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Epicyclic Frequencies, Resonances & QPOs

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The idea that observed HFQPOs arise as a manifestation of some sort of orbital motion in the accretion flow has received a number of supporting arguments. The omnipresent signature of a special 3:2 ratio in HFQPO frequencies led to the idea of a resonance between two modes of disk epicyclic frequencies and still today that idea represents one of the most favourite explanations of the phenomenon. This article reviews some observational facts and results that have been obtained within the epicyclic resonance theory and draw down some ideas about directions of the future research with respect to high precision timing measurements.

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1. Introduction (what's the story)

The idea of an orbital resonance as an explanation for the high-frequency QPOs has been put forward by Kluzniak & Abramowicz (2001) and Abramowicz & Kluźniak (2001) who were motivated by a discovery of one HFQPO peak in a black-hole source GRO 1655−40 (Sobczak et al., 1999). Soon after that, a second QPO to the pair has been detected from that source and the ratio of the two frequencies turned out to be 3:2 (Strohmayer, 2001). Another observations of another sources followed, not only black-hole sources but neutron-star sources as well, which showed a similar thing although with somewhat different properties. Some properties of the double peak QPOs reflect signatures of a kind of non-linear resonance between oscillation modes in accretion flows in strong gravity, possibly those modes can be epicyclic oscillations of accretion disks. Several works showed (Horak, 2004; Rebusco, 2004; Horák et al., 2004; Abramowicz et al., 2003, ...) that a few generic types of non-linear orbital resonances consistent with the $3/2$ ratio are possible to excite and persist in the flow. In particular a parametric resonance between ω_r and ω_v, the two epicyclic frequencies, has been identified as the most interesting one (necessarily gives 3:2 ratio and can be excited easily – the two modes couple).

In strong gravity, the typical length-scale is the gravitational radius, $r_\mathrm{G} \sim M$, and the typical velocity is the speed of light, $c = 1$. Thus, the typical frequency is $\nu \sim c/r_\mathrm{G} \sim 1/M$ and scales inversely proportional with mass. This is also the case for the Keplerian orbital frequency and for all other frequencies of a test particle motion in a gravitational field. Perturbations to a perfectly circular test particle geodesic orbit give rise to a epicyclic motion in radial and vertical direction with frequencies given by

- $\Omega_\mathrm{K} = 1/M\left(r^{3/2}+a\right)^{-1}$
- $\omega_\mathrm{r} = \Omega_\mathrm{K}\left(1-6/r+8ar^{-3/2}-3a^2r^{-2}\right)^{1/2}$
- $\omega_\mathrm{v} = \Omega_\mathrm{K}\left(1-4ar^{-3/2}+3a^2r^{-2}\right)^{1/2}$

Newtonian gravity is scale free, with no preferred frequency. If we had a static star with a thin accretion disk around it, the only observable characteristic frequency would be $\Omega_\mathrm{K}(R_\star)$, that is the Keplerian orbital frequency in the disk at the radius of the star surface. General relativity, however, does have a characteristic scale, GM/c^2 and characteristic frequencies: the radial epicyclic frequency ω_r has a maximum and goes to zero at ISCO, thus $\Omega_\mathrm{K}(r_{ISCO})$ and $\omega_\mathrm{r}|_\mathrm{max}$ are two (fixed) characteristic frequencies for a relativistic object of mass M and having angular momentum J. In sufficiently complicated coupled systems we can also have resonances (forced/parametric) which can give rise to other characteristic frequencies apart from these two.

Coming back to quasi-periodic oscillation and the epicyclic resonance theory, the simplest possibility to obtain a 3:2 pair is that the two observed QPO frequencies correspond directly to the vertical and radial epicyclic frequencies in 3:2 ratio. This condition holds at a specific radius determined by the condition $3\omega_r(r_{3:2},a) = 2\omega_\theta(r_{3:2},a)$ and the spin $a = J/M$ is a free parameter here. From the theory (Mathieu equation) one knows that the parametric resonance is excited when $\omega_\mathrm{r}/\omega_\mathrm{v} = 2/n$, where n is a natural number, $n = 1,2,3,...$; and since it has to be that $\omega_\mathrm{r} < \omega_\mathrm{v}$ we arrive to a restriction for $n \geq 3$ and $3:2$ comes out as the strongest resonance allowed. A direct

