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Mass effect on the photon structure functions

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I discuss the one-gluon-exchange ($\alpha\alpha_s$) corrections to the four real photon structure functions $F_2^\gamma(x,Q^2)$, $F_L^\gamma(x,Q^2)$, $g_1^\gamma(x,Q^2)$ and $W_3^\gamma(x,Q^2)$ in the massive parton model. A technique based on the Cutkosky rules and the reduction of Feynman integrals to master integrals are employed. Except for F_L^γ, the NLO contributions are noticeable at large x and do not vanish at $x_{\rm max}$ which corresponds to the threshold of the massive quark pair production due to the Coulomb singularity. It is found that the first moment sum rule of g_1^γ is satisfied up to the NLO.

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1. Introduction

It is well known that, in high energy e^+e^- collision experiments, the cross section of the two-photon processes $e^+e^- \to e^+e^- +$ hadrons dominates over other processes such as the annihilation process $e^+e^- \to \gamma^* \to$ hadrons. In particular, the two-photon process in which one of the virtual photon is very far off shell (large $Q^2 \equiv -q^2$), while the other is close to the mass shell (small $P^2 \equiv -p^2$), can be viewed as a deep-inelastic electron-photon scattering where the target is a photon rather than a nucleon. In this deep-inelastic scattering off a photon target, we can study the photon structure functions, which are the analogs of the nucleon structure functions.

For a real photon ($P^2 = 0$) target, there exists four structure functions $F_2^\gamma(x,Q^2)$, $F_L^\gamma(x,Q^2)$, $g_1^\gamma(x,Q^2)$ and $W_3^\gamma(x,Q^2)$, where $x = Q^2/(2p\cdot q)$. The photon structure functions are defined in the lowest order of the QED coupling constant $\alpha = e^2/4\pi$ and they are of order α. The QCD analysis of the unpolarized structure function F_2^γ was performed in the leading order (LO) [1], and in the next-to-leading order (NLO) (the order $\alpha\alpha_s$) [2], where $\alpha_s = g^2/4\pi$ is the QCD coupling constant. On the other hand, the polarized structure function g_1^γ was calculated in QCD in the LO [3] and in the NLO [4]. However, in these analyses all the active quarks were treated as massless. At high energies the heavy charm and bottom quarks also contribute to the photon structure functions. The NLO QCD corrections due to heavy quarks have been calculated for the unpolarized photon structure functions F_2^γ and F_L^γ [5]. The heavy quark mass effects on g_1^γ were analysed at NLO in QCD in Ref.[6] by using the LO result of the massive parton model (PM). But the complete heavy quark mass effects have not yet been computed for g_1^γ and W_3^γ at NLO.

In this talk I discuss the real photon structure functions in the massive PM at NLO in QCD. In order to compute structure functions at NLO, a technique based on the Cutkosky rules [7] and the reduction of Feynman integrals to master integrals is employed. The phase space integrals of these master integrals are expressed in analytical form as much as possible so that they may serve as useful tools for the analyses of the future ILC physics.

The polarized real photon structure function g_1^γ satisfies a remarkable sum rule [8, 9, 10, 11, 12]

$$\int_0^1 g_1^\gamma(x,Q^2)dx = 0 \ . \tag{1.1}$$

In particular, applying the Drell-Hearn-Gerasimov sum rule [13] to the case of a virtual photon target and using the fact that the photon has zero anomalous magnetic moment, the authors of Ref. [12] argue that the sum rule (1.1) holds to all orders in perturbation theory in both QED and QCD. We examine whether the NLO result of g_1^γ in the massive PM satisfies this sum rule. Numerically, it is found that the sum rule (1.1) is indeed satisfied at this order. This work has been done in collaboration with Yuichiro Kiyo and Ken Sasaki [14].

