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Two-nucleon Scattering in Nuclear Effective Field Theory: How to Construct the Power Counting

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The effective field theory formulation of nuclear forces grants the possibility of making systematic calculations in few-nucleon systems, that is, we can know in advance the uncertainty of theoretical predictions. However, until recently, this promise has remained unfulfilled. A precondition for systematicity is the existence of a power counting at the level of observables, a goal that has not been easy to achieve. Here we will review the ideas opening the path to the formulation of power counting for two-nucleon scattering. We will concentrate on the conceptual aspects rather than on the technical ones. While one pion exchange is non-perturbative in the lower partial waves and we iterate it at all orders, the chiral two pion exchange diagrams can be treated as a perturbation. The additional requirement of renormalizability (cut-off independence) guarantees the consistency, sistematicity and model independence of the theory and determines the power counting (i.e. the relative size) of the contact range operators. The number of counterterms is larger than what is expected from naive dimensional analysis. Finally the quality of the phase shifts is as good (if not better) than in the Weinberg counting at the same order.

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1. Introduction

The nature and derivation of nuclear forces is a central problem of nuclear physics that still remains open after six decades of research. While there are many phenomenological descriptions of the nuclear force that are able to reproduce two-nucleon scattering data and the properties of light nuclei with great accuracy, what theorists ultimately want is to derive nuclear forces from quantum chromodynamics (QCD), the fundamental theory of strong interactions. The direct strategy – to compute nuclear physics in lattice QCD – is full of promise and is starting to explore the two [1, 2] and three [3] nucleon sector, but currently only for large pion masses as they require less computational power. Even though lattice QCD will eventually reach the physical pion mass, in the meantime there is a more indirect way to do the job: to formulate an effective field theory (EFT) description of few nucleon systems that incorporates the low energy symmetries of QCD, in particular chiral symmetry. In this contribution we will follow the EFT approach, which also has the advantage of complementing lattice QCD calculations via chiral extrapolations.

The standard EFT for hadronic processes is chiral perturbation theory (ChPT). However, its application to the two-nucleon sector is not straightforward owing to the appearance of non-perturbative phenomena (e.g. the existence of the deuteron). To solve this problem Weinberg proposed to expand the potential (instead of the scattering amplitude) within the ChPT framework [4, 5]. The resulting potential is then iterated at all orders by plugging it into the Schrödinger equation for obtaining theoretical predictions. This simple and powerful idea has been followed with enthusiasm (see Refs.[6, 7] for reviews), leading to the development of potentials that are able to reproduce the two-nucleon scattering data with a $\chi^2/d.o.f. \sim 1$ for laboratory energies below 300 MeV [8, 9]. However, rather than a definitive solution, the Weinberg prescription is a smart workaround for avoiding the (formerly unsolved) issue of the non-perturbative renormalizability of the EFT potential, which is in turn instrumental for the formulation of nuclear EFT. We now know that the Weinberg counting is inconsistent [10, 11], but we also know the ingredients that are necessary for averting the theoretical limitations of the Weinberg approach (in particular, a solid understanding of the renormalization of singular interactions [12, 13, 14, 15]).

In this contribution we will explain how to construct a consistent power counting for nuclear EFT. The advantage of power counting is that it makes the calculations systematic: at each order in the EFT expansion we know the *a priori* error of the theoretical predictions in advance. The discussion will be qualitative and we will present the conditions for having power counting at the level of observables as a recipe. Then we will show the specific application of this recipe to the two-nucleon sector and discuss a bit the nuclear EFT we obtain and its results for the phase shifts.

2. How Do We Build a Power Counting?

The EFT formulation of nuclear forces is based on symmetries and power counting. Chiral symmetry provides the connection with QCD, while power counting is the ordering principle that grants predictive power to EFT (it sorts out the infinite number of interactions compatible with the low energy symmetries). Thanks to power counting the EFT amplitudes are organized as a power series in terms of a small expansion parameter $x_0 = Q/\Lambda_0$, which can be identified with the ratio of the characteristic low energy scale Q over the high energy scale Λ_0. Weinberg realized that the two-nucleon potential is amenable to a power counting expansion. However, the potential is not an observable. Therefore the problem in nuclear EFT is to ensure that the power counting expansion of the potential V translates into an expansion of the scattering amplitude T

$$V_{\rm EFT} = \sum_{\nu \geq \nu_0}^{\nu_{\rm max}} V^{(\nu)} + \mathscr{O}(x_0^{\nu_{\rm max}+1}) \quad \Longrightarrow \quad T_{\rm EFT} = \sum_{\nu \geq \nu_0}^{\nu_{\rm max}} T^{(\nu)} + \mathscr{O}(x_0^{\nu_{\rm max}+1}) \,, \tag{2.1}$$

with x_0 the expansion parameter. In this regard, the iteration of the potential in the Lippmann-Schwinger equation ($T = V + V G_0 T$), which is the way to obtain the scattering amplitude T from the potential in the Weinberg counting, does not guarantee a good expansion for T. The problem lies in the loops: they may probe the high-energy structure of the EFT potential, which is not meaningful (the expansion of the potential only makes sense at low energies) but can dominate the calculations. Even after counterterms have been added, the loop contributions from subleading pieces of the potential can spoil the power counting properties of observables [16, 17].

