PROCEEDINGS OF SCIENCE

Definition of seismic and tsunami hazard scenarios by exploiting EU-India Grid e-infrastructures

A. Magrin[a], **A. Peresan**[∗ ab], **F. Vaccari**[ab], **S. Cozzini**[c], **B.K. Rastogi**[d], **I.A. Parvez**[e], **G.F. Panza**[ab]

[a] *Department of Mathematics and Geosciences, University of Trieste, via Weiss 4, 34127. Trieste - Italy.*
[b] *The Abdus Salam International Centre for Theoretical Physics, SAND Group. Trieste - Italy*
[c] *Consiglio Nazionale delle Ricerche, CNR/IOM uos Democritos. Trieste - Italy.*
[d] *Institute of Seismological Research, Raisan, Gandhinagar, Gujarat, India*
[e] *CSIR Centre for Mathematical Modelling and Computer Simulation (C-MMACS), NAL Belur Campus, Bangalore, India*
E-mail: aperesan@units.it

Seismic hazard assessment can be performed following a neo-deterministic approach (NDSHA), which allows to give a realistic description of the seismic ground motion due to an earthquake of given distance and magnitude. The approach is based on modelling techniques that have been developed from a detailed knowledge of both the seismic source process and the propagation of seismic waves. This permits us to define a set of earthquake scenarios and to simulate the associated synthetic signals without having to wait for a strong event to occur. NDSHA can be applied at the regional scale, computing seismograms at the nodes of a grid with the desired spacing, or at the local scale, taking into account the source characteristics, the path and local geological and geotechnical conditions. Synthetic signals can be used as seismic input in subsequent engineering analyses aimed at the computation of the full non-linear seismic response of the structure or simply the earthquake damaging potential. The same approach can be applied to tsunami hazard assesment: for a specific coastal area one has to characterize the seismic sources and select the earthquake scenarios that can drive the hazard; afterward the associated synthetic tsunamigrams are computed using the modal method. Massive parametric tests, to explore the influence not only of deterministic source parameters and structural models but also of random properties of the same source model, enable realistic estimate of seismic hazard and their uncertainty. The use of the EU-India Grid infrastructure allows to conduct massive parametric tests for evaluating the uncertainties in the computed hazard maps. In the framework of cooperation project between three Italian partners (DMG, University of Trieste; ICTP SAND Group; CNR/IOM uos Democritos) and two Indian partners (ISR, Gujarat; CSIR C-MMACS, Bangalore) a system will be set up for the scenario-based characterization of the seismic and tsunami hazard, integrated with the e-infrastructures of the project EU-India Grid.

The International Symposium on Grids and Clouds (ISGC) 2012,
Febraury 26 - March 2, 2012
Academia Sinica, Taipei, Taiwan

∗Speaker.