2. Structure functions at NLO

We calculate the cross sections for the two photon annihilation to the heavy quark $q_H\overline{q}_H$ pairs

$$\gamma^*(q) + \gamma(p) \longrightarrow q_H + \overline{q}_H \ , \tag{2.1}$$

with one-loop gluon corrections and to the gluon bremsstrahlung processes

$$\gamma^*(q) + \gamma(p) \longrightarrow q_H + \overline{q}_H + g \ . \tag{2.2}$$

We employ the technique developed in [15]. First, following the Cutkosky rules [7], the delta-functions which appear in the phase space integrals are replaced with differences of two propagators

$$2\pi i\delta(r^2-m^2) \to \frac{1}{r^2-m^2+i0} - \frac{1}{r^2-m^2-i0}\ , \tag{2.3}$$

where m is the heavy quark mass. Then the cross sections for the virtual corrections to the processes (2.1) and for the bremsstrahlung processes (2.2) are described by the two-loop diagrams shown in Fig.1 and Fig.2, respectively, where a cut propagator should be understood as the r.h.s. of Eq.(2.3). We regularize the amplitudes by dimensional regularization $D = 4-2\varepsilon$. Then we

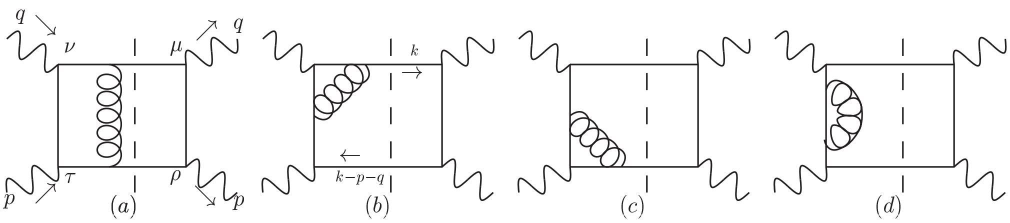

Figure 1: Two-loop diagrams with virtual corrections. Graphs with virtual corrections to the right of the cut lines and graphs with (q,μ) and (p,ρ) interchanged are added. Graphs with the external quark self-energies are not shown in the Figure, but should be included in the calculation.

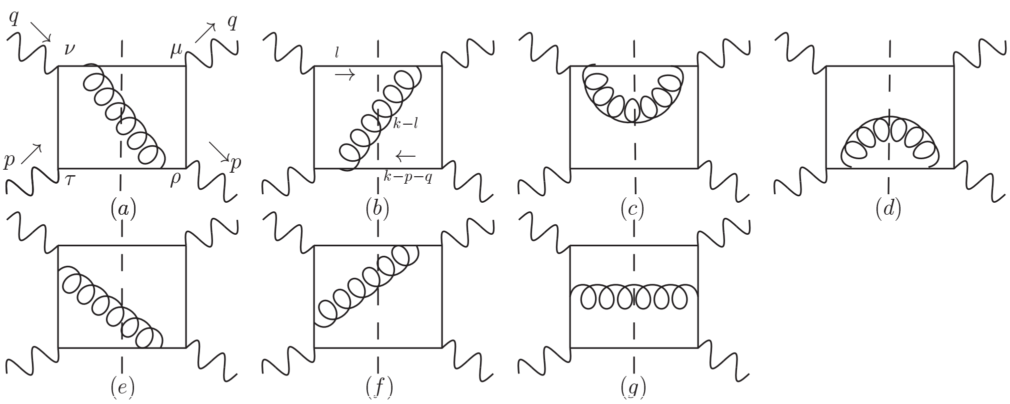

Figure 2: Two-loop diagrams with a real gluon emission. Similar graphs corresponding to (e) and (f) are included. Also graphs with (q,μ) and (p,ρ) interchanged are added.

apply the D-dimensional projection operators to these diagrams to extract the contributions to the corresponding structure functions. They are expressed in terms of a large number of two-loop scalar integrals and the coefficients of these integrals are written as functions of x, Q^2, m^2 and D. For the diagrams in Fig.1, we arrange the integration variables k and l so that the cut propagators are $1/[k^2-m^2]$ and $1/[(k-p-q)^2-m^2]$. Among many integrals, there appear those with one or both of the cut propagators eliminated. Those integrals do not contribute to the structure functions and are discarded. A similar procedure is applied to the diagrams in Fig.2. We choose $1/[l^2-m^2]$, $1/[(k-p-q)^2-m^2]$ and $1/(k-l)^2$ for the cut propagators.

The number of the relevant integrals is still large. Then, following the reduction procedure [16] which is based on the method of integration by parts [17] and the use of the Lorentz invariance of scalar integrals [18], these relevant integrals can be expressed in terms of fewer number of master integrals. We make use of FIRE [19] which is one of the public codes available today.