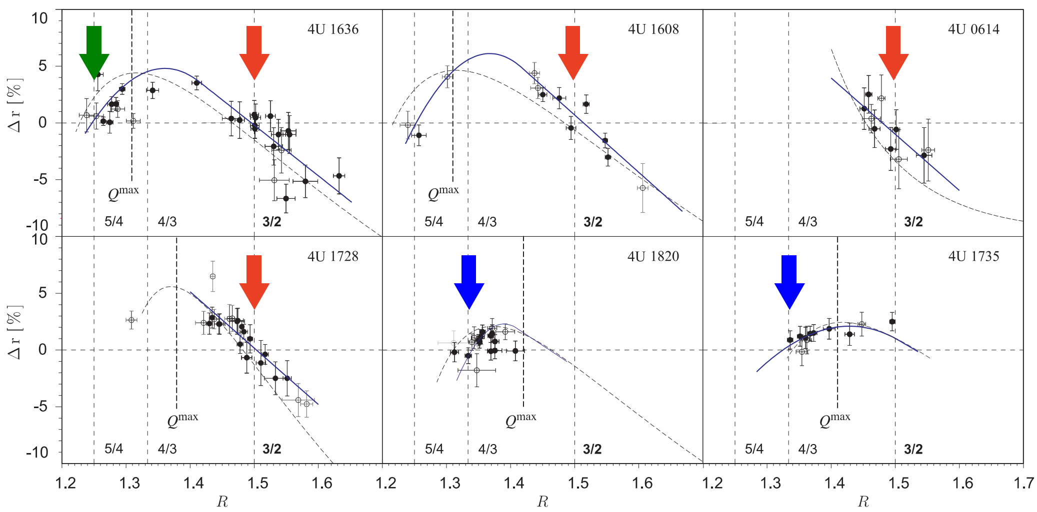

Figure 1: The amplitude difference $A(\nu_U) - A(\nu_L)$ as a function of frequency ratio plotted for several NS atol sources. Arrows show where the amplitudes equal and quite remarkably this happens at prominent frequency ratios. (Adopted from Török 2009.)

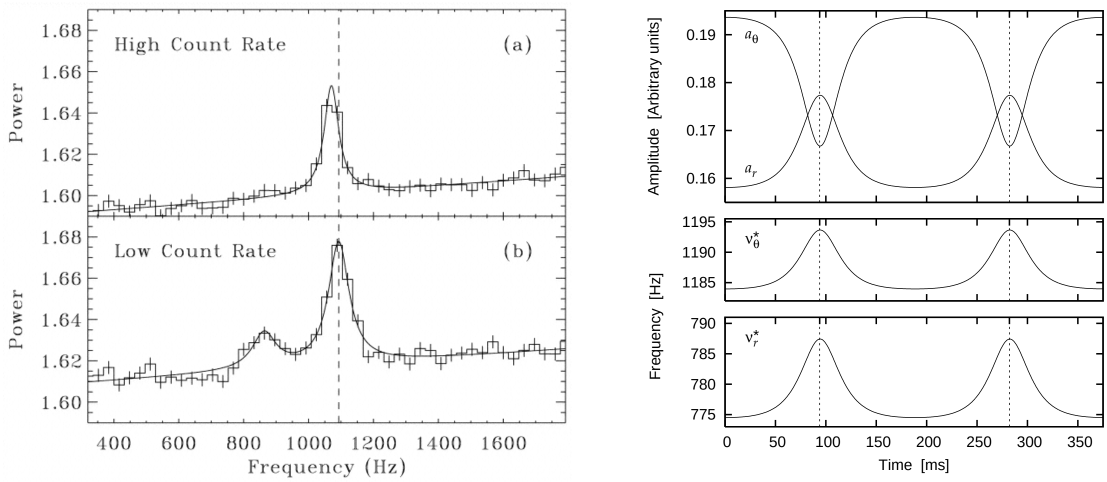

Figure 2: The observation of a slight HFQPO frequency shift observed in Sco X-1 (left; adopted from van der Klis 2002), which resembles low-frequency modulation phenomenon in a non-linear resonance (right; adopted from Horák 2004).

resonant forcing of ω_v by ω_r can also be realized through a pressure coupling (which produces combination frequencies with ratios 2:1 or 3:1). Another possibility are Keplerian resonances between ω_r and Ω_K, however physical coupling of those is more difficult to imagine (co-rotation resonance or vortex flow have been discussed in this context).