PoS(ISGC 2012)030

1. Introduction

Seismic risk mitigation is a worldwide concern and the development of effective mitigation strategies requires sound seismic hazard assessment. The performances of the classical probabilistic approach to seismic hazard assessment (PSHA), currently in use in several countries worldwide, turned out fatally inadequate when considering the earthquakes occurred worldwide during the last decade, including the recent destructive earthquake in Japan (M 9.0, 11 March 2011). Nowadays it is recognised by the engineering community that probabilistic Peak Ground Acceleration estimates alone are not sufficient for the adequate design of special buildings and infrastructures, since displacements play a critical role and the dynamical analysis of the structure response requires complete time series of ground motion. Moreover, when dealing with the protection of critical structures (e.g. nuclear power plants) and cultural heritage, where it is necessary to consider extremely long time intervals, the standard PSHA estimates are by far unsuitable, due to their basic heuristic limitations.
Current computational resources and physical knowledge of the seismic waves generation and propagation processes, along with the improving quantity and quality of geophysical data (spanning from seismological to satellite observations), allow nowadays for viable numerical and analytical alternatives to the use of probabilistic approaches. A set of scenarios of expected ground shaking due to a wide set of potential earthquakes can be defined by means of full waveforms modelling, based on the possibility to efficiently compute synthetic seismograms in complex laterally heterogeneous anelastic media. In this way a set of scenarios of ground motion can be defined, either at national and local scale, the latter considering the 2D and 3D heterogeneities of the medium travelled by the seismic waves.
The considered scenario-based approach to seismic hazard assessment, namely the NDSHA approach (neo-deterministic seismic hazard assessment), builds on rigorous theoretical basis and exploits the currently available computational resources that permit to compute realistic synthetic seismograms. The integrated NDSHA approach intends to provide a fully formalized operational tool for effective seismic hazard assessment, readily applicable to compute complete time series of expected ground motion (i.e the synthetic seismograms) for seismic engineering analysis and other mitigation actions.
The NDSHA methodology has been successfully applied to strategic buildings, lifelines and cultural heritage sites, and for the purpose of seismic microzoning in several urban areas worldwide. Several international projects have been carried out and are still in progress based on the NDSHA methodology, including: the "MAR VASTO" project, with the participation of Italian (ENEA, Universities of Ferrara and Padua, ICTP) and Chilean (University Federico Santa Maria in Valparaiso, University of Chile in Santiago) partners; the UNESCO/IUGS/IGCP projects "Realistic Modelling of Seismic Input for Megacities and Large Urban Areas", "Seismic Hazard and Risk Assessment in North Africa" and "Seismic microzoning of Latin America cities"; the multilateral-oriented network project "Unified seismic hazard mapping for the territory of Romania, Bulgaria, Serbia and Republic of Macedonia", supported by the CEI (Central European Initiative).
The use of the NDSHA methodologies has been recently enabled on the EU-IndiaGrid2 computational infrastructures, based on GRID computing paradigms. Such e-Infrastructure provides an innovative and unique approach to address this problem. They demonstrated to be an efficient way

to share and access resources of different types, which can effectively enhance the capability to define realistic scenarios of seismic ground motion. Intensive usage of these infrastructure may enable scientists to compute a wide set of synthetic seismograms, dealing efficiently with variety and complexity of the potential earthquake sources, and the implementation of parametric studies to characterize the related uncertainties.
A Cooperation Project, aimed at the definition of seismic and tsunami hazard scenarios in the Gujarat region (India), by means of indo-european e-infrastructures, has been recently funded by the Friuli Venezia Giulia Region (Italy). The project aims to set up a system for the seismic characterization, integrated with the e-infrastructures distributed amongst India and Europe, to allow for the optimization of the computation of the ground shaking and tsunami scenarios. This goal will be attained thanks to the strict connection with the European project EU-IndiaGrid2 that provides the necessary infrastructure. Thus, the project will permit developing an integrated system, with high scientific and technological content, for the definition of scenarios of ground shaking, providing in the same time to the local community (local authorities and engineers) advanced information for seismic and tsunami risk mitigation in the study region.

2. Neo-deterministic seismic hazard assessment and tsunami hazard scenarios

The typical seismic hazard problem lies in the determination of the ground motion characteristics associated with future earthquakes, at both the regional and the local scale. Seismic hazard assessment can be performed in various ways, e.g. with a description of the groundshaking severity due to an earthquake of a given distance and magnitude ("groundshaking scenario"), or with probabilistic maps of relevant parameters describing ground motion.
A neo-deterministic approach to seismic hazard assessment (NDSHA), which allows to give a realistic description of the seismic ground motion due to an earthquake of given distance and magnitude has been developed [13], based on the calculation of synthetic seismograms. It can be applied also to areas that have not yet been hit by a catastrophic event in historical times, but are potentially prone to it. The neo-deterministic method allows to quantitatively model the effects of an earthquake which may happen in the future and therefore is a very effective technique in seismic hazard assessment, even in the regions with scarce or no historical or instrumental information available.
Starting from the available information on the Earth's structure, seismic sources, and the level of seismicity of the investigated area, it is possible to compute complete synthetic seismograms and the related estimates on peak ground acceleration (PGA), velocity (PGV) and displacement (PGD) or any other parameter relevant to seismic engineering (such as design ground acceleration, DGA) which can be extracted from the computed theoretical signals.
NDSHA can be applied at the regional scale, computing seismograms at the nodes of a grid with the desired spacing, or at the local scale, taking into account the source characteristics, the path and local geological and geotechnical conditions.
In the NDSHA approach, the definition of the space distribution of seismicity accounts only for the largest events reported in the earthquake catalogue at different sites, as follows. Earthquake epicenters reported in the catalogue are grouped into $0.2^\circ \times 0.2^\circ$ cells, assigning to each cell the maximum magnitude recorded within it. A smoothing procedure is then applied to account for spa-