Finally we perform the phase space integrations by taking the discontinuities of these master integrals. There appear 61 master integrals in total in this analysis. However, the choice of a set of master integrals is not unique. We are at liberty to replace a master integral with one of the other scalar integrals. We choose a set of master integrals such that each corresponding coefficient function is finite in the limit $D \to 4$ [20]. With this choice of the set, the phase space integrations for master integrals need only be evaluated up to the finite terms in the series expansion in ε.

When the virtual correction diagrams in Fig.1 are concerned, the ultraviolet (UV) singularities appear in the graphs (b), (c) and (d), while the infrared (IR) singularity emerges from the graph (a). Both the UV and IR singularities are regularized by dimensional regularization. The UV singularities are removed by renormalization. We adopt the on-shell scheme both for the wave function renormalization of the external quark and the mass renormalization. For the latter, we replace the bare mass in the Born cross section by the renormalized mass m,

$$m_{\rm bare} \to m \left[1+\frac{\alpha_s}{4\pi}C_F S^{\varepsilon}\left(\frac{\mu^2}{m^2}\right)^{\varepsilon}\left\{-\frac{3}{\varepsilon}-4\right\}\right] , \tag{2.4}$$

where $C_F = \frac{4}{3}$ is the Casimir factor, $S^{\varepsilon} = (4\pi)^{\varepsilon} e^{-\varepsilon\gamma_E}$ with Euler constant γ_E and μ is the arbitrary reference scale of dimensional regularization.The renormalization of the QCD gauge coupling constant is not necessary at this order. The IR singularities appear also in the real gluon emission graphs (a),(b), (c) and (d) of Fig.2. However, the IR singularities cancel when the both contributions from the virtual correction graphs and the real gluon emission graphs are added.

3. Numerical results

In Figs.3, 4, 5 and 6 we plot the real photon structure functions $F_2^{\gamma}(x,Q^2)$, $F_L^{\gamma}(x,Q^2)$, $g_1^{\gamma}(x,Q^2)$ and $W_3^{\gamma}(x,Q^2)$, respectively, which are predicted by the massive PM up to the NLO for the case of $Q^2 = 20$ GeV^2 and $\alpha_s = 0.22$. We choose charm, c, as a heavy quark and take $m_c = 1.3$ GeV and $e_c = \frac{2}{3}$. Here we show three curves: the LO result(dotted line), the sum of LO and NLO corrections(solid line) and the NLO corrections alone(dashed line). The allowed x region is $0 \le x \le x_{\rm max}$ with

$$x_{\rm max} = \frac{1}{1+\frac{4m^2}{Q^2}} . \tag{3.1}$$

We observe that the radiative corrections to F_2^{γ}, g_1^{γ} and W_3^{γ} shown in the Figs.3, 5 and 6, respectively, are large near the threshold (near $x_{\rm max}$) and the corresponding NLO curves do not vanish at $x_{\rm max}$. This is due to the well-known Coulomb singularity, which appears when the Coulomb gluon is exchanged between the quark and anti-quark pair near threshold. The diagram Fig.1(a) is responsible for this threshold behaviour. The virtual correction to the left of the cut line in Fig.1(a) gives rise to a factor $1/\beta$ while a factor β comes out from the phase space integration. They are combined and yield a finite but non-zero result at $x_{\rm max}$. On the other hand, radiative corrections to F_L^{γ} evade the Coulomb singularity and vanish at threshold since there is no s-wave contribution.

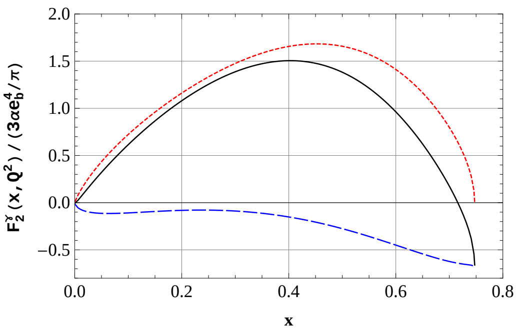

Figure 3: Charm quark effects on the unpolarized structure function $F_2^\gamma(x,Q^2)$ for $Q^2 = 20$ GeV^2

