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Generalized threshold resummation for semi-inclusive e^+e^- annihilation

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Recently methods have been developed to extend the resummation of large-x double logarithms in inclusive deep-inelastic scattering (DIS) to terms not addressed by the soft-gluon exponentiation. Here we briefly outline our approach based on fixed-order results, the general large-x structure in dimensional regularization and the all-order factorization of mass singularities, which is directly applicable also to semi-inclusive e^+e^- annihilation (SIA). We then present some main results for the corresponding timelike splitting functions and transverse and longitudinal fragmentation functions. The close relation between DIS and SIA facilitates the determination of additional third-order results for the latter function which is fully known only at the next-to-leading order. Therefore all above quantities can be resummed at next-to-next-to-leading logarithmic accuracy.

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1. Introduction

In the past years quite a few studies have addressed the threshold behaviour of higher-order splitting functions and hard-process coefficient functions in perturbative QCD beyond the quantities or contributions covered by the standard soft-gluon exponentiation [1–13]. Except for the gluon contributions to the longitudinal structure and fragmentation functions F_L which include an additional factor $(1-x)$, the dominant terms are of the divergent but integrable 'double-logarithmic' form

$$\alpha_s^n \ln^{2n-n_0-\ell}(1-x)\ . \tag{1.1}$$

Here n_0 depends on the quantity under consideration, with $n_0 = 2$ for the 'off-diagonal' splitting functions $P_{ij}(x)$ with $i \neq j$ – recall that the diagonal splitting functions $P_{ii}(x)$ do not exhibit any n-dependent large-x enhancement in the $\overline{\mathrm{MS}}$ scheme adopted in the present contribution [14, 15] – and $n_0 = 1$ for the structure functions in deep-inelastic scattering (DIS) and fragmentation functions in semi-inclusive e^+e^- annihilation (SIA) with the exception of the functions F_L where $n_0 = 2$. $\ell = 0,\ 1,\ 2,\ \ldots$ in Eq. (1.1) represent the leading logarithmic (LL), next-to-leading logarithmic (NLL), next-to next-to-leading logarithmic (NNLL = N^{2}LL), ... large-x contributions.

Using the third-order results of Refs. [16, 17], the LL, NLL and NNLL coefficients of the (non-singlet) quark coefficient functions for the structure functions F_2, F_3 and F_L in DIS and their counterparts F_T, F_A and F_L in SIA have been obtained in Refs. [8, 9], together with the corresponding LL and NLL results for the Drell-Yan process, from the single-logarithmic behaviour of the physical evolution kernels of these observables. The corresponding approach is not sufficient for all-order predictions in flavour-singlet cases, but was used to obtain the NNLL approximations of the fourth-order contributions to the off-diagonal spacelike splitting functions P_{ij}^S for the parton distribution and the longitudinal coefficient function $C_{L,g}$ in DIS [10, 11].

Those DIS results have been confirmed and extended by studying the D-dependence of the unfactorized structure functions in dimensional regularization [12, 13]. This approach facilitates the determination of the one previously missing parameter in the N^{3}LL non-singlet coefficients for F_2 and F_3 – a result that has been obtained independently in Ref. [6] – and the extension of the NNLL all-order resummation to the off-diagonal splitting functions and gluon coefficient functions. In the present contribution we briefly report on the generalization of those results to the off-diagonal timelike splitting functions P_{ij}^T for parton fragmentation and the singlet coefficient functions for the fragmentation functions in SIA; a detailed account of our results will be published elsewhere [18].

For this purpose we consider the transverse and longitudinal gauge-boson exchange fragmentation functions F_T^T and F_L^T as defined in Ref. [19] together with the corresponding Higgs-decay quantity in the heavy top-quark limit F_ϕ^T. Schematically our notation for the corresponding timelike coefficient functions $C_{a,i}^T$ and the evolution of the fragmentation distributions D_i at the physical scale Q^2 with $a_s = \alpha_s(Q^2)/(4\pi)$ reads (note the transposition of the splitting-function matrix)

$$F_a^T(Q^2) \,=\, C_{a,i}^T(a_s) \otimes D_i(Q^2)\ , \quad \frac{d}{d\ln Q^2} D_j \,=\, P_{ij}^T(a_s) \otimes D_i \tag{1.2}$$

where $\otimes$ represent the Mellin convolution and the appropriate summations over i are understood.

