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BEC, the τ-model, and jets in e+e– annihilation

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Bose-Einstein correlations of pairs of identical charged pions produced in hadronic Z decays are analyzed in terms of various parametrizations. A good description is achieved using a Lévy stable distribution in conjunction with a model where a particle's momentum is highly correlated with its space-time point of production, the τ-model. However, an elongation of the particle emission region along the event axis is observed in the Longitudinal Center of Mass frame, which is not accommodated in the τ-model. Further, for three-jet events the region is found to be larger in the event plane than out of the plane.

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1. Introduction

We have recently published[1] a study of Bose-Einstein correlations (BEC) in hadronic Z decay where we found good agreement with parametrizations arising in the τ-model.[2,3] This work is summarized in Section 2, and some new (preliminary) results are presented in Section 3.

The data were collected by the L3 detector at an e^+e^- center-of-mass energy of $\sqrt{s} \simeq 91.2$ GeV. Approximately 36 million like-sign pairs of well-measured charged tracks from about 0.8 million hadronic Z decays are used.[4] Events are classified as two- or three-jet events using calorimeter clusters with the Durham jet algorithm with jet resolution parameter $y_{\mathrm{cut}} = 0.006$, yielding about 0.5 million two-jet events and 0.3 million events having more than two jets. There are few events with more than three jets, and they are included in the three-jet sample. To determine the event (thrust) axis we also use calorimeter clusters.

Two-particle BEC are measured by the correlation function $R_2(p_1,p_2) = \rho_2(p_1,p_2)/\rho_0(p_1,p_2)$, the ratio of the two-particle number density to that which would occur in the absence of BEC. An event mixing technique is used to construct ρ_0.

2. Summary of Previous Results[1]

With a few assumptions, R_2 is related to the Fourier transform, $\tilde{f}(Q)$, of the (configuration space) density distribution of the source, $f(x)$:

$$R_2(Q) = \gamma\left[1+\lambda|\tilde{f}(Q)|^2\right](1+\delta Q) \ , \tag{2.1}$$

where $Q = \sqrt{-(p_1-p_2)^2}$. The parameter γ and the $(1+\delta Q)$ term are introduced to parametrize possible long-range correlations inadequately accounted for in ρ_0, and λ to measure the strength of the BEC. However, (2.1) is ruled out by the data, which show that R_2 has a significant dip below unity in the region 0.6–1.5 GeV, indicative of an anti-correlation.

2.1 The τ-model

This anti-correlation region is predicted in the τ-model.[2,3] In this model it is assumed that in the overall center-of-mass system the average production point $\overline{x} = (\overline{t}, \overline{r}_x, \overline{r}_y, \overline{r}_z)$, of particles with a given four-momentum p is given by $\overline{x}^\mu(p^\mu) = a\tau p^\mu$. In the case of two-jet events, $a = 1/m_{\mathrm{t}}$, where m_{t} is the transverse mass, and $\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2}$ is the longitudinal proper time; for the case of three-jet events the relation is more complicated. The second assumption is that the distribution of $x^\mu(p^\mu)$ about its average is narrower than the proper-time distribution, $H(\tau)$. Then R_2 is found[3] to depend only on Q, the values of a of the two pions, and the Fourier transform of $H(\tau)$. Since there is no particle production before the onset of the collision, $H(\tau)$ should be a one-sided distribution. We choose a one-sided Lévy distribution, which has three parameters: the index of stability α, which is related to the strong coupling constant α_{s},[5,6] the proper time of the start of particle emission τ_0, and $\Delta\tau$, which is a measure of the width of $H(\tau)$. Then[3]

$$\begin{aligned} R_2(Q,a_1,a_2) = \gamma\Bigg\{1+\lambda\cos\left[\frac{\tau_0 Q^2(a_1+a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^\alpha \frac{a_1^\alpha + a_2^\alpha}{2}\right] \\ \cdot\exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^\alpha \frac{a_1^\alpha + a_2^\alpha}{2}\right]\Bigg\}(1+\varepsilon Q) \ . \end{aligned} \tag{2.2}$$

Note that the cosine factor generates oscillations corresponding to alternating correlated and anti-correlated regions, a feature clearly seen in the data. Note also that since $a = 1/m_\mathrm{t}$ for two-jet events, the τ-model predicts a decreasing effective source size with increasing m_t.

