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Quantum optical approach to Bose-Einstein correlations at fixed multiplicities

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Data on negatively charged multiplicity distribution and Bose-Einstein correlations of identical particles at fixed multiplicities at $\sqrt{s} = 900$ GeV in pp collisions are analyzed by a model in the quantum optical approach.

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1. Introduction

In high energy nucleus-nucleus collisions or hadron-hadron collisions, Bose-Einstein correlations of identical particles are considered as one of the possible measures for the space-time domain where identical particles are produced. If multiplicity distribution (MD) and Bose-Einstein correlations (BEC) are constructed from the same observed data sample, some information on BEC would be contained in the MD. Therefore, we can estimate parameters on BEC precisely from the observed MD and BEC at fixed multiplicities.

One of the theoretical approaches to BEC is made on the analogy of the quantum optics [1], where two types of sources, chaotic and coherent are introduced. In Ref.[2], formulae for MD and BEC in semi-inclusive events are derived in the QO approach, and a diagrammatic representation of the cumulants is proposed.

Recently, new data on BEC and MDs in pp collisions are reported from the LHC experiments. In the present paper, MD and BEC are analyzed by the formulae derived in the QO approach.

2. Momentum densities in semi-inclusive events

The n-particle momentum density in semi-inclusive events in the QO approach is defined by,

$$\rho_n(p_1,\cdots,p_n) = c_0\Big\langle |f(p_1)|^2\cdots|f(p_n)|^2\Big\rangle_a, \quad f(p)=\sum_{i=1}^{M} a_i\phi_i(p)+f_c(p). \tag{2.1}$$

In Eq.(2.1), c_0 denotes a normalization factor, $f(p)$ is an amplitude composed of that of the i-th chaotic source, $\phi_i(p)$, and that of the coherent source, $f_c(p)$, and a_i is a random complex number attached to the i-th chaotic source. The number of independent chaotic sources, M, is assumed to be infinite[2]. Parenthesis $\langle F\rangle_a$ in Eq.(2.1) denotes the average of F over the random number a_i with a Gaussian weight [3];

$$\langle F\rangle_a = \Big(\prod_{i=1}^{M}\frac{1}{\pi\lambda_i}\int \exp\big[-\frac{|a_i|^2}{\lambda_i}\big]\, d^2a_i\Big)F. \tag{2.2}$$

The generating functional (GF) of momentum densities in semi-inclusive events is defined by

$$Z_{\rm sm}[h(p)] = c_0\sum_{n=0}^{\infty}\frac{1}{n!}\Big\langle\big[\int |f(p)|^2 h(p)\frac{d^3p}{E}\big]^n\Big\rangle_a, \tag{2.3}$$

where $h(p)$ is an arbitrary function. The n-particle momentum density and the n-th order cumulant in semi-inclusive events are given respectively by

$$\rho_n(p_1,\cdots,p_n) = E_1\cdots E_n\frac{\delta^n Z_{\rm sm}[h(p)]}{\delta h(p_1)\cdots\delta h(p_n)}\Big|_{h(p)=0}, \tag{2.4}$$

$$g_n(p_1,\cdots,p_n) = E_1\cdots E_n\frac{\delta^n \ln Z_{\rm sm}[h(p)]}{\delta h(p_1)\cdots\delta h(p_n)}\Big|_{h(p)=0}. \tag{2.5}$$

For a review of semi-inclusive events, see [4]. From Eqs.(2.5) and (2.6), we have an iteration relation for momentum densities,

$$\begin{aligned}\rho_1(p_1) &= c_0 g_1(p_1) = c_0[r(p_1,p_1)+c(p_1,p_1)],\\ \rho_2(p_1,p_2) &= g_1(p_1)\rho(p_2)+c_0\Big\{|r(p_1,p_2)|^2+2\mathrm{Re}[r(p_1,p_2)c(p_2,p_1)]\Big\}, \qquad (2.6)\\ .\rho_n(p_1,\cdots,p_n) &= g_1(p_1)\rho_{n-1}(p_2,\cdots,p_n)+c_0 g_n(p_1,\cdots,p_n)\\ &+\sum_{i=1}^{n-2}\sum g_{i+1}(p_1,p_{j_1},\cdots,p_{j_i})\rho_{n-i-1}(p_{j_{i+1}},\cdots,p_{j_{n-1}}),\ \text{for } n\geq 2. \qquad (2.7)\end{aligned}$$

In Eq.(2.6), $r(p_1,p_2)$ is a correlation caused by the chaotic sources, $r(p_1,p_2)=\sum_{i=1}^{M}\lambda_i\phi_i(p_1)\phi_i^*(p_2)$, and $c(p_1,p_2)$ is a correlation by the coherent source, $c(p_1,p_2)=f_c(p_1)f_c^*(p_2)$.

