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Precise dispersive determination of the f0(600) and f0(980) resonances

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We review our recent dispersive and model independent determination of the lightest two scalars, in terms of poles and residues – or mass, width and coupling–by means of once and twice subtracted Roy equations, using as input constrained fits to data, including the most recent ones from Kl4 decays. We find the $f_0(600)$ pole at $(457^{+14}_{-13}) - i(279^{+11}_{-7})$ MeV and that of the $f_0(980)$ at $(996\pm 7) - i(25^{+10}_{-6})$ MeV, whereas their respective couplings to two pions are $3.59^{+0.11}_{-0.13}$ GeV and 2.3 ± 0.2 GeV.

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1. Introduction

The existing experimental information from $\pi\pi$ scattering has many conflicting data sets at intermediate energies and no data at all close to the interesting threshold region. For many years this fact has made it very hard to obtain conclusive results on $\pi\pi$ scattering at low energies or in the σ (or $f_0(600)$) and $f_0(980)$ region. However, recent [1] and precise experiments on kaon decays, related to $\pi\pi$ scattering at very low energies, have renewed the interest on this process.

The dispersive integral formalism is model independent, just based on analyticity and crossing, and relates the $\pi\pi$ amplitude at a given energy with an integral over the whole energy range, increasing the precision and providing information on the amplitude at energies where data are poor, or where there is no data, like the complex plane. In addition, it makes the parametrization of the data irrelevant once it is included in the integral and relates different scattering channels among themselves.

Roy equations (RE), based on twice subtracted dispersion relations and crossing symmetry conditions for $\pi\pi \to \pi\pi$ amplitudes were obtained in 1971 [2]. In recent years, these equations have been used either to obtain predictions for low energy $\pi\pi$ scattering, either using Chiral Perturbation Theory (ChPT)[3, 4], or to test ChPT [5, 6, 7], as well as to solve old data ambiguities [8]. The RE are relevant for the sigma pole, whose position has also been predicted very precisely with the help of ChPT [9]. Our group [6, 7] has also used RE with Forward Dispersion Relations (FDR) to obtain a precise determination of $\pi\pi$ scattering amplitudes from data consistent with analyticity, unitarity and crossing. On purpose, we have not included ChPT constraints, so that we can use our results as tests of the ChPT predictions. Unfortunately, the large experimental error of the scattering length a_0^2 of the isospin 2 scalar partial wave, becomes a very large error for the sigma pole determination using RE. For this reason, a new set of once-subtracted RE, called GKPY eqs. for brevity, have been derived [10]. Both the RE and GKPY equations provide analytic extensions for the calculation of poles in the complex plane. Actually, we review here our recent results [10] on simple data fits constrained to satisfy these dispersive representations as well as our results [11] for the σ and $f_0(980)$ poles in the S0 wave obtained from GKPY eqs.

2. Overview of the analysis

The approach we have followed throughout a series of works [6, 7, 10, 12] can be summarized as follows:

We first obtain simple fits to data for each $\pi\pi$ scattering partial wave (the so called *Unconstrained Fits to Data*, or *UFD* for short). These fits are uncorrelated, therefore they can be very easily changed when new, more precise data become available. Let us remark that for our latest results we have used previous fits for all waves except the S0 wave, that we improve in [10]. For this wave, below 850 MeV, we have included the very precise K_{l4} data [1], we got rid of the controversial $K \to \pi\pi$ point, and we have included the isospin correction to K_{l4} data from [13]. Above 850 MeV we have updated the S0 wave using a polynomial fit to improve the intermediate matching between parametrizations (with a continuous derivative) and the flexibility of the $f_0(980)$ region, which is of particular importance for the discussion of the “dip” and “no-dip” scenarios that we will comment below.. At different stages of our approach we have also fitted Regge theory [14] to

$\pi\pi$ high energy data, and as our precision was improving, we have improved some of the UFD fits with more flexible parametrizations.

Then, these UFD are checked against FDR, several sum rules, RE and GKPY eqs. The UFD fit does not satisfy very well these dispersion relations. Particularly the GKPY eqs. for the S0 wave in the $f_0(980)$ region are satisfied very poorly [10].

Finally, we impose these dispersive relations in the previous fits as additional constraints. These new *Constrained Fits to Data* (*CFD* for short) are much more precise and reliable than the UFD set, being consistent with analyticity, unitarity, crossing, etc. The price to pay is that now all the waves are correlated.

