PROCEEDINGS OF SCIENCE

Deconfinement transition at neutron star cores: the role of fluctuations

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The deconfinement of hadronic matter into quark matter in a protoneutron star is studied focusing on finite size effects on the formation of just-deconfined droplets. We employ the Nambu-Jona-Lasinio and the Walecka models in the description of quark and hadron matter respectively. We show that fluctuations in the energy-density of protoneutron star matter are much more relevant for deconfinement than fluctuations in the temperature and the neutrino density. We calculate the critical size spectrum of energy-density fluctuations that allows deconfinement as well as the nucleation rate of each critical bubble. It is shown that drops with any radii smaller than 800 fm can be formed at a huge rate when matter achieves the bulk transition limit of 5−6 times the nuclear saturation density. This paper is a compilation and discussion of the main results published by Lugones & Grunfeld in Phys. Rev. D84, 085003 (2011) that were presented in the XXXIV edition of the Brazilian Workshop on Nuclear Physics.

XXXIV edition of the Brazilian Workshop on Nuclear Physics,
5-10 June 2011
Foz de Iguaçu, Parana state, Brasil

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PoS(XXXIV BWNP)040

1. Introduction

The high-density environment at the core of neutron stars may allow the nucleation of small droplets of quark matter, that under appropriate conditions may grow converting a large part of the star into the quark phase [1, 2, 3, 4, 5, 6].

The nucleation of a small droplet is driven by strong interactions; thus, quark and lepton flavors must be conserved during the process leaving just deconfined quark matter that is transitorily out of equilibrium with respect to weak interactions [1]. When color superconductivity is included in the analysis together with flavor conservation, it is found that the most likely configuration of the just deconfined phase is a two-flavor color superconductor (2SC) provided the pairing gap is large enough [7]. In this work, we show how finite size effects enter in the description of the just deconfined drops. To this end we employ the Nambu-Jona-Lasinio (NJL) model for quark matter and include finite size effects within the multiple reflection expansion (MRE) framework [8, 9, 10, 1]. Through this analysis we can determine the density of hadronic matter at which deconfinement is possible for different radii of the just formed quark drops using typical conditions expected in the interior of protoneutron stars (PNSs), i.e. temperatures in the range of $0-60$ MeV and chemical potentials of the trapped neutrino gas up to 200 MeV. Notice that to form a finite size drop some over-density is needed with respect to the bulk transition density in order to compensate the surface and curvature energy cost. Since this energy cost depends on the drop radius R, so does the necessary over-density necessary to nucleate it. Thus, we can derive a critical fluctuation spectrum ($\delta\rho/\rho$ versus R) delimiting which over-densities can deconfine and grow unlimitedly and which ones will shrink back to hadronic matter.

2. Equations of state

We adopt a two-phase model in which hadronic and quark matter are described by different equations of state.

The hadronic phase is modeled by the non-linear Walecka model including the whole baryon octet, electrons and neutrinos with the parametrizations GM1 and GM4 given in [1]. The equation of state is rather stiff and gives a maximum mass of 2 $M_\odot$, that seems to be adequate in light of the recently determined mass of the pulsar PSR J1614-2230 with $M = 1.97 \pm 0.04 M_\odot$ [11]. This is the largest mass reported ever for a pulsar with a high precision.

The just deconfined quark matter phase is described by a $SU(3)_f$ NJL effective model with the inclusion quark-quark interactions, which are responsible for color superconductivity [1]. The effect of finite size is included in the thermodynamic potential adopting the MRE formalism [1]. The modified density of states of a finite spherical droplet is given by

$$\rho_{MRE}(k, m_f, R) = 1 + \frac{6\pi^2}{kR} f_S + \frac{12\pi^2}{(kR)^2} f_C \tag{2.1}$$

where $f_S = -\frac{1}{8\pi}\left(1 - \frac{2}{\pi}\arctan\frac{k}{m_f}\right)$ and $f_C = \frac{1}{12\pi^2}\left[1 - \frac{3k}{2m_f}\left(\frac{\pi}{2} - \arctan\frac{k}{m_f}\right)\right]$ are the surface and curvature contributions to the new density of states respectively.