tial uncertainty and for source dimensions [13]. Only cells located within the seismogenic zones are retained. This procedure for the definition of earthquake locations and magnitudes for NDSHA makes the method pretty robust against uncertainties in the earthquake catalogue, which is not required to be complete for magnitudes lower than 5. A double-couple point source is placed at the center of each cell, with a focal mechanism consistent with the properties of the corresponding seismogenic zone and a depth, which is a function of magnitude.
To define the physical properties of the source-site paths, the territory is divided into $1.0^\circ \times 1.0^\circ$ cells, each characterized by a structural model composed of flat, parallel anelastic layers that represent the average lithosphere properties at regional scale [4]. Synthetic seismograms are then computed by the modal summation technique for sites placed at the nodes of a grid with step $0.2^\circ \times 0.2^\circ$ that covers the national territory, considering the average structural model associated to the regional polygon that includes the site. Seismograms are computed for an upper frequency content of 1 Hz, which is consistent with the level of detail of the regional structural models, and the point sources are scaled for their dimensions using the spectral scaling laws proposed by [6], as reported in [1].

From the set of complete synthetic seismograms, various maps of seismic hazard describing

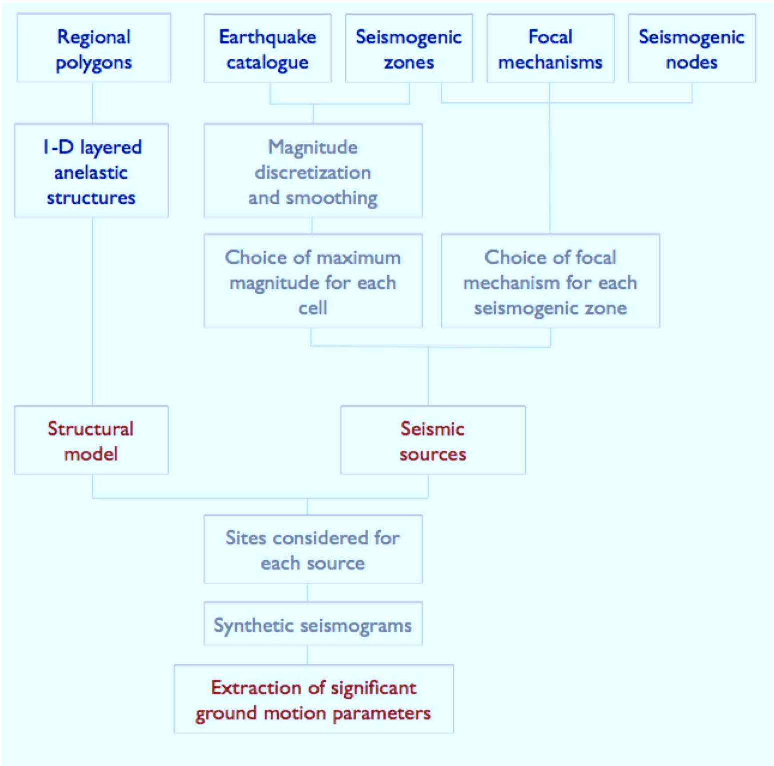

Figure 1: flow chart of national scale hazard package.

the maximum ground shaking at the bedrock can be produced. The parameters representative of earthquake ground motion are maximum displacement, velocity, and acceleration. The acceleration parameter in the NDSHA is given by the design ground acceleration (DGA). This quantity is obtained by computing the response spectrum of each synthetic signal for periods of 1 s and longer (the periods considered in the generation of the synthetic seismograms) and extending the spectrum, at frequencies higher than 1 Hz, using the shape of the Italian design response spectrum for soil A (Norme Tecniche D.M. 14/09/ 2005), which defines the normalized elastic acceleration response spectrum of the ground motion for 5% critical damping. For more details see [11].