The largest support for the resonance theory have come perhaps from two findings that have been made analysing data of NS sources. Török (2009) reported that the amplitudes of the the QPO frequencies become equal when pair passes through a certain point – usually a point where $\nu_U/\nu_L = 3{:}2$, but also $\nu_U/\nu_L = 5{:}3$ or $4:3$ in few cases (Figure 1). Such behaviour is exactly what is expected in a non-linear resonant system (Horák et al., 2009). Second evidence comes from low-frequency variations of HFQPOs that have been observed in Sco X-1, 4U 1608-52 and possibly also in BH source XTE J1150−564 (Yu et al., 2001; Yu & van der Klis, 2002; Yu et al., 2003), which again is a natural consequence of non-linear resonance (Figure 2; Horak 2004).

2. Methods of Estimating BH Spins

In principle, there are several independent ways which could be used to estimate BH spin. They mostly involve various properties of X-ray signal which comes to us from an accretion disk.

The problem with measuring spin is that spin is a strong-gravity effect which dribbles out quickly as r^3, much more rapidly than the effect of gravity. This is the reason why we can use Newtonian limit for measuring mass of BHs by observing their environment even if it is quite far (thousands of gravitational radii) from BH, but for measuring BH spins we need to deliver test particles to the very vicinity of the central compact object. This is why accretion disks are so nice – they do this though job of removing angular momentum and transporting test particles (gas) throughout the disk to its inner edge. By observing this innermost area we can gain useful information about the environment there and from fluid motion we can deduce properties of BH through spacetime characteristics. This is also why most methods work in X-ray part of electromagnetic spectrum. During the process of accretion, the potential energy of particles is continuously dissipated to heat and temperatures in inner parts of accretion disk are high enough that the gas radiates in X-rays. Thus by selecting X-ray band we effectively apply a filter which windows only radiation coming from the most interesting part of accretion disk of few gravitational radii from the central compact object.

Currently, the two mostly used methods of estimating BH spins involve either fitting a fluorescent line emission of iron K-alpha line or fitting the whole X-ray continuum. But there are other possibilities too. We could use QPO timing for that, X-ray polarimetry, direct imaging and few more indirect methods. Except for the timing, all other methods still wait for future use when necessary instrumentation will become available.

The situation with QPO timing is much more interesting. It seems that each source showing HFQPO has a unique nature-given pair frequencies that are stable in time. The most striking example is black hole candidate source GRO 1655−40, which exhibited 300 and 450 Hz pair of QPOs in its 1996 outburst and then, 9 years later in 2005, it showed the same frequencies in another outburst. Moreover, these QPOs are also stable over a huge range of luminosities. And it is mainly the stability of QPOs supporting the argument that those QPOs should be connected to a property which stays unchanged over long period of time and over range of mass accretion rates. And the most likely possibility here is that QPOs are a strong gravity effect, a manifestation of some kind of orbital motion or a disko-seismology mode in accretion disk, because they are given by mass and BH spin only - the two fundamental parameters describing a black hole.

We have already discovered a number of BH sources that show high-frequency QPOs. In principle we could directly and precisely measure BH spins from high-frequency QPOs if only we new how they originated and had a physical model for their production.

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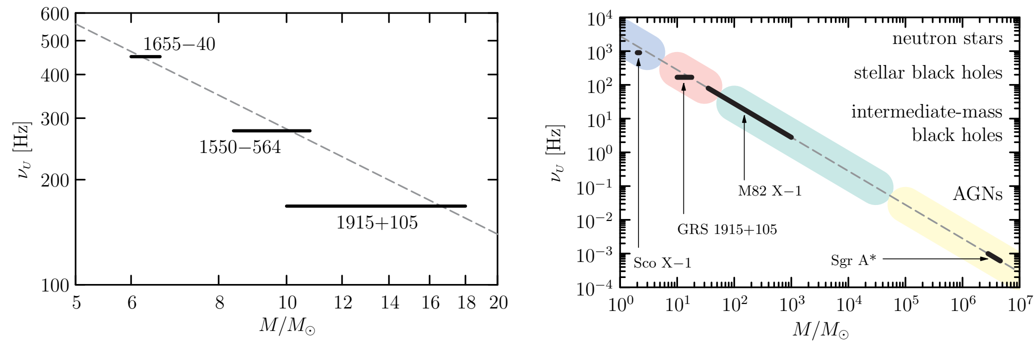