The full thermodynamic potential reads

$$\frac{\Omega^Q_{MRE}}{V} = 2\int_{\Lambda_{IR}}^{\Lambda} \frac{k^2 dk}{2\,\pi^2} \rho_{MRE} \sum_{i=1}^{9} \omega(x_i, y_i) + \frac{1}{4G}(\sigma_u^2 + \sigma_d^2 + \sigma_s^2) + \frac{|\Delta|^2}{2H} - P_e - P_{\nu_e} + P_{\text{vac}} \tag{2.2}$$

where Λ_{IR} and Λ are the cutoffs of the model and $\omega(x,y)$ is defined by

$$\omega(x,y) = -[x + T\ln[1+e^{-(x-y)/T}] + T\ln[1+e^{-(x+y)/T}]], \tag{2.3}$$

with

$$\begin{aligned} x_{1,2} &= E, \quad x_{3,4,5} = E_s, \\ x_{6,7} &= \sqrt{\left[E \pm \frac{(\mu_{ur}+\mu_{dg})}{2}\right]^2 + \Delta^2}, \quad x_{8,9} = \sqrt{\left[E \pm \frac{(\mu_{ug}+\mu_{dr})}{2}\right]^2 + \Delta^2}, \\ y_1 &= \mu_{ub}, \quad y_2 = \mu_{db}, \quad y_3 = \mu_{sr}, \quad y_4 = \mu_{sg}, \quad y_5 = \mu_{sb}, \\ y_{6,7} &= \frac{\mu_{ur}-\mu_{dg}}{2}, \quad y_{8,9} = \frac{\mu_{ug}-\mu_{dr}}{2}. \end{aligned} \tag{2.4}$$

Here, $E = \sqrt{k^2+M^2}$ and $E_s = \sqrt{k^2+M_s^2}$, where $M_f = m_f + \sigma_f$. Note that in the isospin limit we are working $\sigma_u = \sigma_d = \sigma$ and, thus, $M_u = M_d = M$. To obtain the total thermodynamic potential of the quark matter phase the contribution of electrons $-P_e$, neutrinos $-P_{\nu_e}$, as well as a vacuum constant $+P_{\text{vac}}$ has been added. The values of the quark masses, the coupling constants, and the cutoffs Λ_{IR} and Λ are given in Ref. [1].

From the grand thermodynamic potential Ω^Q_{MRE} we can readily obtain the number density of quarks of each flavor and color $n_{fc} \equiv -V^{-1}\partial\Omega^Q_{MRE}/\partial\mu_{fc}$, the number density of electrons $n_e = -V^{-1}\partial\Omega^Q_{MRE}/\partial\mu_e$, and the number density of electron neutrinos $n_{\nu_e} = -V^{-1}\partial\Omega^Q_{MRE}/\partial\mu_{\nu_e}$. The corresponding number densities of each flavor, n_f, and of each color, n_c, in the quark phase are given by $n_f = \sum_c n_{fc}$ and $n_c = \sum_f n_{fc}$ respectively. The baryon number density reads $n_B = \frac{1}{3}\sum_{fc} n_{fc} = (n_u+n_d+n_s)/3$. The Gibbs free energy per baryon is $g = \frac{1}{n_B}(\sum_{fc}\mu_{fc}\, n_{fc} + \mu_e\, n_e + \mu_{\nu_e}\, n_{\nu_e})$.

Finally, the pressure P^Q is given by

$$P^Q \equiv -\left.\frac{\partial\Omega^Q_{MRE}}{\partial V}\right|_{T,\mu,S,C} = -2\int_{\Lambda_{IR}}^{\Lambda} \frac{k^2 dk}{2\,\pi^2} \sum_{i=1}^{9} \omega(x_i,y_i) - \frac{1}{4G}(\sigma_u^2+\sigma_d^2+\sigma_s^2) - \frac{|\Delta|^2}{2H} + P_e + P_{\nu_e} - P_{\text{vac}}, \tag{2.5}$$

the surface tension is

$$\alpha \equiv \left.\frac{\partial\Omega^Q_{MRE}}{\partial S}\right|_{T,\mu,V,C} = 2\int_{\Lambda_{IR}}^{\Lambda} k dk f_S \sum_{i=1}^{9} \omega(x_i,y_i), \tag{2.6}$$

and the curvature energy density is

$$\gamma \equiv \left.\frac{\partial\Omega^Q_{MRE}}{\partial C}\right|_{T,\mu,V,S} = 2\int_{\Lambda_{IR}}^{\Lambda} dk f_C \sum_{i=1}^{9} \omega(x_i,y_i). \tag{2.7}$$

Here we are considering a spherical drop, i.e. the area is $S = 4\pi R^2$ and the curvature is $C = 8\pi R$.