2. Outline of the resummation

The primary objects of the present resummation are the unfactorized SIA fragmentation functions

$$\widehat{F}^{T}_{a,k} \;=\; \widetilde{C}^{T}_{a,i} \otimes Z^{T}_{ik} \quad \text{for} \quad a,k \,=\, T,g\,,\ \phi,q\,,\ L,q \ \text{and}\ L,g \tag{2.1}$$

in $D = 4-2\varepsilon$ dimensions. The functions $\widetilde{C}^{T}_{a,i}$ are given by Taylor series in ε, with the ε^k terms including k more powers in $\ln(1-x)$ than the 4-dimensional coefficient functions. The timelike transition matrix Z^T consist of only negative powers of ε and can be written in terms of

$$P^{T} \;=\; a_{\rm s} P_0 \,+\, a_{\rm s}^2 P_1 \,+\, a_{\rm s}^3 P_2 \,+\, \ldots \tag{2.2}$$

and the corresponding coefficients β_m of the beta function. This dependence can be summarized as

$$a_{\rm s}^n \varepsilon^{-n} :\ P_0\,,\beta_0\ ,\quad a_{\rm s}^n \varepsilon^{-n+1} :\ +P_1\,,\beta_1\ ,\quad \ldots\ ,\quad a_{\rm s}^n \varepsilon^{-1} :\ P_{n-1}\ . \tag{2.3}$$

Hence fixed-order knowledge at N^{m}LO (i.e., of the splitting functions to P_m and the corresponding coefficient functions) fixes the first $m+1$ coefficients in the ε expansion of $\widehat{F}^{T}_{a,k}$ at all orders in $a_{\rm s}$. The large-x expansions of $\widehat{F}^{T}_{a\neq L,k}$ (the corresponding relation for F_L is slightly different) are given by

$$\widehat{F}^{\,T}\big|_{a_{\rm s}^n \varepsilon^{-n+\ell}} \;=\; \mathscr{F}^{(0)}_{n,\ell} \ln^{n+\ell-1}(1-x) \,+\, \mathscr{F}^{(1)}_{n,\ell} \ln^{n+\ell-2}(1-x) \,+\, \ldots\ . \tag{2.4}$$

If the constants up to $\mathscr{F}^{(m)}_{n,\ell}$ are known for all n and ℓ, then the splitting functions and coefficient functions can be determined at N^{m}LL accuracy at all orders of the strong coupling.

As in DIS, the $n^{\rm th}$ order large-x contributions to $\widehat{F}^{T}_{a\neq L,k}$ are built up from n term of the form

$$\big(A_{n,k}\, \varepsilon^{-2n+1} + B_{n,k}\, \varepsilon^{-2n+2} + C_{n,k}\, \varepsilon^{-2n+3} + \ldots\big)(1-x)^{-k\varepsilon}\ ,\quad k = 1,\ldots,n \tag{2.5}$$

which arises from the phase-space integrations for the undetected final-state partons and the loop integrals of the virtual corrections [21–23]. Since the terms with $\varepsilon^{-2n+1}, \ldots, \varepsilon^{-n-1}$ have to cancel in sum (2.1), there are $n-1$ relations between the LL coefficients $A_{n,k}$ which lead to the constants $\mathscr{F}^{(0)}_{n,\ell}$ in Eq. (2.4), $n-2$ relations between the NLL coefficients $B_{n,k}$ etc. As discussed above, a N^{m}LO calculation fixes the (non-vanishing) coefficients of $\varepsilon^{-n}, \ldots, \varepsilon^{-n+m}$ at all orders n, adding $m+1$ more relations between the coefficients in Eq. (2.5). Consequently the highest $m+1$ double logarithms, i.e., the N^{m}LL approximation, can be determined in this manner from the N^{m}LO results.

The coefficient functions for $F_T \equiv F_T^T$, F_L^T and F_ϕ^T are known at the second order [23–26]. The third-order timelike splitting functions have been determined, up to an uncertainty which is irrelevant in the present context, in Refs. [26–28]; see also Ref. [7] for their large-x logarithms. The all-order factorization of the quantities (2.1) requires corresponding large-x results for the quantities $F^T_{T,q}$ and $F^T_{\phi,g}$. These are available from the soft-gluon exponentiation to a far higher accuracy than required here, cf. Ref. [29]. The calculations are carried out in Mellin-N space with

$$\ln^k(1-x) \,\triangleq\, (-1)^k N^{-1}\big(\ln^k\widetilde{N} + \tfrac{1}{2}\,k(k-1)\zeta_2 \ln^{k-2}\widetilde{N} + \ldots\big)\ ,\ \ \ln\widetilde{N} = \ln N + \gamma_{\rm e} \tag{2.6}$$

where $\gamma_{\rm e}$ is the Euler-Mascheroni constant. The required formalism is completely analogous to that in Ref. [13]. Our symbolic manipulations have been performed using FORM and TFORM [30, 31].