The second summation on the right hand side of Eq.(2.7) indicates that all possible combinations of $(j_1,\cdots,j_i)$ and $(j_{i+1},\cdots,j_{n-1})$ are taken from $(2,3,\cdots,n)$.

In order to calculate momentum densities at fixed multiplicity, following equations are defined;

$$\begin{aligned}\rho_n^{(k)}(p_1,\cdots,p_k) &= \frac{1}{(n-k)!}\int\cdots\int\rho_n(p_1,\cdots,p_k,p_{k+1},\cdots,p_n)\frac{d^3p_{k+1}}{E_{k+1}}\cdots\frac{d^3p_n}{E_n},\\ g_n^{(k)}(p_1,\cdots,p_k) &= \frac{1}{(n-k)!}\int\cdots\int g_n(p_1,\cdots,p_k,p_{k+1},\cdots,p_n)\frac{d^3p_{k+1}}{E_{k+1}}\cdots\frac{d^3p_n}{E_n}. \qquad (2.8)\end{aligned}$$

The MD is given by $P(0)=Z_{\mathrm{sm}}[0]=c_0$ and

$$P(n)=\rho_n^{(0)}=\frac{(n-k)!}{n!}=\int\cdots\int\rho_n^{(k)}(p_1,\cdots,p_k)\frac{d^3p_1}{E_1}\cdots\frac{d^3p_k}{E_k}. \qquad (2.9)$$

For a review on MD, see for example [5]. From Eq.(2.7), we have [2]

$$\rho_n^{(1)}(p_1) = \sum_{j=1}^{n} j g_j^{(1)}(p_1)P(n-j), \quad g_j^{(1)}(p_1)=R_j(p_1,p_1)+\sum_{l=0}^{j-1}T_{l,j-l-1}(p_1,p_1),\ \text{for } n\geq 1, \qquad (2.10)$$

$$\begin{aligned}\rho_n^{(2)}(p_1,p_2) &= \sum_{j=1}^{n-1}(n-j)g_j^{(1)}(p_1)\rho_{n-j}^{(1)}(p_2)+\sum_{j=2}^{n}g_j^{(2)}(p_1,p_2)P(n-j),\\ g_j^{(2)}(p_1,p_2) &= \sum_{l=1}^{j-1}R_j(p_1,p_2)R_{j-l}(p_2,p_1)\\ &+\sum_{l=0}^{j-2}\sum_{m=0}^{l}\{T_{m,l-m}(p_1,p_2)R_{j-l-1}(p_2,p_1)+R_{j-l-1}(p_1,p_2)T_{m,l-m}(p_2,p_1)\}, \qquad (2.11)\end{aligned}$$

where, with $R_0(p_1,p_2)=E_1\delta^3(p_1,p_2)$,

$$\begin{aligned}R_j(p_1,p_2) &= \int r(p_1,p')R_{j-1}(p',p_2)\frac{d^3p'}{E'},\\ T_{j,l}(p_1,p_2) &= \iint R_j(p_1,p_1')c(p_1',p_2')R_l(p_2',p_2)\frac{d^3p_1'}{E_1'}\frac{d^3p_2'}{E_2'}. \qquad (2.12)\end{aligned}$$