Figure 3: *Left*: The frequency–mass scaling for XTE J1550−564, GRO J1655−40, and GRS 1915+105. The upper frequencies of 3 : 2 pairs are plotted vs. measured masses. The dashed line shows the best fit relation, $\nu_U = 2.8\,\mathrm{kHz}\ (M/M_\odot)^{-1}$. (Adopted from McClintock & Remillard (2005).) *Right*: The global frequency–mass scaling across the range of masses. This illustration shows the microquasar 'best fit' relation (*dashed line*), extended to the range of masses going from 1 up to $10^7\ M_\odot$. Colour bars represent different classes of objects and thick black lines show some individual objects including the neutron star Sco X−1, the microquasar GRS 1915+105, the speculative intermediate-mass black hole M 82 X−1 and the Galactic super-massive black hole Sgr A*. The line lengths indicate the uncertainties in the mass and/or frequency estimates for the objects.

3. Mass Scaling Across Masses

If HFQPOs originate in the fluid motion in the accretion flow, their frequencies should also scale as $1/M$, assuming that they stay around a same place. Indeed, initially Remillard et al. (2002) and later McClintock & Remillard (2005) reported that for three microquasars showing the 3 : 2 QPO frequency pairs, where the mass is known independently from optical measurements, the relationship between the HFQPO frequencies and BH masses scale as $1/M$ (Figure 3) and can be well fitted by a formula $\nu_U = 2.8\,\mathrm{kHz}\ (M/M_\odot)^{-1}$. The same trend seems to be the same also for Cyg X-1 and XTE J1859+226 (see R.Remillard's contribution elsewhere in this volume).

It is a very interesting finding, because there is an ambiguity in the angular momentum of black holes, which affects orbital frequencies and can destroy the $1/M$ scaling. If a natural assumption is made that HFQPOs are produced by the same type of mechanism in each source, the fact that within uncertainties in mass measurements the formula $\nu_U = 2.8\,\mathrm{kHz}\ (M/M_\odot)^{-1}$ can fit all three sources means that all sources should have a similar spin.

A confirmation of the scaling law has the crucial importance for possible explanations of HFQPOs. It strongly supports models that identify the oscillations with some type of orbital motion in the accretion disc. If the origin of oscillations is the same in neutron stars and black holes, it should be possible (with respect to differences in spin, spacetime structure and magnetic fields) to roughly rescale the basic QPO properties between the two classes of sources.

4. BH HFQPO Mass and Spin

Let us assume for a moment that indeed HFQPOs are produced in the accretion disk as a resonance or other interplay of two kinds of orbital modes. For each combination of two particular