Before proceeding to fits of (2.2), we first consider a simplification of the equation obtained by assuming (a) that particle production starts immediately, *i.e.*, $\tau_0 = 0$, and (b) an average a-dependence, which is implemented by introducing an effective radius, R, defined by

$$R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2} \ . \tag{2.3}$$

This results in

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_\mathrm{a}Q)^{2\alpha}\right) \exp\left(-(RQ)^{2\alpha}\right)\right](1+\varepsilon Q) \ , \tag{2.4}$$

where R_a is related to R by

$$R_\mathrm{a}^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha} \ . \tag{2.5}$$

Fits of (2.4) are first performed with R_a as a free parameter. The fits for both two- and three-jet events have acceptable confidence levels (CL), and describe well the dip in the 0.6–1.5 GeV region, as well as the peak at low values of Q. The estimates of some fit parameters are rather highly correlated. For example, for two-jet events the estimated correlation coefficients from the fit for α, R and R_a are $\rho(\alpha,R) = -0.62$, $\rho(\alpha,R_\mathrm{a}) = -0.92$, and $\rho(R,R_\mathrm{a}) = 0.38$. Taking the correlations into account, the fit parameters satisfy (2.5), the difference between the left- and right-hand sides of the equation being less than 1 standard deviation

Fits are also performed imposing (2.5). For two-jet events, the values of the parameters are comparable to those with R_a free. For three-jet events, the imposition of (2.5) results in values of α and R closer to those for two-jet events, but the χ^2 is noticeably worse, though acceptable, than with R_a free.

For two-jet events, $a = 1/m_\mathrm{t}$, while for three-jet events the situation is more complicated. We therefore limit fits of (2.2) to the two-jet data. For each bin in Q the average values of $m_{\mathrm{t}1}$ and $m_{\mathrm{t}2}$ are calculated, where $m_{\mathrm{t}1}$ and $m_{\mathrm{t}2}$ are the transverse masses of the two particles making up a pair, requiring $m_{\mathrm{t}1} > m_{\mathrm{t}2}$. Using these averages, (2.2) is fit to $R_2(Q)$, which results in a good fit with a value of α consistent with that from fitting (2.4).

Since the τ-model describes the m_t dependence of R_2, its parameters, α, $\Delta\tau$, and τ_0, should not depend on m_t. However, λ, which is not a parameter of the τ-model, but rather a measure of the strength of the BEC, can depend on m_t. The large correlation between the fit estimates of λ, α, and $\Delta\tau$ complicate the testing of m_t-independence. We perform fits in various regions of the $m_{\mathrm{t}1}$-$m_{\mathrm{t}2}$ plane keeping α and $\Delta\tau$ fixed at the values obtained in the fit to the entire m_t plane. The CLs are reasonably uniformly distributed between 0 and 1. The data are thus in agreement with the hypothesis of m_t-independence of the parameters of the τ-model.

2.2 Test of dependence of BEC on components of Q

The τ-model predicts that the two-particle BEC correlation function R_2 depends on the two-particle momentum difference only through Q, not through components of Q separately. However, R_2 has been found to depend on components of Q,[7–11] the shape of the region of homogeneity being elongated along the event (thrust) axis. The question is whether this is an artifact of the Edgeworth or Gaussian parametrizations used in these studies or shows a defect of the τ-model.

This is investigated in the Longitudinal Center of Mass System[1] (LCMS), where

$$Q^2 = Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2 - (\Delta E)^2 \tag{2.6}$$

$$= Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2\left(1-\beta^2\right) , \qquad \beta = \frac{p_{1\mathrm{out}} + p_{2\mathrm{out}}}{E_1 + E_2} . \tag{2.7}$$

Assuming azimuthal symmetry about the event axis suggests that the region of homogeneity have an ellipsoidal shape with the longitudinal axis along the event axis. In (2.4) R^2Q^2 is then replaced by

$$R^2Q^2 \Longrightarrow A^2 = R_{\mathrm{L}}^2 Q_{\mathrm{L}}^2 + R_{\mathrm{side}}^2 Q_{\mathrm{side}}^2 + \rho_{\mathrm{out}}^2 Q_{\mathrm{out}}^2 , \tag{2.8}$$

which results in

$$R_2(Q) = \gamma\left[1 + \lambda \cos\left(\tan\left(\frac{\alpha\pi}{2}\right) A^{2\alpha}\right)\exp\left(-A^{2\alpha}\right)\right]\left(1 + \varepsilon_{\mathrm{L}} Q_{\mathrm{L}} + \varepsilon_{\mathrm{side}} Q_{\mathrm{side}} + \varepsilon_{\mathrm{out}} Q_{\mathrm{out}}\right) . \tag{2.9}$$

