, Australia

^{*}Speaker.

[†]This research was supported in part by US Department of Energy contracts DE-AC02-06CH11357 and DE-FG02-12ER41871

1. Introduction

We are interested in extensions of the standard model with a composite/strongly-interacting Higgs sector. Technicolor theories [1, 2] – QCD-like theories with massless techni-quarks where the techni-pions play the rôle of the Higgs field, giving masses to the W and Z – show the most promise. It is difficult to suppress flavour-changing neutral currents in extended Technicolor while giving large enough masses to the fermions. Technicolor theories which are simply scaled-up QCD fail the precision electroweak tests [4]. Walking Technicolor theories [5, 6, 7, 8], where the fermion content of the gauge theory is such that the running coupling constant evolves very slowly over a considerable range of length/energy-momentum scales, can potentially avoid these problems [9, 10, 11, 12].

QCD with $1\frac{28}{125} < N_f < 3\frac{3}{10}$ flavours of massless colour-sextet quarks is expected to be either a Walking or a Conformal field theory. (First term in the β function is negative, second positive.) The $N_f = 3$ theory is presumably conformal. The $N_f = 2$ theory could be either Walking or Conformal. In addition, if walking, it is minimal, having just the right number of Goldstone bosons (3) to give masses to the W and Z. For other arguments as to why this theory might be of interest see [13, 14].) We simulate the $N_f = 2$ theory to see if it walks. We also study the $N_f = 3$ theory for comparison. We simulate these theories at finite temperature, using the evolution of the lattice bare coupling at the chiral transition with N_t to determine if it is governed by asymptotic freedom – walking – or if it approaches a non-zero constant (bulk transition) – conformal. The deconfinement transition occurs at appreciably smaller $\beta = 6/g^2$ (stronger coupling), and it is unlikely to give useful information on QCD-like versus conformal behaviour at the N_t values we use.

We simulate the $N_f = 2$ theory on lattices with $N_t = 4, 6, 8, 12$ and hope to extend this to larger N_t . (For our earlier work on this theory see [15, 16, 17] Preliminary results indicate that $\beta_\chi(N_t = 12)$ is significantly larger than $\beta_\chi(N_t = 8)$, but by less than what the 2-loop β -function would predict. We are also performing preliminary runs with β fixed at a value above $\beta_\chi(N_t = 12)$ on lattices with fixed N_s varying N_t , keeping $N_t \leq N_s$ (we thank Julius Kuti for suggesting this). Here we look for the transition to the chiral-symmetry restored state as N_t increases.

We simulate the $N_f = 3$ theory on lattices with $N_t = 4, 6, 8$ and hope to extend this to $N_t = 12$. (For earlier work, see [18].) Preliminary results indicate that $\beta_\chi(N_t = 8)$ is probably significantly greater than $\beta_\chi(N_t = 6)$ which would indicate that we are not yet at weak enough coupling.

We are using unimproved staggered quarks and a simple Wilson plaquette action for all our simulations. The RHMC algorithm is used to implement the required fractional powers of the fermion determinant.

DeGrand *et al.* are studying the 2-flavour theory using improved Wilson fermions [19, 20, 21, 22, 23]. Most of their simulations have been at zero temperature. So far their results are inconclusive, although they tend to favour the conformal field theory interpretation. The Lattice Higgs Collaboration are also studying this theory at zero temperature using improved staggered quarks. [24, 25, 26, 27, 28]. Their results favour a QCD-like (and hence walking) interpretation.

2. Simulations of lattice QCD with 2 colour-sextet quarks at finite temperature

2.1 Simulations at $N_t = 6$

We have extended our simulations for quark mass $m = 0.005$ on a $12^3 \times 6$ lattice, such that the β spacing is now 0.02 in the range $6.5 \leq \beta \leq 6.7$, which encompasses the chiral transition. In this range we have accumulated 100,000 trajectories for each β .

At this quark mass, it is impossible to determine the position of the chiral-symmetry restoration transition with any precision from the chiral condensate itself. We determine the position of this transition from the peak in the disconnected chiral susceptibility. This is defined by:

$$\chi_{\bar{\psi}\psi} = \frac{V}{T} [\langle (\bar{\psi}\psi)^2 \rangle - \langle \bar{\psi}\psi \rangle^2] \quad (2.1)$$

where V is the spatial volume of the lattice and T is the temperature. $\bar{\psi}\psi$ is a lattice averaged quantity. Because we only have stochastic estimators for $\bar{\psi}\psi$ (5 per trajectory), we obtain unbiased estimators of $(\bar{\psi}\psi)^2$ as the products of 2 different estimators of $\bar{\psi}\psi$ for the same gauge configuration.