3. Formulae for MD and BEC

In the followings, rapidity $y_i = \tanh^{-1}(p_{iL}/E_i)$ and transverse momentum $\boldsymbol{p}_{iT}$ $(i = 1, 2, \cdots)$ are used. Correlations $r(p_1, p_2)$ and $c(p_1, p_2)$ are both assumed to be real and parametrized as,

$$\begin{aligned} r(y_1, \boldsymbol{p}_{1T}; y_2, \boldsymbol{p}_{2T}) &= p_{\rm sm}\sqrt{\rho(y_1, \boldsymbol{p}_{1T})\rho(y_2, \boldsymbol{p}_{2T})}\,\exp[-\gamma_L(\Delta y)^2 - \gamma_T(\Delta \boldsymbol{p}_T)^2], \\ c(y_1, \boldsymbol{p}_{1T}; y_2, \boldsymbol{p}_{2T}) &= (1 - p_{\rm sm})\sqrt{\rho(y_1, \boldsymbol{p}_{1T})\rho(y_2, \boldsymbol{p}_{2T})}, \\ \rho(y_1, \boldsymbol{p}_{1T}) &= \langle n_0\rangle\sqrt{\pi/\alpha}(\pi/\beta)\exp[-\alpha y_1^2 - \beta\, \boldsymbol{p}_{1T}^2], \end{aligned}$$

where $\Delta y = y_2 - y_1$ and $\Delta \boldsymbol{p}_T = \boldsymbol{p}_{2T} - \boldsymbol{p}_{1T}$. The chaoticity parameter in semi-inclusive events is assumed to be constant, and is denoted by $p_{\rm sm}(= r(p_i, p_i)/\rho(p_i))$. As the longitudinal momentum transfer squared, $Q_L^2 = (E_1 - E_2)^2 - (p_{1L} - p_{2L})^2$, is approximately written as $Q_L^2 \approx \langle m_T\rangle^2(\Delta y)^2$ for $|\Delta y| << 1$ with average transverse mass $\langle m_T\rangle$, $\sqrt{\gamma_L}/\langle m_T\rangle$ roughly equal to the longitudinal source size, and $\sqrt{\gamma_T}$ is the transverse souce size. Parameter $\langle n_0\rangle$ is related to an average multiplicity. If $p_{\rm sm} = 0$, the MD defined by Eq.(3.1) becomes a Poisson distribution with an average $\langle n_0\rangle$. Parameters α are β are related to the width of rapidity distribution and that of p_T distribution, respectively.

Then, function $R_j(y_1, \boldsymbol{p}_{1T}, y_2, \boldsymbol{p}_{2T})$ in Eq.(2.12) is written as,

$$R_j(y_1, \boldsymbol{p}_{1T}, y_2, \boldsymbol{p}_{2T}) = N_j \exp[-A_j(y_1^2 + y_2^2) + 2C_j y_1 y_2 - U_j(\boldsymbol{p}_{1T}^2 + \boldsymbol{p}_{2T}^2) + 2W_j \boldsymbol{p}_{1T}\boldsymbol{p}_{2T}],$$

where [6, 7]

$$\begin{aligned} A_j &= \frac{r_2 - r_1}{2}\frac{1 + (r_1/r_2)^j}{1 - (r_1/r_2)^j}, \quad C_j = (r_2 - r_1)\frac{(r_1/r_2)^{j/2}}{1 - (r_1/r_2)^j}, \\ U_j &= \frac{t_2 - t_1}{2}\frac{1 + (t_1/t_2)^j}{1 - (t_1/t_2)^j}, \quad W_j = (t_2 - t_1)\frac{(t_1/t_2)^{j/2}}{1 - (t_1/t_2)^j}, \\ N_j &= \frac{{r_2}^{1/2} t_2}{\pi^{3/2}}\Big(\frac{p_{\rm sm}\langle n_0\rangle \alpha^{1/2}\beta}{{r_2}^{1/2} t_2}\Big)^j \Big\{\frac{1 - (r_1/r_2)}{1 - (r_1/r_2)^j}\Big\}^{1/2}\frac{1 - (t_1/t_2)}{1 - (t_1/t_2)^j}, \\ r_1 &= \frac{1}{2}\big[\alpha + 2\gamma_L - \sqrt{\alpha^2 + 4\alpha\gamma_L}\big], \quad r_2 = \frac{1}{2}\big[\alpha + 2\gamma_L + \sqrt{\alpha^2 + 4\alpha\gamma_L}\big], \\ t_1 &= \frac{1}{2}\big[\beta + 2\gamma_T - \sqrt{\beta^2 + 4\beta\gamma_T}\big], \quad t_2 = \frac{1}{2}\big[\beta + 2\gamma_T + \sqrt{\beta^2 + 4\beta\gamma_T}\big]. \end{aligned}$$