3. Results for the timelike splitting functions

As their spacelike counterparts, the LL and NLL contributions to the off-diagonal timelike splitting functions can be expressed in terms of functions $\mathscr{B}_n(x)$ introduced and discussed in Refs. [12, 13],

$$\mathscr{B}_k(x) \;=\; \sum_{n=0}^{\infty} \frac{B_n}{n!(n+k)!}\, x^n \quad \text{and} \quad \mathscr{B}_{-k}(x) \;=\; \sum_{n=k}^{\infty} \frac{B_n}{n!(n-k)!}\, x^n \tag{3.1}$$

for $k = 0, 1, 2, \ldots$, where B_n are the Bernoulli numbers in the normalization of Ref. [20]. We obtain

$$\begin{aligned} NP^T_{\rm qg}(N,\alpha_{\rm s}) \;=&\; 2a_{\rm s}\, n_f\, \mathscr{B}_0(-\tilde{a}_{\rm s}) \\ &+ a_{\rm s}^2 \ln\widetilde{N}\, n_f \Big[(12C_F - 6\beta_0)\, \frac{1}{\tilde{a}_{\rm s}}\, \mathscr{B}_{-1}(-\tilde{a}_{\rm s}) - \frac{\beta_0}{\tilde{a}_{\rm s}}\, \mathscr{B}_{-2}(-\tilde{a}_{\rm s}) + (6C_F - \beta_0)\, \mathscr{B}_1(-\tilde{a}_{\rm s}) \Big] \\ &+\; \text{NNLL contributions} \;+\; \ldots\,, \end{aligned} \tag{3.2}$$

$$\begin{aligned} NP^T_{\rm gq}(N,\alpha_{\rm s}) \;=&\; 2a_{\rm s}\, C_F\, \mathscr{B}_0(\tilde{a}_{\rm s}) \\ &+ a_{\rm s}^2 \ln\widetilde{N}\, C_F \Big[(12C_F - 2\beta_0)\, \frac{1}{\tilde{a}_{\rm s}}\, \mathscr{B}_{-1}(\tilde{a}_{\rm s}) + \frac{\beta_0}{\tilde{a}_{\rm s}}\, \mathscr{B}_{-2}(\tilde{a}_{\rm s}) + (8C_A - 2C_F - \beta_0)\, \mathscr{B}_1(\tilde{a}_{\rm s}) \Big] \\ &+\; \text{NNLL contributions} \;+\; \ldots\,. \end{aligned} \tag{3.3}$$

Here C_A and C_F are the standard $SU(n_c)$ colour factors with $C_A = n_c = 3$ and $C_F = 4/3$ in QCD, n_f represents the number of effectively massless quark flavours, $\beta_0 = 11/3\, C_A - 2/3\, n_f$ is the first coefficient of the (four-dimensional) beta function, and we have used the shorthand

$$\tilde{a}_{\rm s} \;\equiv\; 4a_{\rm s}\, C_{AF}\, \ln^2\widetilde{N} \quad \text{with} \quad C_{AF} \equiv C_A - C_F \tag{3.4}$$

reflecting the vanishing of the double logarithms for the ‘supersymmetric’ case $C_A = C_F$.

The first and second lines of Eqs. (3.2) and (3.3) are the LL and NLL results, respectively. They differ from their spacelike counterparts, $n_f/C_F\, P^S_{\rm gq}$ and $C_F/n_f\, P^S_{\rm qg}$ only by coefficients of $\mathscr{B}_1$ proportional to $C_A - C_F$. These differences are due to different $(1-\varepsilon)^{-1}$ prefactors of the spacelike and timelike unfactorized structure functions, e.g., the absence of the DIS gluon spin-averaging in the corresponding SIA quantity, which are otherwise related by a (at this level) simple analytic continuation. As in the spacelike case, we have no closed all-order expression for the NNLL contributions to Eqs. (3.2) and (3.3) which therefore will be presented via tables to a sufficiently high order in $\alpha_{\rm s}$ in Ref. [18]. After transformation back to x-space the fourth-order results read