The neo-deterministic method has been recently adapted to account for the extended source process by including higher frequency content (up to 10 Hz) as well as the rupture process at the source and the consequent directivity effect (obtained by means of the PULSYN algorithm by [7].

The seismic waves due to an extended source are obtained by approximating the fault with a rectangular plane surface, on which the main rupture process is assumed to occur. The source is represented as a grid of point subsources, and their seismic moment rate functions are generated considering each of them as realizations (sample functions) of a non-stationary random process. Specifying in a realistic way the source length and width, as well as the rupture velocity, one can obtain realistic source time functions, valid in the far-field approximation. Furthermore, assuming a realistic kinematic description of the rupture process, the stochastic structure of the accelerograms can be reproduced, including the general envelope shape and peak factors. The extended earthquake source model allows us to generate a spectrum (amplitude and phase) of the source time function that takes into accounts both the rupture process and directivity effects, also in the near source region.

At local scale the neo-deterministic method is based on a hybrid technique that combines two methods: the analytical modal summation [10, 5, 13] and the numerical finite difference methods [19, 20, 8] taking advantage of the characteristics of both. Each of the two methods is applied in that part of the structural model where it works most efficiently. The modal summation is applied to simulate wave propagation from the source to the sedimentary basin of interest. Being a pure analytical technique, there is no penalty applied in this part of the modelling associated with the model size. Finite difference method is applied to propagate the incoming wavefield in the laterally heterogeneous part of the structural model that contains the sedimentary basin.

The path from the source position to the local structure of interest is approximated by a structure composed of laterally homogeneous layers. The coupling of the two methods is carried out by introducing the resulting time series obtained with the modal summation into the finite-difference computations, used to model wave propagation in the laterally heterogeneous part of the model.

As for NDSHA, to build scenario-based tsunami hazard maps for a specific coastal area one has first to characterize the seismic sources and select the earthquake scenarios that can drive the hazard [2, 3]. By means of the modeling we then calculate the maximum amplitude of the vertical displacement of the water particles on the sea surface and the travel time of the maximum amplitude peak, since they are the most relevant aspects of the tsunami wave and also are the only characteristics always recorded in the chronicles and therefore in catalogues. The horizontal displacement field is calculated too, and, on average, it exceeds the vertical one by an order of magnitude approximately (this accounts for the great inundating power of tsunami waves with respect to wind driven ones). It is important to point out that the analytical modeling techniques [12] is extremely efficient (computation times are of the order of seconds) for real time simulations can be very useful also for integration in a Tsunami Warning System, since they can be compared with real time incoming open-sea level data, in order to validate, or close, an impending alarm.

3. Porting and optimization of codes

E-infrastructures represent a critical mean to provide access to important computing resources and specialized software to worldwide seismological community. In fact, e-science removes some of the infrastructural barriers that prevent collaborative work at the international level. Accordingly, the proposed scientific and computational tools and networking will permit a widespread application of the advanced methodologies for seismic hazard assessment, particularly useful for urban planning and risk mitigation actions in developing countries, and, in turn, will allow for a faster development and verification of the models.

Preliminary studies have been devoted to expose seismologists and seismic engineers to modern e-infrastructures (which includes both HPC and Grid environments) so that the potential provided by this infrastructure for seismic hazard assessment research can be assessed and exploited. These activities aim to enable the computational seismology user community to the use of modern e-infrastructure and acquire the core innovations emerging in this framework, for example the development of an European and worldwide e-infrastructure for advanced applications in seismic hazard assessment driven by European Union projects such as Prace, EGI, EU-IndiaGrid2, EUMEDGRID-Support and Chain. The major goals of this new collaboration are to:

- facilitate the development and application of a scientifically consistent approach to seismic hazard assessment;
- disseminate, in scientific and in engineering practice, advanced reliable tools for seismic hazard estimates;
- exploit, as much as possible, the advantages provided by computational resources and e-Infrastructures.