3. Conditions for deconfinement

In order to study just deconfined matter, a suitable number of conditions must be imposed on the variables $\{\mu_{fc}\}, \mu_e, \mu_{\nu_e}, \sigma, \sigma_s$ and Δ in the formulae of the previous section. Three of these conditions are consequences from the fact that the thermodynamically consistent solutions correspond to the stationary points of Ω^Q_{MRE} with respect to σ, σ_s, and Δ. Thus, we have

$$\frac{\partial \Omega^Q_{MRE}}{\partial \sigma} = 0, \qquad \frac{\partial \Omega^Q_{MRE}}{\partial \sigma_s} = 0, \qquad \frac{\partial \Omega^Q_{MRE}}{\partial |\Delta|} = 0. \tag{3.1}$$

The condition of flavor conservation between hadronic and deconfined quark matter is written as

$$Y^H_f = Y^Q_f \qquad f = u, d, s, e, \nu_e \tag{3.2}$$

being $Y^H_f \equiv n^H_f / n^H_B$ and $Y^Q_i \equiv n^Q_f / n^Q_B$ the abundances of each particle in the hadron and quark phase respectively. It means that the just deconfined quark phase must have the same "flavor" composition than the β-stable hadronic phase from which it has been originated. Notice that, since the hadronic phase is assumed to be electrically neutral, flavor conservation ensures automatically the charge neutrality of the just deconfined quark phase.

Additionally, the deconfined phase must be locally colorless; thus it must be composed by an equal number of red, green and blue quarks: $n_r = n_g = n_b$. Also, ur, ug, dr, and dg pairing will happen provided that $|\Delta|$ is nonzero, leading to $n_{ur} = n_{dg}$ and $n_{ug} = n_{dr}$. In order to have all Fermi levels at the same value, we consider [7] $n_{ug} = n_{ur}$ and $n_{sb} = n_{sr}$ leading to $n_{ur} = n_{ug} = n_{dr} = n_{dg}$ and $n_{sr} = n_{sg} = n_{sb}$ [7].

For a spherical droplet the condition for mechanical equilibrium reads [1]:

$$P^Q - \frac{2\alpha}{R} - \frac{2\gamma}{R^2} - P^H = 0. \tag{3.3}$$

Notice that in the bulk limit $R \to \infty$ we find the standard bulk condition $P^H = P^Q$.

Finally, we assume thermal and chemical equilibrium, i.e. the Gibbs free energy per baryon are the same for both hadronic matter and quark matter at a given common temperature. Thus, we have

$$g^H = g^Q \, , \qquad T^H = T^Q \, . \tag{3.4}$$

If we fix the radius R of the deconfined drop for a given temperature T^H and neutrino chemical potential of the trapped neutrinos in the hadronic phase $\mu^H_{\nu_e}$, there is an unique hadronic pressure P^H at which the equilibrium conditions are fulfilled. In the present work we want to describe thermodynamic conditions analogous to those encountered in protoneutron stars; thus, we use $T \lesssim 60$ MeV and $\mu^H_{\nu_e} \lesssim 200$ MeV.

4. Fluctuations and deconfinement

According to the theory of homogeneous nucleation, the free energy involved in the formation of a spherical quark bubble of radius R is given by [1]

$$\Delta\Omega = -\frac{4\pi}{3} R^3 \Delta P + 4\pi\alpha R^2 + 8\pi\gamma R, \tag{4.1}$$