$$\begin{aligned} P^T_{\rm qg}(x)\Big|_{a_{\rm s}^4} =&\; \ln^5(1-x)\, C_{AF}^2 n_f \Big[\tfrac{22}{27}\, C_{AF} - \tfrac{14}{27}\, C_F - \tfrac{4}{27}\, n_f \Big] \\ &+ \ln^4(1-x)\, C_{AF} n_f \Big[\Big(\tfrac{1432}{81} + \tfrac{64}{9}\, \zeta_2 \Big) C_{AF}^2 + \Big(\tfrac{1471}{54} - 8\zeta_2 \Big) C_{AF} C_F \\ &- \tfrac{16}{3}\, C_{AF} n_f - \tfrac{49}{81}\, C_F^2 + \tfrac{17}{81}\, C_F n_f + \tfrac{32}{81}\, n_f^2 \Big] + \mathscr{O}\left(\ln^3(1-x)\right)\,, \end{aligned} \tag{3.5}$$

$$\begin{aligned} P^T_{\rm gq}(x)\Big|_{a_{\rm s}^4} =&\; \ln^5(1-x)\, C_{AF}^2 C_F \Big[-\tfrac{26}{27}\, C_{AF} - \tfrac{14}{27}\, C_F - \tfrac{4}{27}\, n_f \Big] \\ &+ \ln^4(1-x)\, C_{AF} C_F \Big[\Big(\tfrac{469}{27} - \tfrac{128}{9}\, \zeta_2 \Big) C_{AF}^2 + \Big(\tfrac{5317}{162} - 8\zeta_2 \Big) C_{AF} C_F \\ &- \tfrac{212}{81}\, C_{AF} n_f - \tfrac{13}{81}\, C_F^2 + \tfrac{17}{81}\, C_F n_f - \tfrac{4}{81}\, n_f^2 \Big] + \mathscr{O}\left(\ln^3(1-x)\right)\,. \end{aligned} \tag{3.6}$$

The LL coefficients vanish at all even orders in $\alpha_{\rm s}$ from the fourth due to $B_{2n+1} = 0$ for $n \geq 1$. The above results are illustrated in Fig. 1 below for $\alpha_{\rm s} = 0.12$ and $n_f = 5$, i.e., at a scale $Q^2 \simeq M_Z^2$.