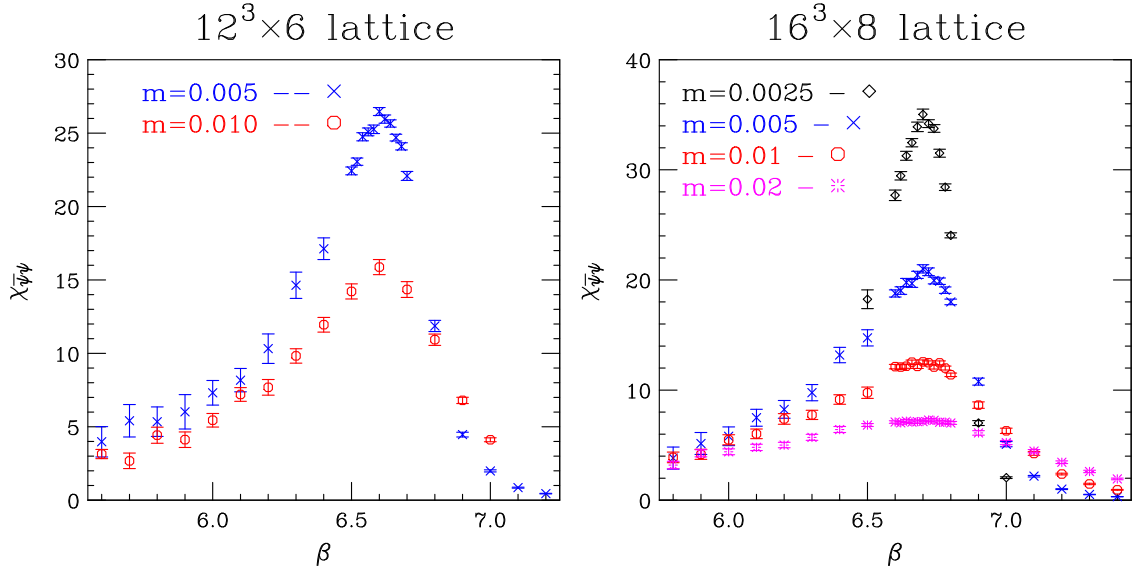


Figure 1: Chiral susceptibilities on a $12^3 \times 6$ lattice, **Figure 2:** Chiral susceptibilities on a $16^3 \times 8$ lattice, $N_f = 2$.

Figure 1 shows these chiral susceptibilities for our $12^3 \times 6$ simulations. The $m = 0.005$ susceptibility shows a clear peak at $\beta \approx 6.60$. Under the assumption that the position of this peak has little mass dependence (which is not inconsistent with this data), we thus conclude that the position of the chiral transition in the chiral ($m \rightarrow 0$) limit is at $\beta = \beta_\chi = 6.60(2)$

2.2 Simulations at $N_t = 8$

We have performed simulations on $16^3 \times 8$ lattices in the vicinity of the chiral transition. Quark masses $m = 0.0025$, $m = 0.005$, $m = 0.01$ and $m = 0.02$ were used, to enable extrapolation to the chiral limit. For $6.6 \leq \beta \leq 6.8$ which spans the neighbourhood of the chiral transition we

performed simulations at β values spaced by 0.02 to enable accurate determination of the position of this transition. At $m = 0.0025$ we performed simulations of 100,000 trajectories in length for each β in this range. For each of the 3 larger masses we simulated 50,000 trajectories at each of the β values in this range.

Again, the dependence of the chiral condensates on β is too smooth to accurately determine the position of the chiral transition, even though the decrease in the chiral condensate with decreasing mass does indicate that there *is* such a transition above which the condensate vanishes in the chiral limit. Thus we again turn to the chiral susceptibility, which has a peak that diverges in the chiral limit, to accurately determine β_χ . A graph of these chiral susceptibilities for each of the 4 masses is shown in figure 2.

Because the β values used are sufficiently close together, it is possible to use Ferrenberg-Swendsen reweighting to determine the position of the peak for $m = 0.0025$ more precisely. This yields $\beta = \beta_{peak} = 6.690(5)$. Since β_{peak} clearly has very little mass dependence, we deduce that $\beta_\chi = 6.69(1)$. Hence

$$\beta_\chi(N_t = 8) - \beta_\chi(N_t = 6) \approx 0.09, \quad (2.2)$$

consistent with the 2-loop perturbative prediction of ≈ 0.087 .