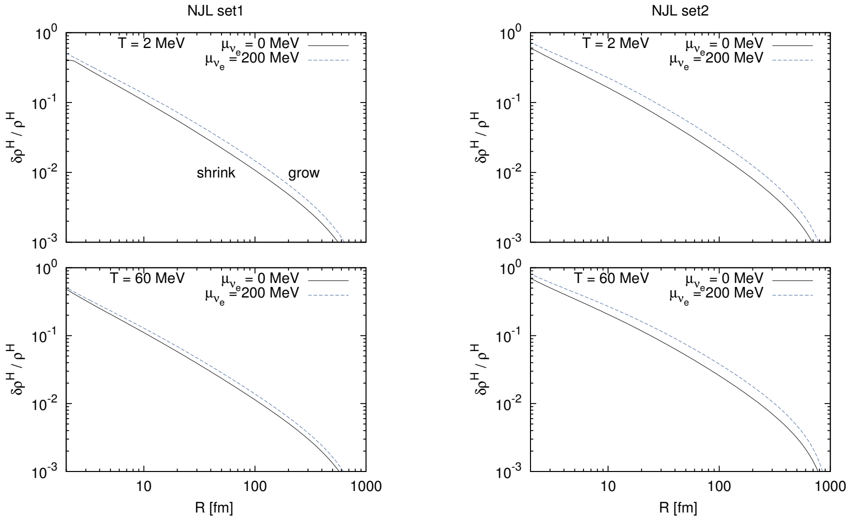

Figure 1: Critical spectrum for fluctuations that allow deconfinement of hadronic matter in a protoneutron star. The results were calculated using GM4 + NJL *set1* on the left panel and GM4 + NJL *set 2* on the right panel. The spectrum does not depend significantly on the temperature and the chemical potential of trapped neutrinos. Fluctuations in hadronic matter having a given $\delta\rho^H/\rho^H$ are able to grow if they have a size R larger than the here-shown critical one.

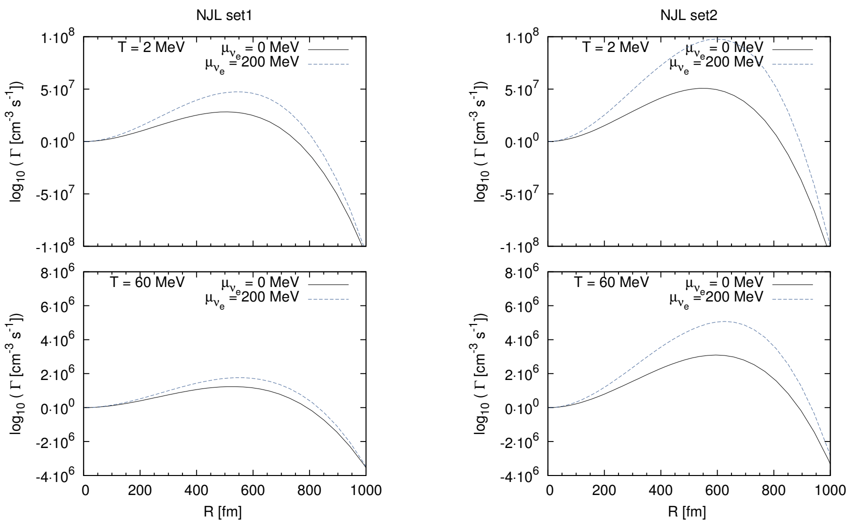

Figure 2: Nucleation rate for bubbles of the critical size.

where $\Delta P = P^Q - P^H$ is the pressure difference between internal and external parts of the bubble. For given ΔP, α and γ, the extremal points (maximum or minimum) of $\Delta\Omega$ are obtained from $\partial\Delta\Omega/\partial R = 0$, which leads to Eq. (3.3). Thus, the critical radii are given by:

$$R_{\pm} = \frac{\alpha}{\Delta P}\left(1 \pm \sqrt{1+b}\right), \tag{4.2}$$

with $b \equiv 2\gamma\Delta P/\alpha^2$.

For $b < -1$ both solutions are complex and $\Delta\Omega$ is a monotonically decreasing function of R. This means that any small fluctuation of one phase into the other will gain energy by expanding and a rapid phase transition is likely to occur. For $b \geq -1$, $\Delta\Omega$ has a local minimum at R_- and a local maximum at R_+. For $\Delta\Omega(R_+) < 0$ we have again that any small fluctuation is energetically favored. For $\Delta\Omega(R_+) > 0$ bubbles with radii larger than R_+ gain energy by growing unlimitedly, while those below the critical size gain energy by shrinking to zero (if $R_- < 0$) or to R_- (if $R_- > 0$). In this case, the standard assumption in the theory of bubble nucleation in first order phase transitions is that bubbles form with a critical radius R_+ [1].

The approach we adopted in the previous section is closely related to what we explained in the above paragraph. Instead of finding the critical radius for arbitrary values of ΔP, α and γ, we fixed R and found the corresponding ΔP, α and γ that satisfy the conditions presented in the previous section. Since Eq. (3.3) is satisfied by construction, the radius R is precisely the critical radius R_+ introduced in Eq. (4.2), because we choose the solution that verifies $\partial^2\Delta\Omega/\partial R^2 < 0$.