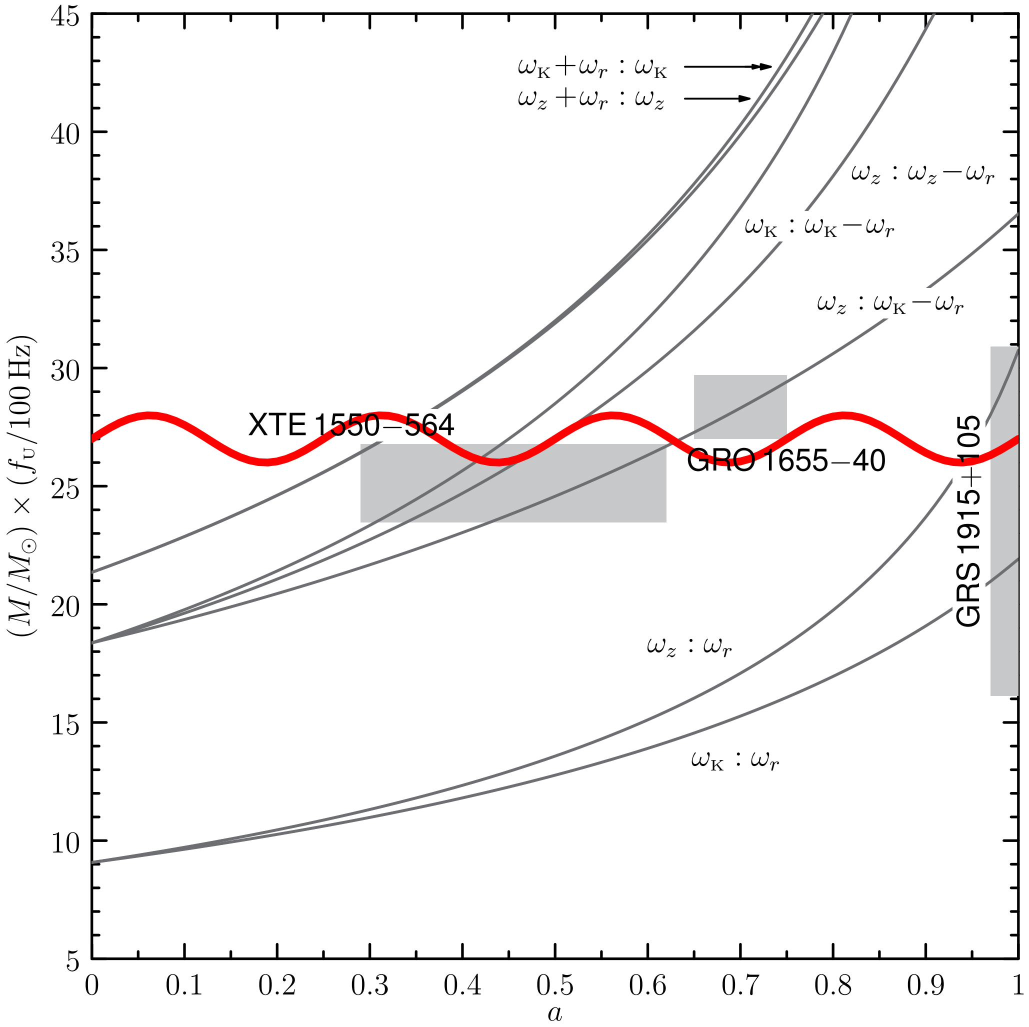

Figure 4: Constraints on masses and spins given by either observations (boxes) or by theoretical resonance models (lines). For detailed explanation of the plot see the text.

modes we can write down formulae that prescribe frequencies of those modes as functions of radius and BH mass and spin. Mass dependence can be moved to the left hand side of the formulae and what remains on the right is just a function of spin and radius. We know that the two frequencies should be in 3:2 ratio, which fixes the radius and finally we have a function of spin only for frequency which we compare to an actually observed QPO frequency (whether upper or lower is a matter of choice) scaled by assumed source mass.

Then we can construct a diagram where we plot the QPO frequency times mass of the source against spin of the source. In such plot each combination of modes draws a single theoretical line. Data of sources can be added too if we have an estimate for their masses (from radial velocities) and their spins (obtained either from iron line measurements or from fitting continuum spectra). Typically, estimates come with their errors so a source is represented by a box in the plot, where the horizontal extend of the box corresponds to an error in spin estimate and the vertical extend corresponds to an error in mass determination (error in QPO frequency is usually much smaller