In order to quantify how well the dispersion relations are satisfied, we define six quantities Δ_i as the difference between the left and right sides of each dispersion relation whose uncertainties we call $\delta\Delta_i$. Next, we define the average discrepancies

$$\overline{d_i^2} \equiv \frac{1}{\text{number of points}} \sum_n \left(\frac{\Delta_i(s_n)}{\delta\Delta_i(s_n)} \right)^2, \tag{2.1}$$

where the values of s_n are taken at intervals of 25 MeV. Note the similarity with an averaged $\chi^2/(d.o.f)$ and thus $\overline{d_i^2} \leq 1$ implies fulfillment of the corresponding dispersion relation within errors. In Table 1 we show the average discrepancies of the UFD for each FDR eq. up to 1420 MeV, and for each RE and GKPY eq. up to 1100 MeV. Since the total average discrepancies lie between 1 and 1.6 standard deviations, they can be clearly improved by imposing simultaneous fulfillment of dispersion relations. This is actually done in the CFD set, which is obtained by minimizing:

$$\chi^2 \equiv \left\{ \overline{d_{00}^2} + \overline{d_{0+}^2} + \overline{d_{I_t=1}^2} + \overline{d_{S0}^2} + \overline{d_{S2}^2} + \overline{d_P^2} \right\} W \tag{2.2}$$

$$+ \overline{d_I^2} + \overline{d_J^2} + \sum_i \left(\frac{p_i - p_i^{\text{exp}}}{\delta p_i} \right)^2, \tag{2.3}$$

where p_i^{exp} are all the parameters of the different UFD parametrizations for each wave or Regge trajectory, thus ensuring the data description, and d_I and d_J are the discrepancies for a couple of crossing sum rules, see reference [10] for details. Note that we choose $W \simeq 9-12$ for the effective number of degrees of freedom needed to parametrize curves like those appearing in the S0, P and S2 waves.

From Table 1 it is clear that the CFD set satisfies remarkably well all dispersion relations within uncertainties, and hence can be used directly if one needs a consistent parametrization. But, in addition, it can be used inside certain sum rules to obtain precise predictions for the threshold parameters of the effective range expansion for $\pi\pi$ scattering [10, 15]. At least for the scattering lengths and slopes, they are remarkably compatible with the prediction of [16], and seem to be easily accommodated within two-loop ChPT [15]. However, the description of the shape parameters (third order in the effective range expansion) seem to call for even higher orders of ChPT [15].

A relevant remark about the use of GKPY Eqs. is that it has allowed us [10] to settle a longstanding controversy between a "dip" and "non-dip" solution for the inelasticity of the S0 wave right above the $K\bar{K}$ threshold. The "dip" solution is clearly favored by the GKPY Eqs., and this is of relevance for the precise determination of the $f_0(980)$ resonance properties from scattering data.

In summary the CFD set provides a model independent and very precise description of the $\pi\pi$ scattering data consistent with analyticity and crossing.

	(UFD)	(CFD)
	$s^{1/2} \leq 1420\,\mathrm{MeV}$	$s^{1/2} \leq 1420\,\mathrm{MeV}$
$\pi^0\pi^0$ FDR	2.13	0.51
$\pi^+\pi^0$ FDR	1.11	0.43
$I_{t=1}$ FDR	2.69	0.25
	$s^{1/2} \leq 1100\,\mathrm{MeV}$	$s^{1/2} \leq 1100\,\mathrm{MeV}$
Roy eq. S0	0.56	0.04
Roy eq. S2	1.37	0.26
Roy eq. P	0.69	0.12
GKPY eq. S0	2.42	0.24
GKPY eq. S2	1.14	0.11
GKPY eq. P	2.13	0.60

Table 1: Average discrepancies $\overline{d^2}$ of the UFD and CFD for each FDR and RE. Note the remarkable CFD consistency.

3. Position of the σ and $f_0(980)$ poles

The mass and width of the σ meson quoted in the Particle Data Table are very widely spread [17]

$$M_\sigma - i\frac{\Gamma_\sigma}{2} \approx (400-1200) - i(250-500)\mathrm{MeV}. \tag{3.1}$$

The main reason of these uncertainties is that $\pi\pi$ scattering data are few and sometimes contradictory. Moreover, all quoted theoretical models are not equally reliable, and less so when extending the amplitude to the complex plane. Thus the position of the sigma pole in various models differ significantly [17], although, with a couple of exceptions, they tend to agree roughly around $\sim (450\pm 50) - i\,(250\pm 50)$, particularly those results based on dispersion theory.