The MD in the QO approach is written as [7],

$$P(n) = \frac{1}{n}\sum_{j=1}^{n}\Big(\Delta_j^{(R)} + j\Delta_{j-1}^{(S)}\Big)P(n - j), \tag{3.1}$$

where

$$\begin{aligned} \Delta_j^{(R)} &= p_{\rm sm}\langle n_0\rangle\Big(\frac{p_{\rm sm}\langle n_0\rangle\sqrt{\alpha}\,\beta}{\sqrt{r_2}\,t_2}\Big)^{j-1}\frac{1 - \sqrt{r_1/r_2}}{1 - (r_1/r_2)^{j/2}}\Big\{\frac{1 - \sqrt{t_1/t_2}}{1 - (t_1/t_2)^{j/2}}\Big\}^2, \\ \Delta_{j-1}^{(S)} &= (1 - p_{\rm sm})\langle n_0\rangle\Big(\frac{p_{\rm sm}\langle n_0\rangle\sqrt{\alpha}\,\beta}{\sqrt{r_2}\,t_2}\Big)^{j-1}\Big\{\frac{1 - (r_1/r_2)}{1 - (r_1/r_2)^j}\Big\}^{1/2}\frac{1 - (t_1/t_2)}{1 - (t_1/t_2)^j}. \end{aligned}$$

As can be seen from the above equations, the MD contains four parameters, $p_{\rm sm}$, $\langle n_0\rangle$, $h_L = \gamma_L/\alpha$ and $h_T = \gamma_T/\beta$. The inclusive one-particle rapidity distribution is given by

$$\rho^{(1)}(y) = \sum_{n=1}^{n_{\rm max}}\rho_n^{(1)}(y), \quad \rho_n^{(1)}(y) = \int \rho_n^{(1)}(y, \boldsymbol{p}_T)d^2\boldsymbol{p}_T. \tag{3.2}$$

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The Bose-Einstein correlation function $C_n^{(2)}(\Delta y)$ at n-particle events is defined as

$$C_n^{(2)}(\Delta y) = \frac{nP(n)}{n-1} \frac{\int\int\int \rho_n^{(2)}(y_1, \boldsymbol{p}_{1T}, y_1+\Delta y, \boldsymbol{p}_{1T}+\Delta\boldsymbol{p}_T) dy_1 d^2\boldsymbol{p}_{1T} d^2\Delta\boldsymbol{p}_T}{\int\int\int \rho_n^{(1)}(y_1, \boldsymbol{p}_{1T}) \rho_n^{(1)}(y_1+\Delta y, \boldsymbol{p}_{1T}+\Delta\boldsymbol{p}_T) dy_1 d^2\boldsymbol{p}_{1T} d^2\Delta\boldsymbol{p}_T}. \quad (3.3)$$

4. Analysis of experimental data

The MD for negatively charged particles observed in the pseudo-rapidity region $|\eta| < 1$ in pp collisions at $\sqrt{s} = 900$ GeV [8] is analyzed by Eq.(3.1). It is constructed from the even prongs of observed charged MD. At first, it is analyzed with four parameters, $p_{\rm sm}$, $\langle n_0 \rangle$, h_L and h_T. The result becomes that $h_T \approx 0$. Therefore, the data is re-analyzed with three parameters under the condition that $h_T = 0 (\gamma_T = 0)$. Estimated parameters are shown in Table 1, and comparison of the calculated result with the observed MD is shown in Fig.1.

$p_{\rm sm}$	$\langle n_0 \rangle$	h_L	$\chi^2_{\rm min}/n.d.f$
0.670 ± 0.052	1.100 ± 0.10	0.351 ± 0.294	41.1/(22-3)

Table 1: Estimated parameters in the analysis of negatively charged MD observed in pp collisions [8].