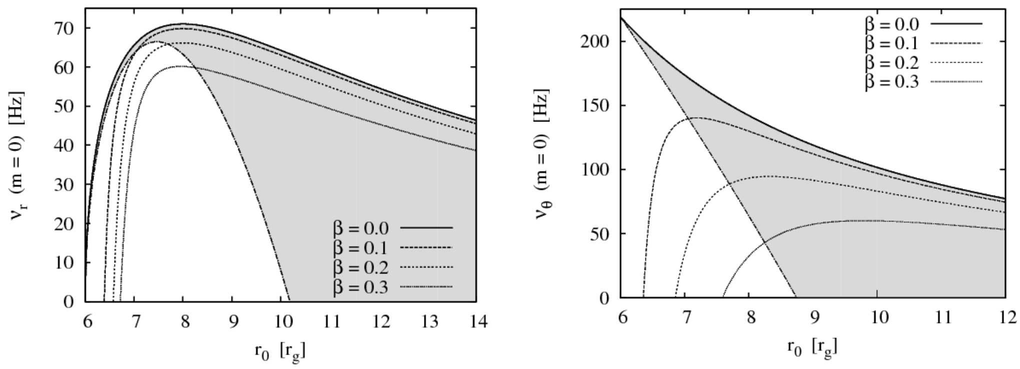

Figure 5: The change of radial *left* and vertical *right* epicyclic frequencies in a pressure supported torus as a function of radius of the torus centre for various values of slenderness parameter β. (Adopted from Straub & Šrámková 2009.)

than mass error and can be neglected).

The above described plot is shown in Figure 4. It contains boxes of three sources which exhibit HFQPOs and at the same time it has been possible to estimate their masses and spins. The three sources are GRS 1915+105, GRO 1655−40, XTE 1550−564. Then there are (resonant) lines which represent a set of combination of orbital modes that might be responsible of HFQPOs. The set is not a full set but rather a selection of most favourable modes.

The point which strikes the eyes is that while the resonance lines are all strong functions of spin and they change a lot with spin value. On the other hand, it seems that the boxes for the three sources are aligned more or less along a horizontal line (red “wave” line in the plot). If we assume that the same type of mechanism (resonance) for producing HFQPOs acts in all sources, which is quite natural with respect to properties of observed QPOs, one can expect that the boxes should follow one of the theoretical lines. Obviously, they do not and there are two ways how to interpret this fact: they are produced by a process which is almost independent of spin, which suggests that simple resonances of basic modes are not enough to explain the phenomenon or that the HFQPO phenomenon have some different origin. Another possibility is that different mechanisms are responsible for QPOs at every source, or that their mechanism does not always operate on the same place but is affected be something else that just gravity. In the later case it is however difficult to imagine that a mechanism without intrinsic 1/M scaling could produce 1/M dependence in QPO frequencies.

5. Non-geodesic corrections

A possible escape within the resonance theory can lie in chasing the right frequency formulae that should be used for setting up resonance conditions. As it has been stressed already in the previous section, the resonances between various orbital modes assume that we are dealing with test particle motion and that frequencies of those modes are based on formulae from Section 1. This may or rather may not be quite correct, because there are additional stresses that are likely to be present in the disk either caused by gas pressure, the presence of magnetic fields or other

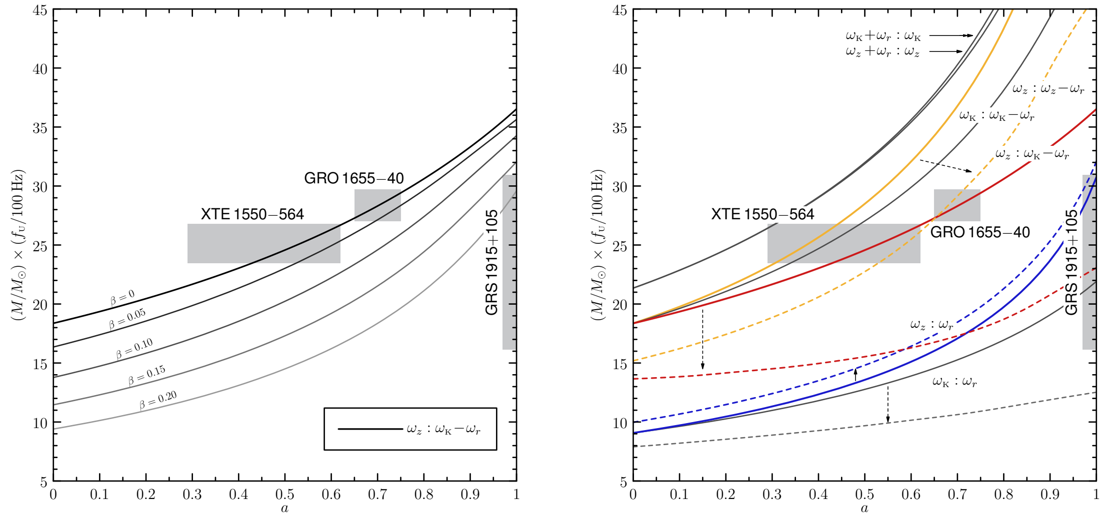

Figure 6: The impact of pressure corrections. *Left*: the change of location of $\Omega_K : \Omega_K - \omega_r$ resonance in response to increasing slederness parameter β of the model torus and a corresponding increase of inner pressure. *Right*: The shift of all resonant lines from Figure 4 calculated for $\beta = 0.1$.