The mass and width of the $f_0(980)$ meson quoted in the Particle Data Table are [17]

$$M_{f_0(980)} - i\frac{\Gamma_{f_0(980)}}{2} \approx (970-990) - i(20-50)\mathrm{MeV}. \tag{3.2}$$

The recent data from E865 collaboration at Brookhaven [18] and from NA48/2 [1] provide us with new and very precise information on the $\pi\pi$ scattering at low energies. Thanks to these new data we are able to construct, with our Constrained Fits to Data, a very reliable description for the $S0$ wave especially near the $\pi\pi$ threshold.

With those precise data parametrizations, we can now use either RE or GKPY eqs. to extend the partial waves analytically to the complex plane and look for poles in the second sheet of the S-matrix. As it is well known, a pole on the second Riemann sheet (unphysical sheet) is associated with a zero on the first—the physical one.

Depending on whether we use Roy or GKPY Eqs. we find a different accuracy in our results, namely:

$$\sqrt{s_\sigma} = (445 \pm 25) - i\,(278^{+22}_{-18})\ \mathrm{MeV}\ \ (\mathrm{RE}) \tag{3.3}$$
$$\sqrt{s_\sigma} = (457^{+14}_{-13}) - i\,(279^{+11}_{-7})\ \mathrm{MeV}\ \ \ (\mathrm{GKPY}) \tag{3.4}$$

and for the $f_0(980)$ pole:

$$\sqrt{s_{f_0(980)}} = (1003^{+5}_{-27}) - i\,(21^{+10}_{-8})\ \mathrm{MeV}\ \ (\mathrm{RE}) \tag{3.5}$$
$$\sqrt{s_{f_0(980)}} = (996 \pm 7) - i\,(25^{+10}_{-6})\ \mathrm{MeV}\ \ (\mathrm{GKPY}). \tag{3.6}$$

These values are in good agreement with each other. Note that for the $f_0(980)$ we have had to add a 4 MeV systematic uncertainty on the imaginary part of the pole position, which comes as the difference between using our isospin symmetric formalism with the charged or the neutral kaon mass.

In the case σ, on the one hand, both the mass and width lie less than 1 standard deviation away from the prediction of twice-subtracted RE combined with ChPT results for the scattering lengths [9]: $\sqrt{s_\sigma} = 441^{+16}_{-8} - i\,272^{+9}_{-14.5}$MeV. On the other hand our pole determination above is roughly two standard deviations from the mass and width in our simple fit of a conformal expansion to low energy data [20] $\sqrt{s_\sigma} = (484 \pm 17) - i\,(255 \pm 10)$MeV.

In the case of the $f_0(980)$, the mass is somewhat higher than that quoted in the PDG 980 ± 10 MeV, although note that ours is the pole position and is model independent. Concerning the width, which once again we obtain from the pole position as $\Gamma = -2Im\sqrt{s_{f_0(980)}} = 50^{+20}_{-12}$MeV it lies within the range given in the PDG, namely, $40 - 100$ MeV.

4. Conclusions

The GKPY equations [20, 21]—Roy-like dispersion relations with one subtraction for the $\pi\pi$ amplitudes—provide stringent constraints for dispersive analysis of experimental data. We have provided simple and ready to use parametrizations, constrained to satisfy these equations as well as Roy equations and froward dispersion relations, that simultaneously describe the existing data.

The main advantage of GKPY eqs. is that, for the same input, in the $0.45\,\mathrm{GeV} \leq \sqrt{s} \leq 1.1$ GeV region they have significantly smaller errors than standard Roy. eqs. Hence, they provide better accuracy tests and analytic extensions of the amplitudes in that region. In particular, using just a data analysis consistent within errors with Forward Dispersion Relations, Roy eqs. and GKPY eqs. (and no ChPT input), we have presented here our recent very precise determination of the σ pole position:

$$\sqrt{s_\sigma} = (457^{+14}_{-13}) - i\,(279^{+11}_{-7})\ \mathrm{MeV}, \tag{4.1}$$

and of the $f_0(980)$:

$$\sqrt{s_{f_0(980)}} = (996 \pm 7) - i\,(25^{+10}_{-6})\ \mathrm{MeV}. \tag{4.2}$$

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