The longitudinal and transverse size of the source are measured by R_{L} and R_{side}, respectively, whereas ρ_{out} reflects both the transverse and temporal sizes.[2] We also investigate two other decompositions of Q:[3]

$$Q^2 = Q_{\mathrm{LE}}^2 + Q_{\mathrm{side}}^2 + Q_{\mathrm{out}}^2 , \qquad Q_{\mathrm{LE}}^2 = Q_{\mathrm{L}}^2 - (\Delta E)^2 , \tag{2.10a}$$

$$A^2 = R_{\mathrm{LE}}^2 Q_{\mathrm{LE}}^2 + R_{\mathrm{side}}^2 Q_{\mathrm{side}}^2 + R_{\mathrm{out}}^2 Q_{\mathrm{out}}^2 ; \tag{2.10b}$$

$$Q^2 = Q_{\mathrm{L}}^2 + Q_{\mathrm{side}}^2 + q_{\mathrm{out}}^2 , \qquad q_{\mathrm{out}}^2 = Q_{\mathrm{out}}^2 - (\Delta E)^2 , \tag{2.10c}$$

$$A^2 = R_{\mathrm{L}}^2 Q_{\mathrm{L}}^2 + R_{\mathrm{side}}^2 Q_{\mathrm{side}}^2 + r_{\mathrm{out}}^2 q_{\mathrm{out}}^2 . \tag{2.10d}$$

The first, (2.10a), corresponds to the LCMS frame where the longitudinal and energy terms are combined; its three components of Q are invariant with respect to Lorentz boosts along the event axis. The second, (2.10c), corresponds to the LCMS frame boosted to the rest frame of the pair; its three components are invariant under Lorentz boosts along the out direction.

Fits of (2.9) with (2.8), (2.10b), and (2.10d) show that R_2 depends differently on the components of Q. Also, the values of $R_{\mathrm{side}}/R_{\mathrm{L}}$ found are consistent with values found previously using Gaussian or Edgeworth parametrizations.[7–11]

3. New (Preliminary) Results

Recent work investigates the dependence of the BEC radius on the 'jettiness' of the event using the simplified τ-model parametrization, (2.4), and its extension (2.9) to dependence on $\vec{Q}$ rather than Q.

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[1] Also known as the Longitudinal Co-Moving System; it is defined as the frame, obtained by a Lorentz boost along the event axis, where the sum of the three-momenta of the two pions ($\vec{p}_1 + \vec{p}_2$) is perpendicular to the event axis.

[2] In the literature[7–12] the coefficient of Q_{out}^2 in (2.8) is usually denoted R_{out}^2. We prefer to use ρ_{out}^2 to emphasize that, unlike R_{L} and R_{side}, ρ_{out} contains a dependence on β, *i.e.*, on the energy difference,. and to differentiate it from R_{out} in (2.10b) below.

[3] Note that in (2.10b) the coefficient of Q_{out}^2 is R_{out}^2, since the energy difference is here incorporated in Q_{LE}^2 rather than in the coefficient of Q_{out}^2 as was the case in (2.8).

Using the Durham algorithm, events can be classified according to the number of jets. The number of jets in a particular event depends on y_{cut}. We define y_{23} as that value of y_{cut} at which the number of jets changes from two to three. The event sample is then split into subsamples according to the value of y_{23}. The subsample with the smallest value of y_{23} corresponds to narrow two-jet events, whereas that with the largest y_{23} consists of three or more very well separated jets. Fits of (2.4) are performed for each subsample. The estimates of α and R are very highly correlated in the fits. Therefore, to stabilize the fits we fix the value of α to the value found in a fit of the entire sample: $\alpha = 0.443$. We see in Fig. 1 that R increases with y_{23}. This is consistent with an earlier observation of OPAL.[13]

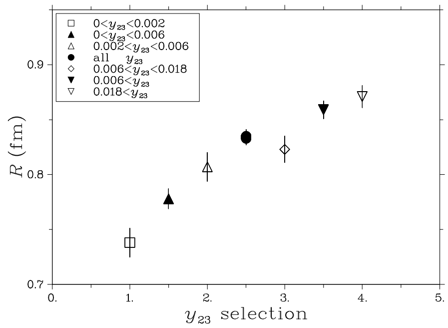

Figure 1: The radius R from fits of (2.4) for various y_{23} subsamples.