There is a fail-safe way to avoid the breakdown of power counting, which is (a) to iterate only a minimal subset of the EFT potential (usually the lowest order diagrams), and (b) to treat the subleading pieces as perturbations. The first point takes into account the non-perturbative nature of the nuclear force and the second guarantees that a subleading contribution in the potential remains subleading in the scattering amplitude. A third condition that is still required to obtain a renormalizable theory is the following: (c) at each step in the construction of the EFT, check for cut-off independence of the results and, if not, include additional counterterms. Renormalizability is necessary if we want power counting to hold for any value of the cut-off, so the calculations are actually model-independent. This last step is simple at the conceptual level, but complex at the technical one: its realization depends on a good understanding of the renormalization of singular interactions [12, 13, 14, 15]. In the next section we will show the result of applying this recipe.

3. The Power Counting in Nuclear EFT

We begin by choosing a minimal set of diagrams to iterate: the obvious candidates are the one pion exchange potential and the two S-wave contact interactions that conform the leading order (LO) potential in the Weinberg counting. Now, if we check the cut-off independence of the LO observables we will find that the two counterterms that we have included are not enough to renormalize the amplitudes. As shown numerically by Nogga el al. [11] (see also Refs. [12, 13, 14] for a formal derivation), there is still a strong cut-off dependence in the $^3P_0$ partial wave and a moderate one in the in the $^3P_2$ and $^3D_2$ waves. Therefore we include one counterterm in each of these lower partial waves to restore cut-off independence at LO [1]

The next step is to explore the next-to-leading and next-to-next-to-leading orders (NLO and NNLO). We include the subleading contribution (chiral two pion exchange) to the EFT potential as a perturbation and study the divergences appearing as a consequence of the singular character of the potential. We will not enter into the specific details on how to analyze the divergences and how to determine the number of counterterms curing them. The interested reader can consult

[1]Notice that for higher partial waves OPE becomes perturbative [18]. Alternatively, they can be non-perturbative renormalized from the lower partial waves without counterterm proliferation by using the adequate techniques [15].

Partial wave	LO	NLO	N^{2}LO	N^{3}LO
$^1S_0$	1	3	3	4
$^3S_1 - {}^3D_1$	1	6	6	6
$^1P_1$	0	1	1	2
$^3P_0$	1	2	2	2
$^3P_1$	0	1	1	2
$^3P_2 - {}^3F_2$	1	6	6	6
$^1D_2$	0	0	0	1
$^3D_2$	1	2	2	2
$^3D_3 - {}^3G_3$	0	0	0	1
All	5	21	21	27
Weinberg	2	9	9	24

Table 1: Power counting (i.e. number of counterterms) in two-nucleon scattering for different partial waves. We notice that the exact number of counterterms depends on the representation of the contact range interaction (the difference being redundant counterterms) – in the table above we have employed energy-dependent contact interactions in coordinate space [19, 20] – and that the counting is still disputed in repulsive triplet channels [22]. We also compare to the standard Weinberg approach and notice that at high orders the number of counterterms converge.

Refs. [18, 19, 20, 21, 22, 23] for a detailed account from different perspectives (renormalization group analysis, coordinate space and momentum space). Instead we merely present a compact summary of the results in Table 1. In general we obtain a larger number of counterterms than the Weinberg approach at lower orders.

Once we have determined the power counting, it is time to see how well we compare with experiment. We present the LO, NLO and NNLO predictions of Refs. [19, 20] for the phase shifts of the lower partial waves in Figure 1. We regularize the EFT potential with a sharp cut-off in coordinate space (i.e. a boundary condition), for which we take the values $r_c = 0.6 - 0.9\,\mathrm{fm}$. The counterterms are determined by fitting the data in the center-of-mass momentum range $k = 40 - 160\,\mathrm{MeV}$ ($k = 100 - 200\,\mathrm{MeV}$) for S-waves (P- and D-waves). As can be seen, they do in general compare well with the phase shifts obtained with the Nijmegen II potential [24] (which are equivalent to the partial wave analysis of Ref. [25]) and show a clear convergence pattern, meaning that the EFT expansion works. For completeness, we also compare with the NNLO results of Refs. [26, 27] in the Weinberg counting and see that we do better.