Now, we consider hadronic matter at the bulk transition density, i.e. the density for which deconfinement is possible if $R \longrightarrow \infty$. While deconfinement is energetically favored in this case, this is not possible in practice because real drops have a finite size, and there is a surface and curvature energy cost for nucleating them. However, energy-density fluctuations with radius R that drive some part of the hadron fluid to a density $\rho^H_* = \rho^H_{bulk} + \delta\rho^H$ may be energetically favored. To quantify this, we construct a critical spectrum $\delta\rho^H/\rho^H$ as a function of R for different values of T and $\mu^H_{\nu_e}$ as seen in Fig. 1. Fluctuations of a given over-density $\delta\rho^H/\rho^H$ must have a size larger than the critical value given in Fig. 1 in order to grow. Equivalently, fluctuations of a given size must have an over-density $\delta\rho^H/\rho^H$ larger than the critical one for that size [1].

We can calculate the formation rate of critical bubbles through

$$\Gamma \approx T^4 \exp(-\delta\Omega_c/T). \tag{4.3}$$

where in our case $\delta\Omega_c$ is the work required to form a quark bubble with the critical radius from hadronic matter at the bulk transition point

$$\delta\Omega_c \equiv -\frac{4\pi}{3}R^3(P^Q - P^H_{bulk}) + 4\pi\alpha R^2 + 8\pi\gamma R, \tag{4.4}$$

Instead of T^4, different prefactors are used for Γ in other works [3]. However, this fact does not affect significantly the results because Γ is largely dominated by the exponent in Eq. (4.3); i.e. we always have $\log_{10}\Gamma \approx \log_{10}(\text{prefactor}) - \delta\Omega_c/[T\ln(10)]$ with the second term much larger than the first.

The results are given in Fig. 2 and show that critical bubbles with $R \gtrsim 800$ fm are strongly disfavored while those with $R \lesssim 800$ fm have a huge rate. In practical situations, i.e. at neutron star cores, this means that if fluctuations lead hadronic matter to the bulk transition point, quark drops with $R \lesssim 800$ fm will nucleate instantaneously.

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5. Summary and Conclusions

When the bulk transition density is reached in the core of a neutron star, it is energetically favored to convert a macroscopically large portion of hadronic matter into quark matter. But in practice, it is needed some over-density with respect to the bulk transition density in order to compensate the surface and curvature energy cost of a finite drop. Since this energy cost depends on the drop radius, so does the necessary over-density and over-pressure necessary to nucleate it. Thus, we can derive a critical fluctuation spectrum $\delta\rho^H/\rho^H$ versus R delimiting which fluctuations are able to grow unlimitedly and which will shrink (see Fig. 1). Typically, fluctuations of $\delta\rho^H/\rho^H \sim 0.001 - 0.1$ above the bulk point are needed for the nucleation of drops with $R \sim 10 - 1000$ fm. However, the nucleation rates Γ vary over several orders of magnitude. Our results show that drops with $R \sim 2 - 800$ fm have a huge nucleation rate while those with $R \gtrsim 800$ fm are strongly suppressed (see Fig. 2).

We can also show [1] that fluctuations in the temperature and in the chemical potential of trapped neutrinos are not very important for deconfinement. This is in contrast with previous results found within the frame of the MIT Bag model (e.g. in [12] it is argued that nucleation is suppressed at $T \lesssim 2$ MeV and in [13] it is found that neutrino trapping precludes deconfinement). Instead, fluctuations in the energy density are the more efficient way to trigger the transition.

Notice that the nucleation rate and the typical radii of deconfined drops are also very different from the values found within the MIT Bag model (see e.g. [3] and references therein). The drops studied in [3] have typically radii less than 10 fm and a long nucleation time, in contrast ours may have much larger radii and nucleate almost instantaneously. This is due to the use of different equations of state for the quark phase as well as for the different treatments of the surface and curvature terms. While in [3] the surface tension is assumed to be constant (α = 30 MeV fm^{-2}), in our work α and γ are calculated self consistently within the MRE formalism resulting non-constant values around 140 MeV fm^{-2} and 110 MeV fm^{-1} respectively. Since our surface tension is larger, larger critical drops are obtained. For $R \lesssim 800$ fm, $\delta\Omega_c$ is negative and the exponents of Eq. (4.3) are large positive numbers that result in huge nucleation rates. In the context of protoneutron stars the main conclusion is that if the bulk transition point is attained near the star centre, quark matter drops with $R \lesssim 800$ fm will nucleate instantaneously. Since the bulk transition density is $\sim 5 - 6\rho_0$, this should happen for stars with masses larger than $\sim 1.5 - 1.6 M_\odot$.

Acknowledgements

G. Lugones acknowledges the financial support received from FAPESP (Brazil).

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