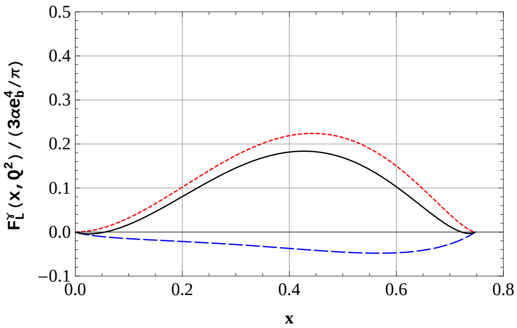

Figure 4: Charm quark effects on the unpolarized longitudinal structure function $F_L^\gamma(x,Q^2)$ for $Q^2 = 20$ GeV^2

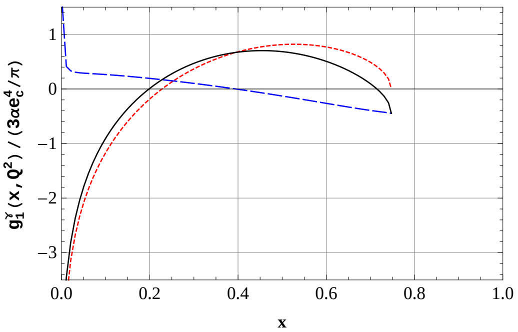

Figure 5: Charm quark effects on the polarized structure function $g_1^\gamma(x,Q^2)$ for $Q^2 = 20$ GeV^2

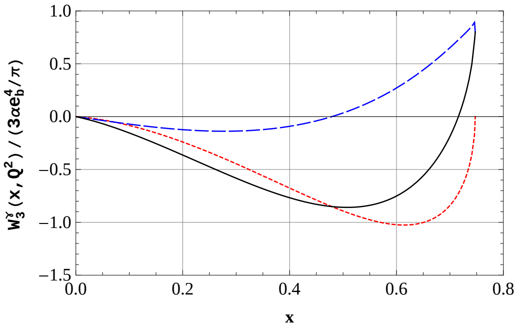

Figure 6: Charm quark effects on the helicity flipped structure function $W_3^\gamma(x,Q^2)$ for $Q^2 = 20$ GeV^2

Finally, the sum rule (1.1) for $g_1^\gamma(x,Q^2)$ is considered. It is known that the sum rule holds [8, 9, 10, 11, 12] at LO. Fig.5 shows that the sum rule also seems to be satisfied by the NLO contribution. Expressing the NLO contribution as $g_1^\gamma(x,Q^2)|_{\rm NLO}$, we find numerically

$$\int_0^{x_{\rm max}} g_1^\gamma(x,Q^2)|_{\rm NLO} dx = 0\ . \tag{3.2}$$

But due to the limitation of accuracy of our numerical integration, we observed

$$\delta \equiv \frac{\int_0^{x_{\rm max}} g_1^\gamma(x,Q^2)|_{\rm NLO} dx}{\int_0^{x_{\rm max}} \left| g_1^\gamma(x,Q^2)|_{\rm NLO} \right| dx} = 2.2\times 10^{-4}\ .$$

So we conclude that the sum rule is satisfied in the massive PM up to the NLO.

However, if we go on further and analyse g_1^γ to higher orders in perturbation theory, we expect that the result will diverge at $x_{\rm max}$ due to the Coulomb singularity. A detail analysis on the structure of the Coulomb singularity tells that $g_1^\gamma|_{\rm NNLO} \sim \beta\times(\alpha_s/\beta)^2$ [21, 22] whose integral for the first moment is ill-defined due to end-point singularity at $x = x_{\rm max}$. The sum rule is not well-defined in the perturbation theory starting at NNLO. To obtain an appropriate threshold behavior for photon structure functions, we may resort to the method of resummation of the Coulomb singularities. A noticeable difference in the resummation is emergence of bound-state poles of $q_H\overline{q}_H$ above $x_{\max}$. Then the left-hand side of the sum rule Eq. (1.1) should include also the bound-state contributions. We will not pursue this issue further here but render it to our future publications.

4. Conclusion

In summary we have calculated the NLO corrections to the four photon structure functions $F_2^\gamma(x,Q^2)$, $F_L^\gamma(x,Q^2)$, $g_1^\gamma(x,Q^2)$ and $W_3^\gamma(x,Q^2)$ in the massive PM. We have found that the NLO contributions are noticeable at large x and do not vanish at $x_{\max}$ due to the Coulomb singularity to F_2^γ, g_1^γ and W_3^γ. We have also found numerically that the sum rule (1.1) is satisfied up to the NLO in the massive PM. It is pointed out that the sum rule may not be well-defined when g_1^γ is analysed to higher orders in perturbation theory.

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