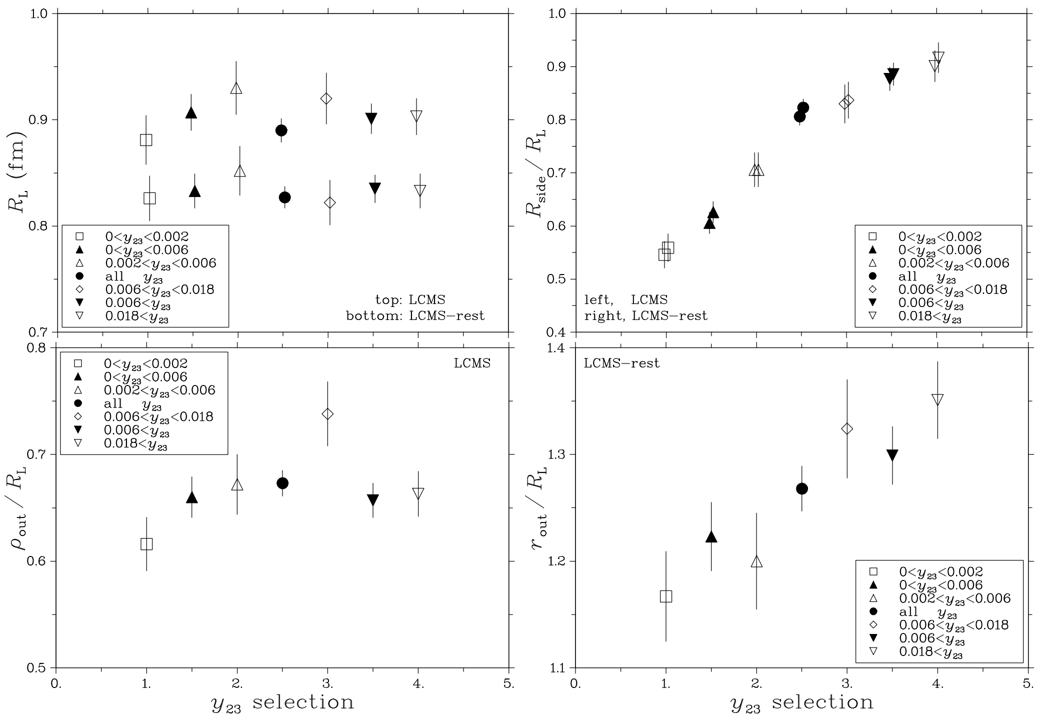

Figure 2: The radii from fits in the LCMS and LCMS-rest frames for various y_{23} subsamples.

The dependence on y_{23} of the radii for components of Q, (2.8) and (2.10d), is shown in Fig. 2. While the values of R_{L} found in the LCMS-rest frame fits are systematically lower than in the LCMS frame, the values of $R_{\mathrm{side}}/R_{\mathrm{L}}$ agree extremely well. Note that at all values of y_{23} $R_{\mathrm{side}} < R_{\mathrm{L}}$ while $r_{\mathrm{out}} > R_{\mathrm{L}}$. Thus we do not observe azimuthal symmetry about the thrust axis, not even for the narrowest two-jet sample. Further, we observe that R_{L} and R_{out} are approximately independent of y_{23}, whereas both R_{side} and r_{out} increase with y_{23}.

We find (*cf.* Fig. 3) that the *out* direction tends to be in the direction of the *major* axis, *i.e.*, that the *out* direction tends to be in the event plane, or equivalently, that the *side* direction tends to be

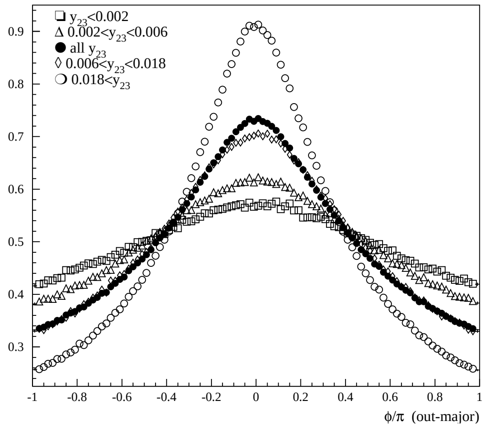

Figure 3: The angle between out and major.

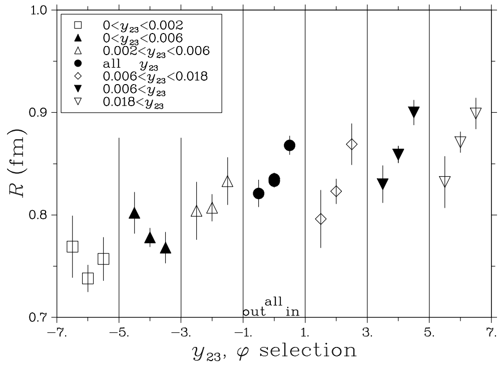

Figure 4: The radius R from fits of (2.4) for various y_{23} subsamples, which are split into 'in-plane' and 'out-of-plane' samples.

out of the event plane. This effect becomes stronger as y_{23} increases.

To further investigate the dependence on the event plane, each y_{23} subsample is divided into 'in-plane' and 'out-of-plane' samples which use, respectively, only particles having azimuthal angle less than or greater than 45° of the major axis.. The values of R from fits of (2.4) are shown in Fig. 4. We see that for small y_{23} there is little dependence of R on whether the tracks are in or out of the event plane, but for large y_{23} R is larger for the in-plane sample.

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