factors. Those additional stresses modify the effective potential acting on fluid elements and thus also change the character of fluid motion.

The non-geodesic (pressure) effects on the epicyclic modes have been investigated by Blaes et al. (2007) and in full GR by Straub & Šrámková (2009) who found that in comparison with the epicyclic frequencies of free test particles, non-slender tori receive negative pressure corrections and thus exhibit lower frequencies (Figure 5) then the ones coming from the test particle motion. Another finding is that corrections for non-axisymmetric ($m > 1$) modes are generally much larger that those of axisymmetric modes.

Figure 6 illustrates the impact of pressure corrections on the resonance curves presented in Figure 4. The left part shows how the curve travel across the graph when pressure increases on the example of non-axisymmetric resonance between vertical and $m = 1$ radial epicyclic mode. The right part then shows also other resonances for torus slenderness parameter $\beta = 0.1$.

The question which still needs to be answered is whether spin measurements using QPOs could be reliable. Even if we new the right combination of modes, we might not know the details of the inner accretion disk, e.g. its central pressure, and thus there might be large uncertaintly in the inferred spin value. On the other hand the same argument about the remarkable stability of QPO frequencies applies also here. If they are stable from one outburst to another and over a wide range of luminosities, it means that their mechanism should be little sensitive to such details like central pressure in the disk.

6. Prospects with future X-ray missions

Finally, let us discuss in brief what prospects we have with future X-ray timing/spectroscopy missions for BH/NS high-frequency QPOs – what are the open questions which may be answered by more advanced missions with larger collecting area and higher sensitivity and which are critical for understanding both neutron-star and black-hole high-frequency QPOs and they relation to each other.

- *Variations of BH QPO frequencies with luminosity.* Neutron star sources show the so called "parallel track" effect – a specific correlation between count rate (luminosity) and QPO frequency. It is interesting to look for such an effect in BH sources too. The idea behind is that if it is connected to NS solid surface and it is an effect of boundary layer radiation back-reaction on the disk, it should not be observed in black-hole sources. Thus it may provide a test whether or not BH do have a solid surface.

- *Variations of BH QPO frequencies with time.* Again, in NS sources we observe QPO pair to travel over some (sometimes extensive) frequency range. Do black-hole HFQPOs change frequencies and cross 3:2 line too?

- *Variations of BH QPO amplitudes with frequencies.* In neutron-star sources it has been observed that QPOs exchange amplitudes when they pass across the resonance point. On one side one QPO is always stronger, on the other side the other QPO of the pair takes over. At the resonance point the amplitude of both is about the same. The queastion is whether in BH sources QPOs exchange their amplitudes as frequencies pass across 3:2?

- *Relations between HF and LF QPOs.* This is still another open question in both types of sources together with the origin of LFQPOs. According to resonance theory, HFQPO properties should be changing on timescales corresponding to LFQPOs. An observation evidence for that (marginally already reported from Sco X-1) would provide a lot of support for the theory.

- *Fourier-resolved spectroscopy.* The use of this technique is very important to constrain variations of various spectral features (blackbody, powerlaw, Fe line) on the QPO timescale. It can also help to find out a solid answer to the question of what does oscillate.

- *Measure other sources with HFQPOs.* Three sources with known QPOs, mass and spin do not form a great statistical sample. We need to make an effort to measure masses/spins of more BH sources with QPOs and put them to mass$-$spin diagram, e.g. H 1743$-$322 (mass,spin), XTE 1859+226 (spin), Cyg X$-$1 (spin). And of course we should be well prepared for a possible next outburst whether it is a repeating one from a known source or a completely new one from a so far unknown source.

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