2.3 Simulations at $N_t = 12$

We are simulating lattice QCD with 2 flavours of light colour-sextet quarks on a $24^3 \times 12$ lattice, in the neighbourhood of the chiral-symmetry restoration transition. We are running at masses $m = 0.0025$, $m = 0.005$ and $m = 0.01$, to enable continuation to the chiral limit. For $6.6 \leq \beta \leq 6.9$, we are running at a set of β s spaced by 0.02. In this range, we have 10,000 – 25,000 trajectories at each β and m (except at $\beta = 6.6$, $m = 0.005$, where we have 62,500 trajectories), and will extend this to 50,000 – 100,000 trajectories.

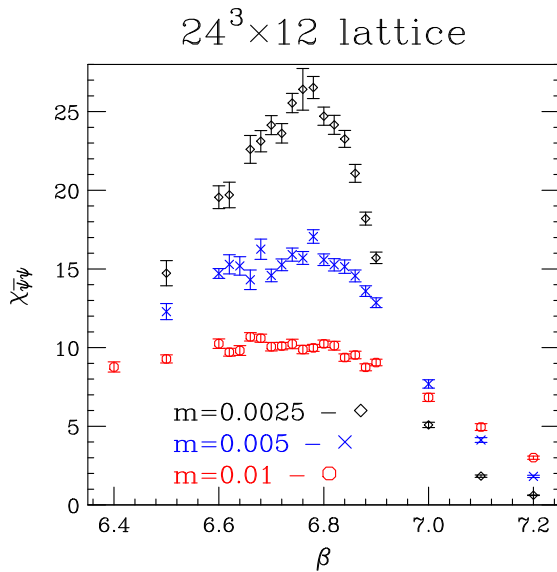


Figure 3: Chiral susceptibilities on a $24^3 \times 12$ lattice, $N_f = 2$.

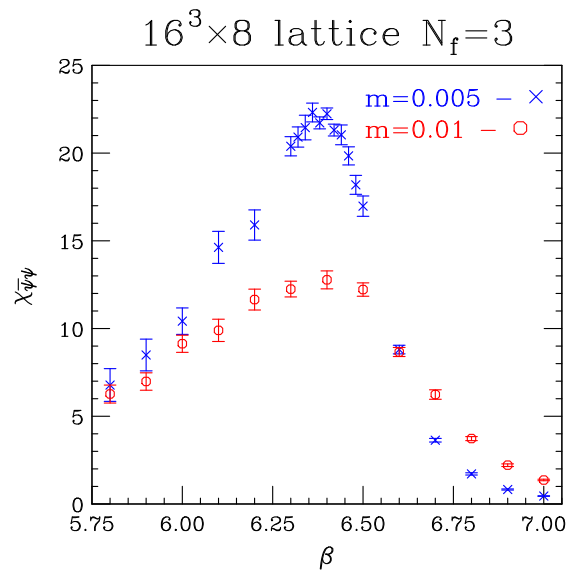


Figure 4: Chiral susceptibilities on a $16^3 \times 8$ lattice, $N_f = 3$.

Figure 3 shows the chiral susceptibilities from these runs. While the $m = 0.0025$ and possibly the $m = 0.005$ susceptibilities show clear peaks, the statistics are not yet adequate to make a precise estimate of the positions of these peaks. The $m = 0.0025$ graph shows a peak at $\beta = \beta_\chi \approx 6.78$. If this were correct, it would mean that $\beta_\chi(N_t = 12) - \beta_\chi(N_t = 8) \approx 0.09$, compared with the predicted 0.12.

2.4 Other simulations and analyses

In addition to measuring the unsubtracted, unrenormalized chiral condensates, we have also used the subtraction scheme of the Lattice Higgs Collaboration, which removes much of the quadratic UV divergence.

$$\langle \bar{\psi}\psi \rangle_{sub} = \langle \bar{\psi}\psi \rangle - \left(m_V \frac{\partial}{\partial m_V} \langle \bar{\psi}\psi \rangle \right)_{m_V=m} \quad (2.3)$$

[25], where m_V is the valence-quark mass. While this reduces the magnitude of $\langle \bar{\psi}\psi \rangle$, making the fact that it vanishes at large β more obvious, it does not help significantly in the precision determination of β_χ at the masses we use.

Finally, we are simulating our theory on $24^3 \times N_t$ lattices for $N_t \leq 24$ at $\beta = 6.9$ to try to observe chiral-symmetry restoration as N_t is increased. Here we look at both the unsubtracted and the subtracted chiral condensates. Our preliminary results look promising but not conclusive.

3. Simulations of lattice QCD with 3 colour-sextet quarks at finite temperature

We are extending our earlier simulations of QCD with 3 colour-sextet quarks to $N_t = 8$. Our runs have been performed on $16^3 \times 8$ lattices with $m = 0.005$ and $m = 0.01$. For $m = 0.005$ and $6.3 \leq \beta \leq 6.5$ (the neighbourhood of the chiral transition), we currently have 20,000 to 50,000 trajectories for each β . Elsewhere we have 10,000 trajectories for each (β, m) .