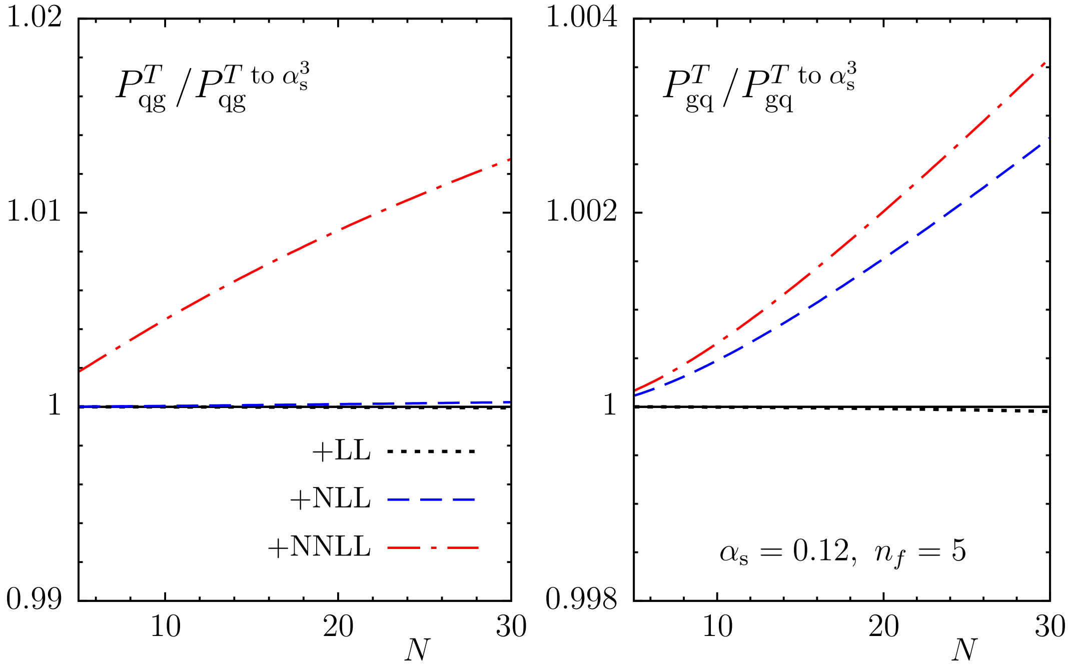

Figure 1: The relative leading-logarithmic (LL), next-to-leading logarithmic (NLL) and next-to-next-to-leading logarithmic (NNLL) higher-order large-x corrections to the third-order off-diagonal timelike splitting functions P^T_{ij} in Mellin-N space at a typical high-scale reference point.

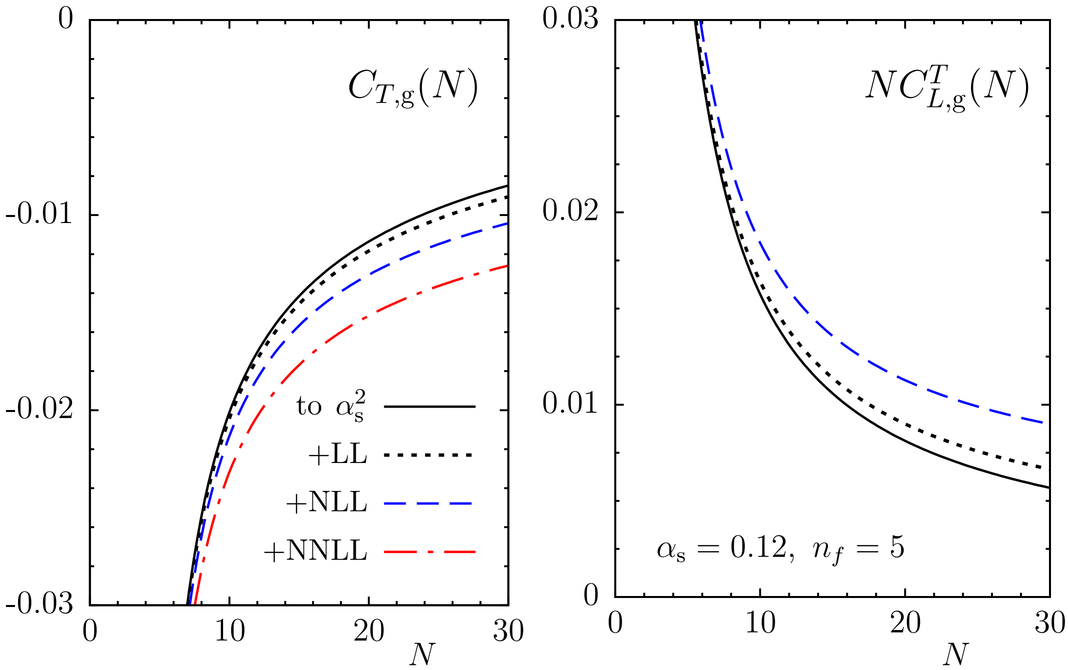

Figure 2: The absolute LL, NLL and (for $C_{T,g}$) NNLL higher-order threshold corrections to the second-order transverse and longitudinal gluon coefficient functions $C_{T,g}$ and $C^T_{L,g}$ in N-space at a scale $Q^2 \simeq M_Z^2$. Note that our normalization of both functions differs by a factor of $\frac{1}{2}$ from that of Refs. [23–25], i.e., here the lowest-order large-x limits are $C_{T,g}(x,a_s) = 2C_F a_s \ln(1-x) + \ldots$ and $C^T_{L,g}(x,a_s) = 4C_F a_s (1-x) + \ldots$.

4. Results for the SIA coefficient functions

The NNLO results facilitate the resummation of also the (more complicated) coefficient functions $C_{T,\mathrm{g}}$ and $C^T_{\phi,\mathrm{q}}$ at NNLL accuracy. Here we show, for brevity, only the NLL resummation of the former quantity (the NNLL contributions and the result for the latter will be given in Ref. [18]),