Activities carried out so far have been dedicated to a general introduction to the e-Infrastructures for Grid and HPC and to the preliminary assessment of their use in seismological research, with special emphasis on methods for advanced definition of ground shaking scenarios based on physical modelling of seismic waves generation and propagation processes. Researchers gained some practice on the use of the e-infrastructure for neo-deterministic seismic hazard assessment at different scales and level of detail, working actively on Italian data and testing the specialized seismological software running on an e-infrastructure environment, leveraging on the work performed within the EU-India Grid projects.

The use of the EU-India Grid infrastructure allows to conduct massive parametric tests, to explore the influence not only of deterministic source parameters and structural models but also of random properties of the same source model, to enable realistic estimate of seismic hazard and their uncertainty. The random properties of the source are especially important in the simulation of the high frequency part of the seismic ground motion.

We have ported and tested seismological codes for national scale on the Grid infrastructure. The first step was the speed optimization of this package by the identification of the critical programs and of the hot spots within programs. The critical point in the algorithm was the computation of synthetic seismograms. The optimization was performed in two ways: first by removing repeated formatted disk I/O; second by sorting of seismograms by source depth, storing in memory depth-depended quantities and therefore removing the need for their repeated computation at different stages of the execution flow.

In the second step, the national scale hazard package was ported on the EU-India GRID infras-

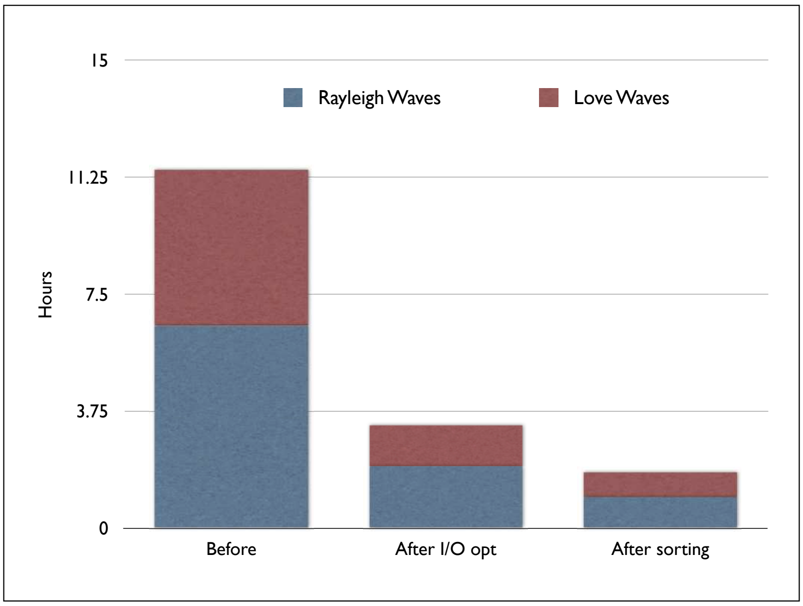

Figure 2: speedup of seismograms computation obtained on our local machine.

tructure. Two different types of parametric tests were developed: the first aimed at the variation of the deterministic source parameters, and the second focussed on the random properties of the source model. The first experiment is performed by perturbing the properties of the seismic sources selected by the algorithm before the computation of synthetic seismograms. In the second test different sets of curves of source spectrum generated by a MonteCarlo simulation of the source model are used for scaling the seismograms. In both cases there are many independent runs to be executed, so a script for the generation of the input and other scripts for checking the status of jobs, retrieving the results and relaunching aborted jobs were developed.

One preliminary test over deterministic source parameter for whole Italy ("persut Italy"), and two different tests over random properties ("seed1Hz" and "seed10Hz") for the whole Italian territory, with different frequency content and different maximum distance for the computation of seismograms, were conducted. The performance of the package over the grid in terms of computational time and number of successful jobs was tested, and submission of job and retrieval of its output were refined.