Figure 4 shows the chiral susceptibilities from these runs. While we need to complete 50,000 trajectories at each β in the above range to accurately determine β_χ , it already appears that $\beta_\chi(N_t = 8) - \beta_\chi(N_t = 6)$ will be appreciable. Since we believe that this theory is conformal, when $\beta_\chi(N_t)$ will approach a finite value as $N_t \rightarrow \infty$, this suggests that we will need a larger N_t to see the weak-coupling limit. Even if this theory were QCD-like, the 2-loop β -function would predict $\beta_\chi(N_t = 8) - \beta_\chi(N_t = 6) \approx 0.0025$, which would be too small for us to see, and hence inconsistent with our preliminary observations.

4. Discussion and Conclusions

We simulate the thermodynamics of QCD with 2 colour-sextet quarks on lattices with $N_t = 4, 6, 8, 12$, in the neighbourhood of the chiral transition. If chiral-symmetry restoration is a finite-temperature phase transition, measuring β_χ as a function of N_t yields the running of the bare lattice coupling β with a at $a = 1/(N_t T_\chi)$. Asymptotic freedom would imply that $\beta_\chi \rightarrow \infty$ ($g_\chi^2 \rightarrow 0$) as $N_t \rightarrow \infty$ and hence $a \rightarrow 0$. At the masses we use, the unsubtracted, unrenormalized chiral condensates become small at large β s, and appear consistent with extrapolating to zero as $m \rightarrow 0$. However, they decrease too smoothly to enable an accurate determination of β_χ . Hence we

have extracted β_χ from the peaks in the chiral susceptibilities. We have also examined the chiral condensates with the subtraction scheme used by the Lattice Higgs Collaboration. While these subtracted condensates show more clearly that the condensates will vanish in the chiral limit at large enough β , they still do not allow for an accurate determination of β_χ .

We present preliminary results indicating that β_χ increases with N_t over the range of N_t s considered. This suggests that the theory does walk. The change in β_χ between $N_t = 6$ and $N_t = 8$ is in excellent agreement with 2-loop perturbation theory. However, the change in β_χ between $N_t = 8$ and $N_t = 12$ appears to be about 25% smaller than would be predicted from the 2-loop β -function. This is of concern, since this lies in the assumed weak-coupling domain ($\beta \gtrsim \beta_\chi(N_t = 6)$). However, we need more statistics to accurately determine $\beta_\chi(N_t = 12)$, and if it remains low, we will need to check that this is not a finite volume effect. It is possible that the 2-loop β -function is inadequate to describe the running of the bare coupling for unimproved staggered lattice QCD at these couplings. This will then require simulations at larger N_t , to distinguish walking from conformal behaviour.

A series of runs performed on $24^3 \times N_t$ lattices for several N_t s ($N_t \leq 24$) at small quark masses, and a fixed coupling intermediate between $\beta_\chi(N_t = 12)$ and the expected value of $\beta_\chi(N_t = 24)$, does show an increase in both the unsubtracted and subtracted chiral condensates. More work is needed to determine if this implies a transition to a chirally broken theory as N_t increases.

The zero temperature properties of this theory need to be studied and the results compared with Fodor *et al* and DeGrand *et al*. In particular we need to determine the masses of the Higgs and the dilaton, to see if either of these could be the Higgs-like particle observed at the LHC. For a recent discussion of what this entails see reference [26].

We are extending our $N_f = 3$ runs to $N_t = 8$. Preliminary results indicate that there is a substantial increase in β_χ between $N_t = 6$ and $N_t = 8$. This would indicate that, for this range of N_t s, β_χ does not lie completely in the weak-coupling domain, and that we will need simulations at larger N_t .

Other theories we plan to study include $SU(2)_{colour}$ with 3 colour-adjoint (symmetric) Majorana/Weyl quarks, and $SU(4)_{colour}$ with colour-antisymmetric quarks.

Acknowledgements

This research used resources of the National Energy Research Scientific Computing Center, which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231. In particular, these simulations were performed on the Cray XT4, Franklin, the Cray XT6, Hopper, and the Linux cluster, Carver, all at NERSC. In addition this research used the Cray XT5, Kraken at NICS under XSEDE Project Number: TG-MCA99S015. We also acknowledge the computing resources provided on ‘‘Fusion’’, a 320-node computing cluster operated by the Laboratory Computing Resource Center at Argonne National Laboratory.

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