To summarize, we have shown how to construct a consistent power counting for two-nucleon scattering that is successful at the phenomenological level. The power counting is similar to Weinberg's, but contact range physics definitively play a more important role at lower orders. In the future we expect to extend the ideas presented in this contribution to the thee-nucleon sector and the study of electroweak reactions on the deuteron. Even though the technical details are more challenging than for two-nucleon scattering, at the conceptual level the discussion is straightforward and we can anticipate the enhancement of contact operators with respect to the expectations derived from naive dimensional analysis.

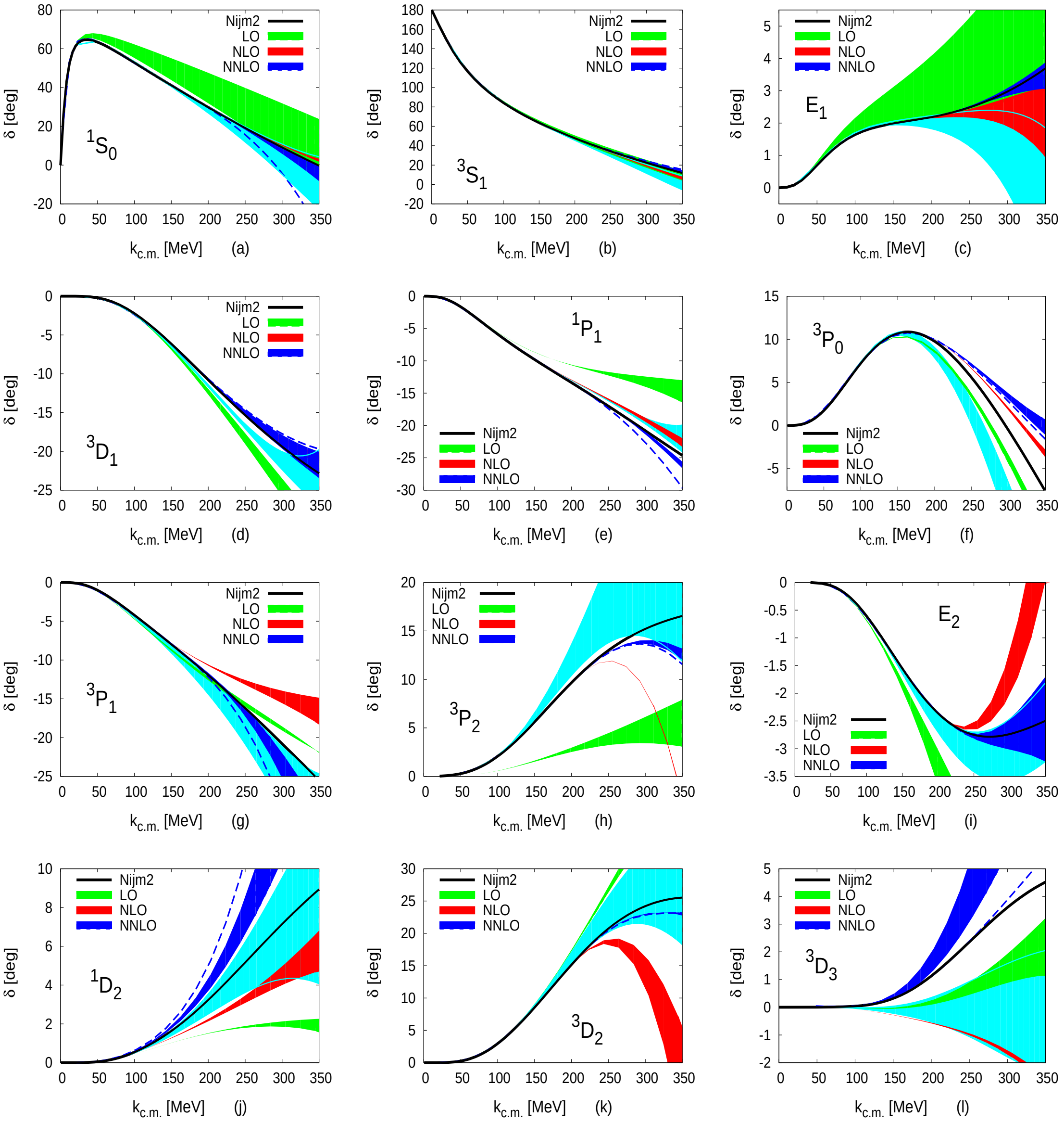

Figure 1: The S-, P- and D-wave phase shifts in two-nucleon scattering within nuclear EFT (δ is the phase shift and $k_{\rm c.m.}$ the center-of-mass momentum). We are using the power counting described in this contribution, in which one-pion exchange is non-perturbative and chiral two-pion exchange is added as a perturbation [19, 20]. The bands reflect the cut-off uncertainty of the results in the range $r_c = 0.6 - 0.9\,{\rm fm}$ (boundary radius). The dashed line (not always visible) represents the the 0.3 fm NNLO results. The phase shifts are compared with the corresponding ones in the Weinberg counting, which are taken from Refs. [26, 27].

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