The number of seismograms that must be computed determines the duration and the storage requirement of the run. This parameter seems critical for the success of the job. The test runs on the random component of the source gave an indication of the effective number of jobs that must be executed to obtain a good estimate of the distribution of the ground shaking peaks at each receiver.

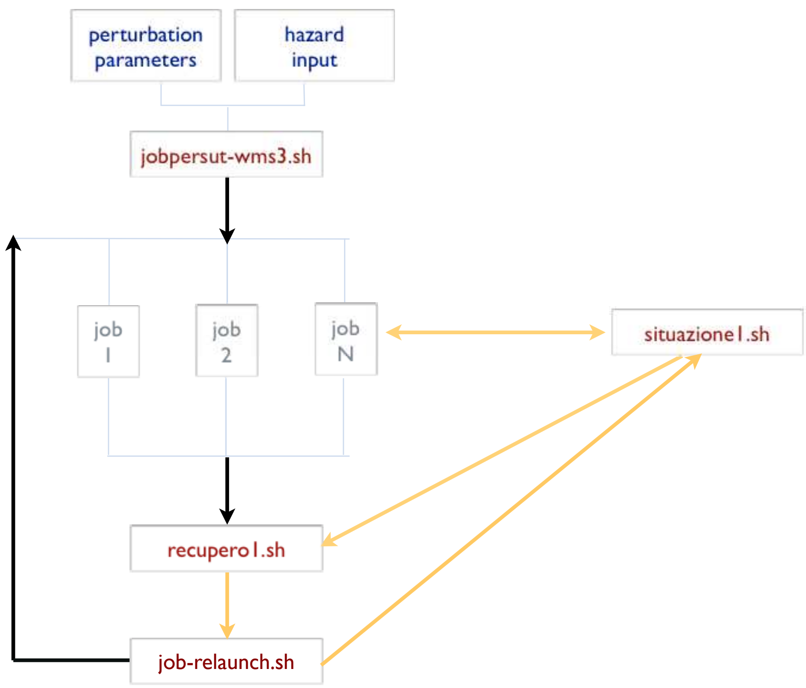

Figure 3: scheme of test over deterministic source parameter.

	seed1Hz	persut Italy	seed10Hz
type of submission	direct to CE	WMS	WMS
total n. of job	216	526	600
% of successful job	26	65	80
total time of computation of successful job	948 h	1200 h	791 h
average time of computation for one job	17 h	3.5 h	1.6 h
number of computed seismograms for one job	197720	91398	29232

Table 1: performance of the three test runs.

4. Application of neo-deterministic seismic hazard assessment for Gujarat region by means of indo-european e-infrastructures

The application of the NDSHA approach to the territory of India started in the framework of long-term bilateral cooperation projects Italy-India, involving DMG and CSIR C-MMACS (Bangalore). In that framework, a neo-deterministic hazard map have been produced for India [15], and specific studies have been performed to estimate the ground motion amplifications along selected profiles in the cities of Delhi [14, 16, 17] and Kolkata [9, 18].
A Cooperation Project, aimed at the definition of seismic and tsunami hazard scenarios by means of indo-european e-infrastructures in the Gujarat region (India), has been recently funded by the Friuli Venezia Giulia Region. This two-years project, starting in November 2011, involves three Italian partners (DMG, University of Trieste; ICTP SAND Group; CNR/IOM uos Democritos) and two Indian partners (ISR, Gujarat; CSIR C-MMACS, Bangalore). The project will contribute to the sustainable development in the intervention area, which already witnessed many destructive earthquakes in the past, by means of the application of advanced methodologies for seismic and tsunami hazard assessment. The planned activities will allow for a characterization of ground shaking adequate to seismic design and territorial planning, essential for the design of critical facilities and infrastructures, as well as for the safeguard of cultural heritage. Specifically, as far as it concerns the territory of Gujarat, the project activities will contribute to the reconstruction and rehabilitation actions following the destructive Bhuj earthquake (M=8.0; 26 January 2001), by means of the seismic characterization (microzoning) of the sites identified for the new urban and industrial settlements and harbours. Gujarat, in fact, is the industrial hub of India, hosting one of the world's largest refineries, chemical industries and large maritime facilities; moreover, it is developing a number of special economic and investment areas.
The project aims to set up a system for the seismic characterization, integrated with the e-infrastructures distributed amongst India and Europe, to allow for the optimization of the computation of the ground shaking and tsunami scenarios. This goal will be attained thanks to the strict connection with the European project EU-India Grid2, that involves ICTP, and that provides the necessary infrastructure. Thus, the project will permit developing an integrated system, with high scientific and technological content, for the definition of scenarios of ground shaking, providing in the same time to the local community (local authorities and engineers) advanced information for seismic and tsunami risk mitigation in the study region.