In our calculations, transverse momentum is integrated. Therefore, parameter β is not included in Eqs.(3.2) and (3.3). Calculated result on inclusive one-particle rapidity distributions is shown in Fig.2. Calculated result on Bose-Einstein correlation functions at fixed multiplicities as a function of Δy is shown in Fig.3. That of Bose-Einstein correlation functions at $\Delta y = 0$ is compared with the data at $Q_{\rm inv} = 0$ in Fig.4. The data is presented in four p_T range [8]. Therefore we use the average value of four cases.

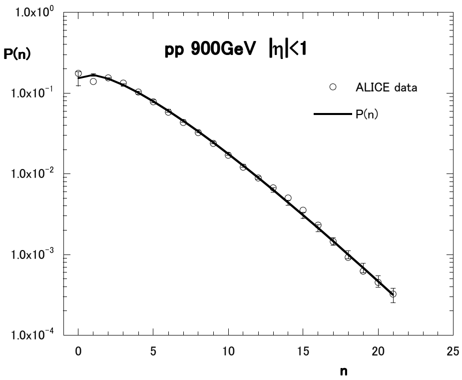

Figure 1: Analysis of negatively charged MD in $|\eta| < 1$ in pp collisions [8] by Eq.(3.1).

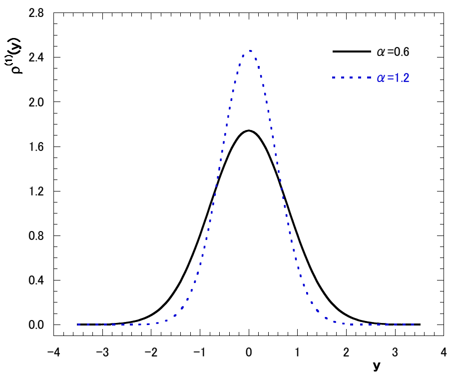

Figure 2: Inclusive one-particle rapidity distributions calculated with $\alpha = 0.6$ and 1.2 by Eq.(3.2).

5. Summary

The observed MD and BEC at $\Delta y = 0$ ($Q_{\rm inv} = 0$) are analyzed by our model in the QO approach. In our formulation, six parameters, $p_{\rm sm}$, $\langle n_0 \rangle$, $h_L = \gamma_L/\alpha$, $h_T = \gamma_T/\beta$, α and β are con-

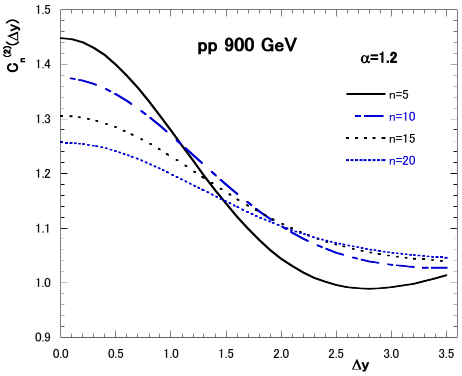

Figure 3: Bose-Einstein correlation function $C_n^{(2)}(\Delta y)$ calculated with $\alpha = 1.2$ by Eq.(3.3).

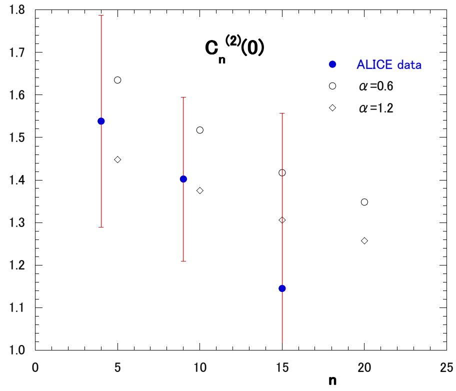

Figure 4: Comparison of $C_n^{(0)}(0)$ calculated with $\alpha = 0.6$ and 1.2 with the data [9].

tained. In the present analysis, parameter γ_T becomes effectively zero and β is not included in Eq.(3.3). Therefore, only parameter α is effective to the analysis of BEC. As can be seen from Fig.4, calculated result of $C_n^{(2)}(0)$ for $0.6 \le \alpha \le 1.2$ would not be inconsistent with the observed data at $\Delta y = 0$ ($Q_{\rm inv} = 0$). If the MD and BEC at fixed multiplicities are constructed from the same data sample, we would obtain more precise information on the production region of like-sign charged particles in the final states of hadronic interactions.

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