$$\begin{aligned}
NC_{T,\mathrm{g}}(N,\alpha_\mathrm{s}) = {} & \frac{1}{2\ln\widetilde{N}}\,\frac{C_F}{C_A-C_F}\Big[\exp(2a_\mathrm{s}C_F\ln^2\widetilde{N})\mathscr{B}_0(\tilde{a}_\mathrm{s})-\exp(2a_\mathrm{s}C_A\ln^2\widetilde{N})\Big] \\
& -\frac{1}{8\ln^2\widetilde{N}}\,\frac{C_F(3C_F-\beta_0)}{(C_A-C_F)^2}\Big[\exp(2a_\mathrm{s}C_F\ln^2\widetilde{N})\mathscr{B}_0(\tilde{a}_\mathrm{s})-\exp(2a_\mathrm{s}C_A\ln^2\widetilde{N})\Big] \\
& -\frac{a_\mathrm{s}}{4}\,\frac{C_F}{C_A-C_F}\,\exp\left(2a_\mathrm{s}C_A\ln^2\widetilde{N}\right)(8C_A+4C_F-\beta_0) \\
& -\frac{a_\mathrm{s}^2}{3}\,\beta_0\ln^2\widetilde{N}\,\frac{C_F}{C_A-C_F}\Big[C_A\exp(2a_\mathrm{s}C_A\ln^2\widetilde{N})-C_F\exp(2a_\mathrm{s}C_F\ln^2\widetilde{N})\mathscr{B}_0(\tilde{a}_\mathrm{s})\Big] \\
& -\frac{a_\mathrm{s}}{4}\,\frac{C_F}{C_A-C_F}\,\exp\left(2a_\mathrm{s}C_F\ln^2\widetilde{N}\right)\Big[-6C_F\mathscr{B}_0(\tilde{a}_\mathrm{s})-(8C_A-2C_F-\beta_0)\mathscr{B}_1(\tilde{a}_\mathrm{s}) \\
& \quad -(12C_F-4\beta_0)\,\frac{1}{\tilde{a}_\mathrm{s}}\,\mathscr{B}_{-1}(\tilde{a}_\mathrm{s})-\frac{\beta_0}{\tilde{a}_\mathrm{s}}\,\mathscr{B}_{-2}(\tilde{a}_\mathrm{s})\Big] + \text{ NNLL contributions } + \ldots\,.
\end{aligned} \tag{4.1}$$

Also this expression differs from its spacelike counterpart in Ref. [13] only in the coefficient of $\mathscr{B}_1$. The third-order contribution, now including the NNLL term, is given by

$$\begin{aligned}
C_{T,\mathrm{g}}(x)\Big|_{a_\mathrm{s}^3} = {} & \ln^5(1-x)\,C_F\left[\tfrac{2}{3}C_A^2+\tfrac{10}{3}C_F^2\right] \\
& +\ln^4(1-x)\,C_F\left[\tfrac{7}{27}C_An_f-\tfrac{269}{54}C_A^2+\tfrac{17}{27}C_Fn_f-\tfrac{338}{27}C_FC_A-\tfrac{97}{18}C_F^2\right] \\
& +\ln^3(1-x)\,C_F\Big[\left(\tfrac{2990}{81}-\tfrac{16}{9}\zeta_2\right)C_A^2+\left(\tfrac{3652}{81}-\tfrac{88}{9}\zeta_2\right)C_FC_A \\
& \quad -\left(\tfrac{41}{9}-\tfrac{112}{9}\zeta_2\right)C_F^2-\tfrac{140}{81}C_An_f-\tfrac{436}{81}C_Fn_f\Big] + \mathscr{O}\left(\ln^2(1-x)\right)\,.
\end{aligned} \tag{4.2}$$

For the longitudinal fragmentation function the second-order results represent only the NLO contribution. The resulting NLL results for the gluon coefficient function read

$$\begin{aligned}
N^2C^T_{L,\mathrm{g}}(N,\alpha_\mathrm{s}) = {} & 4a_\mathrm{s}C_F\exp\left(2a_\mathrm{s}C_A\ln^2\widetilde{N}\right) + 2a_\mathrm{s}C_F\,NC^{\mathrm{LL}}_{T,\mathrm{g}}(N,\alpha_\mathrm{s}) \\
& + 8a_\mathrm{s}^2\ln\widetilde{N}\,n_f\exp(2a_\mathrm{s}C_A\ln^2\widetilde{N})\Big[\left(4C_A-C_F\right)+\tfrac{1}{3}a_\mathrm{s}\ln^2\widetilde{N}\,C_A\beta_0\Big]\,.
\end{aligned} \tag{4.3}$$

where the first terms is the LL contribution. The resulting third-order x-space expression is

$$\begin{aligned}
(1-x)^{-1}\,C^T_{L,\mathrm{g}}(x)\Big|_{a_\mathrm{s}^3} = {} & 8C_FC_A^2\ln^4(1-x) \\
& +\ln^3(1-x)\,C_F\left[\tfrac{20}{3}C_F^2+\tfrac{52}{3}C_FC_A-\tfrac{952}{9}C_A^2+\tfrac{16}{9}C_An_f\right] + \ldots\,.
\end{aligned} \tag{4.4}$$