5. Conclusions

The NDSHA methodology has been successfully applied to strategic buildings, lifelines and cultural heritage sites, and for the purpose of seismic microzoning in several urban areas worldwide. Several international projects have been carried out and are still in progress based on the NDSHA methodology. The very positive outcomes from seismological collaborative research call for an improvement of such interactions; this is attained by integration and formalization of the existing scientific and computing networks. The e-infrastructures provide an innovative and unique approach to address this problem. They demonstrated to be an efficient way to share and access

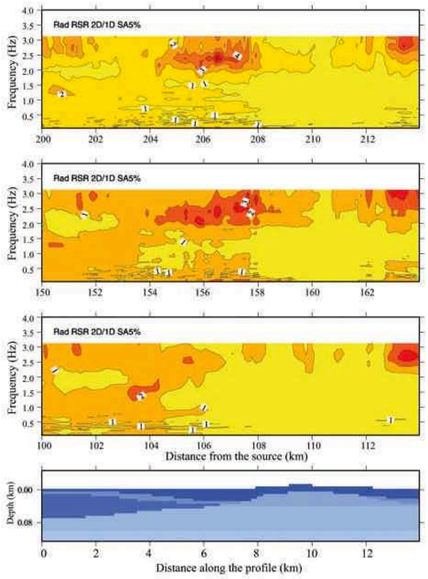

Figure 4: 2D model of a geological cross-section in Delhi city and corresponding plot of the RSR (5% damping) for radial component versus frequency (from [17]).

resources of different types, which can effectively enhance the capability to define realistic scenarios of seismic ground motion, i.e. to compute the reliable seismic input necessary for seismic risk mitigation. Such facilities, in fact, may enable scientists to compute a wide set of synthetic seismograms, dealing efficiently with variety and complexity of the potential earthquake sources, and the implementation of parametric studies to characterize the related uncertainties. A cooperation project, recently funded by the Friuli Venezia Giulia Region and started in November 2011, involves three Italian partners (DMG, University of Trieste; ICTP SAND Group; CNR/IOM uos Democritos) and two Indian partners (ISR, Gujarat; CSIR C-MMACS, Bangalore). In the framework of this project, a system will be set up for the scenario-based characterization of the seismic hazard, integrated with the e-infrastructures of the European project EU-IndiaGrid.

References

[1] K. Aki, *Strong motion seismology*, in: Strong ground motion seismology, *NATO ASI Series, Series C: Mathematical and Physical Sciences*, vol 204 (eds. M. Erdik, and Z.M. Toksö) (D. Reidel Publishing Company, Dordrecht 1987), pp 3-39.

[2] D. Bisignano, F. Romanelli, A. Peresan, *Modeling scenarios of earthquake-generated tsunamis for Vietnam coasts*, in *The International Symposium on Grids and Clouds and the Open Grid Forum Academia Sinica*, Taipei, Taiwan March 19 - 25, 2011

[3] D. Bisignano, F. Romanelli, *Earthquake generated tsunami hazard scenarios along the Vietnam coasts*, in *Proceedings of IAEA International Workshop on External Flooding Hazard at Nuclear Plant Sites in Commemoration of the 5 years of the 2004 Indian Ocean Tsunami*, Kalpakkam (India), 11-15 January 2010

[4] E. Brandmayr, R. Raykova, M. Zuri, F. Romanelli, C. Doglioni, G.F. Panza, (2010). *The lithosphere in Italy: structure and seismicity*. In: (Eds.) M. Beltrando, A. Peccerillo, M. Mattei, S. Conticelli, and C. Doglioni, *Journal of the Virtual Explorer*, volume 36, paper 1, doi: 10.3809/jvirtex.2009.00224.

[5] N. Florsch, D. Fäh, P. Suhadolc, G.F. Panza (1991). *Complete synthetic seismograms for high-frequency multimode SH-waves*, *Pure and Appl. Geoph*. 136, 529-560.

[6] A.A. Gusev, (1983), *Descriptive statistical model of earthquake source radiation and its application to an estimation of short period strong motion*, *Geophys J R Astron Soc*, 74, 787-800

[7] A.A. Gusev (2011) *Broadband kinematic stochastic simulation of an earthquake source: a refined procedure for application in seismic hazard studies*, *Pure Appl Geophys* 168(1-2)

[8] A.R. Levander, (1988). *Fourth-order Finite-difference P-SV Seismograms*. *Geophysics*, 53, 1425-1436.

[9] W.K. Mohanty, M.Y. Walling, F. Vaccari, T. Tripathy, G.F. Panza (2009), *Modeling of SH- and P-SV-wave fields and seismic microzonation based on response spectra for Talchir Basin, India*, *Engineering Geology* 104 80-97

[10] G.F. Panza, (1985). *Synthetic Seismograms: The Rayleigh waves modal summation*, *Journal of Geophysics*, 58, 125-145.

[11] G.F. Panza, F. Vaccari, G. Costa, P. Suhadolc, D. Fäh, (1996), *Seismic input modelling for zoning and microzoning*, *Earthq Spectra* 12, 529-566

[12] G.F. Panza, F. Romanelli, T. Yanovskaya, *Synthetic Tsunami mareograms for realistic oceanic models*, *Geophysical Journal International*, 141, 498-508, 2000.

[13] G.F. Panza, F. Romanelli, F. Vaccari, (2001), *Seismic wave propagation in laterally heterogeneous anelastic media: theory and applications to seismic zonation*, *Advances in Geophysics*, vol. 43. Academic Press, pp. 1-95.

[14] I.A. Parvez, G.F. Panza, A.A. Gusev and F. Vaccari, (2002), *Strong-motion amplitudes in Himalayas and a pilot study for the deterministic first-order microzonation in a part of Delhi city*, *CURRENT SCIENCE*, VOL. 82, NO. 2, 25

[15] I.A. Parvez, F. Vaccari and G.F. Panza, (2003) *A deterministic seismic hazard map of India and adjacent areas*, *Geophys. J. Int.* 155, 489-508

[16] I.A. Parvez, F. Vaccari and G.F. Panza, (2004), *Site-specific Microzonation Study in Delhi Metropolitan City by 2-D Modelling of SH and P-SV Waves*, *Pure appl. geophys.* 161 (2004) 1165-1184

[17] I.A. Parvez, F. Vaccari and G.F. Panza, (2006), *Influence of source distance on site-effects in Delhi city*, *CURRENT SCIENCE*, VOL. 91, NO. 6

[18] F. Vaccari, M.Y. Walling, W.K. Mohanty, S.K. Nath, A. Verma, A. Sengupta, and G.F. Panza (2010) *Site-Specific Modeling of SH and P-SV Waves for Microzonation Study of Kolkata Metropolitan City, India*, *Pure Appl. Geophys.* 168 (2011), 479-493

[19] J. Virieux (1984). *SH - velocity-stress finite-difference method: velocity-stress finite-difference method*, *Geophysics*, 49, pp. 1933-1957.

[20] J. Virieux (1986). *P-SV wave propagation in heterogeneous media: velocity-stress finite-difference method*, *Geophysics*, 51, pp. 889-901.