The third logs for $C^T_{L,\mathrm{g}}$ can not be derived by resumming the NLO results. The coefficient at order α_s^3, however, can be obtained be comparing the physical kernels for (F_T, F_L^T), cf. Ref. [32], to the analogous DIS results [11,13] along the lines of Refs. [7,26], yielding the continuation of Eq. (4.4)

$$\ldots + \ln^2(1-x)\,C_F\Big[(62-32\,\zeta_2)\,C_F^2-\left(\tfrac{784}{3}-32\,\zeta_2\right)C_FC_A+\tfrac{5720}{9}\,C_A^2+\tfrac{16}{3}\,C_Fn_f-\tfrac{224}{9}\,C_An_f\Big] + \ldots$$

where, as in Ref. [11], an additional $d^{abc}d_{abc}$ contribution has been suppressed for brevity. The consequences of this result and its extension to the $\ln(1-x)$ term will be discussed in Ref. [18]. The numerical size of the resummed large-N corrections is shown in Fig. 2, again using $Q^2 \simeq M_Z^2$.

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5. Discussion and Outlook

We have derived the all-order resummation of the three highest (NNL) threshold double logarithms for the off-diagonal timelike splitting functions P_{ij}^T and the coefficient functions $C_{T,g}^T$, $C_{L,g}^T$, $C_{\phi,q}^T$ and $C_{L,q}^T$ for gauge-boson and (in the heavy-top limit) Higgs exchange semi-inclusive e^+e^- annihilation (SIA). Our results for the last quantity confirm the findings obtained in Ref. [9] by a different method, while the others are new. For brevity only a part of these results have been discussed here; for a full account the reader is referred to Ref. [18].

The numerical effect of the double logarithms beyond the third order on the splitting functions in Mellin-N space is very small for a strong coupling $\alpha_s \simeq 0.12$ corresponding to a scale close to the Z-boson mass, amounting to less that 1% for $N \lesssim 20$. Here the contributions of terms beyond the fourth order are negligible. The corresponding corrections are larger, and receive noticeable contributions to order α_s^5, for the coefficient functions.

In (almost) all these cases these is no reason to believe that the N$^\ell$LL corrections for $\ell > 2$ are small compared to the present results. This is hardly surprising, given previous experience with other end-point resummations, but indicates that more terms are required in order to achieve phenomenological relevance. We hope that the present results will provide useful information for the development of more sophisticated approaches in the future.

As done at the fourth-order for non-singlet quark coefficient functions in Ref. [9] and the spacelike splitting functions in Ref. [10], it is possible to extend the present results to all higher orders in the expansion in powers of $(1-x)$, i.e., to all terms of the form $(1-x)^a \ln^{2n-\ell-n_0}(1-x)$. Consequently all large-x double logarithms in inclusive deep-inelastic scattering (DIS) and SIA are fixed by lower-order information, with the coefficient of the N$^\ell$LL contributions determined by the N$^\ell$LO fixed-order results.

Due to the somewhat different structure of the D-dimensional phase-space integrations, the present approach is unfortunately not directly applicable (beyond the leading logarithms) to the Drell-Yan lepton-pair and Higgs-boson production in proton-proton collisions. Hence further, more refined tools are required in these cases to improve upon the physical-kernel constraints of Ref. [9] and to extend those results to the flavour-singlet contributions.

The present approach can be extended, on the other hand, to high-energy (small-x) double logarithms, if neither to all quantities nor to all powers a in the analogous small-x expansion in terms of $x^a \ln^{2n-\ell-n_0} x$ terms. The NNLL resummation of the dominant x^{-1} terms in the timelike splitting functions and the fragmentation functions in SIA has been presented in Ref. [33], the results for the x^0 terms for the corresponding DIS quantities have also been derived and will be presented elsewhere [34]. Clearly more research is required on both the threshold and high-energy logarithms.

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