PoS PROCEEDINGS OF SCIENCE

Leading-order hadronic contribution to g-2 from lattice QCD

Dru B. Renner[*†]
NIC, DESY, Platanenallee 6, D-15738 Zeuthen, Germany
E-mail: dru@jlab.org

Xu Feng[‡]
NIC, DESY, Platanenallee 6, D-15738 Zeuthen, Germany
Universität Münster, Institut für Theoretische Physik, Wilhelm-Klemm-Strasse 9, D-48149, Germany

Karl Jansen
NIC, DESY, Platanenallee 6, D-15738 Zeuthen, Germany

Marcus Petschlies
Institut für Physik, Humboldt-Universitaet zu Berlin, D-12489, Berlin, Germany

We calculate the leading-order hadronic correction to the anomalous magnetic moments of each of the three charged leptons in the Standard Model: the electron, muon and tau. Working in two-flavor lattice QCD, we address essentially all sources of systematic error: lattice artifacts, finite-size effects, quark-mass extrapolation, momentum extrapolation and disconnected diagrams. The most significant remaining systematic error, the exclusion of the strange and charm quark contributions, will be addressed in our four-flavor calculation. We achieve a statistical accuracy of 2% or better for the physical values for each of the three leptons and the systematic errors are at most comparable.

35th International Conference of High Energy Physics - ICHEP2010,
July 22-28, 2010
Paris France

[*]Speaker.
[†]Current address: Jefferson Laboratory.
[‡]Current address: KEK.

PoS(ICHEP 2010)371

1. Introduction

The lingering 3σ discrepancy between the measured [1] and calculated [2] values of the anomalous magnetic moment of the muon a_μ raises the possibility of physics beyond the Standard Model. However, a clear understanding of the significance of this discrepancy is complicated by the fact that the Standard Model calculation uses additional experimental inputs and models to account for the hadronic contributions to a_μ. Additionally, the hadronic corrections give rise to the dominant sources of error in the Standard Model computation. Thus a non-perturbative first-principles calculation of the hadronic contributions to a_μ could clarify the significance of the current 3σ effect and potentially reduce the error of the Standard Model result.

The hadronic contributions to a_μ are organized as an expansion in the electromagnetic coupling α. The leading-order contribution is responsible for the largest single source of error in the Standard Model prediction for a_μ. In this work, we calculate the leading-order hadronic correction to a_μ using two-flavor lattice QCD. We use the gauge field ensembles of the European Twisted Mass Collaboration [3]. By introducing a new method to overcome the difficulties of previous calculations, we achieve a statistical precision of less than 2%. All sources of systematic error have been examined and appear to be no larger than the statistical error. As a check, we calculate the leading-order correction for the electron and tau leptons and reach similar conclusions. The final results of our two-flavor calculation, including an estimate of the systematic errors, will be presented in [4], so here we summarize only the recent advances in our calculation.

2. Leading-order hadronic contribution

The expression for the leading-order correction to the anomalous magnetic moment a_l^{hvp} of a lepton l with mass m_l was given by Blum [5] as

$$a_l^{\mathrm{hvp}} = \alpha^2 \int_0^\infty dQ^2 \, \frac{1}{Q^2} w(Q^2/m_l^2) \, \Pi_R(Q^2) \tag{2.1}$$

where $\Pi_R(Q^2) = \Pi(Q^2) - \Pi(0)$ is the renormalized vacuum polarization $\Pi(Q^2)$, Q^2 is the Euclidean momentum and $w(Q^2/m_l^2)$ is a known function. The computation of $\Pi(Q^2)$ is now a standard calculation in lattice QCD. The details of our approach were recently described in [6].

3. Extrapolation and interpolation

The lattice calculation of $\Pi(Q^2)$ is restricted to discrete values of Q^2 and must be interpolated and extrapolated to $Q^2 = 0$ to perform the integration in Eq. 2.1. To describe the high Q^2 region of $\Pi(Q^2)$, we use

$$\Pi_{\mathrm{high}}(Q^2) = c + \ln Q^2 \sum_{n=0}^{O} b_n (Q^2)^n \, .$$

The constant c is necessary because $\Pi(Q^2)$ by itself is ultraviolet divergent. The expected $\ln Q^2$ behavior is explicitly accounted for, but the polynomial sum is added to ensure that Π_{high} provides a complete basis in which to expand. The low Q^2 region of $\Pi(Q^2)$ is dominated by the contributions from the lightest vector-meson states. Therefore, we take the simple tree-level form used

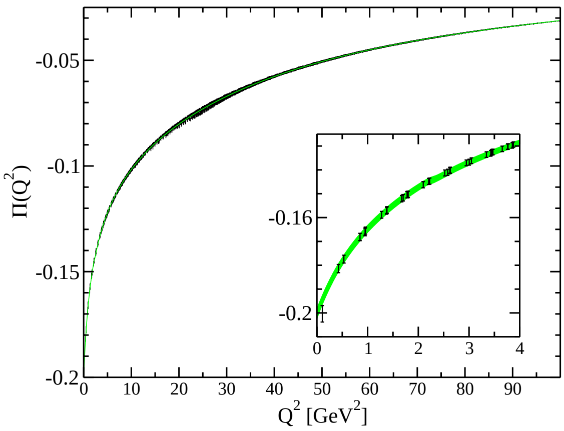

Figure 1: Vacuum polarization function, $\Pi(Q^2)$. This is an example of our calculation of $\Pi(Q^2)$ and the interpolation of the entire range of Q^2 that is calculated on the lattice.

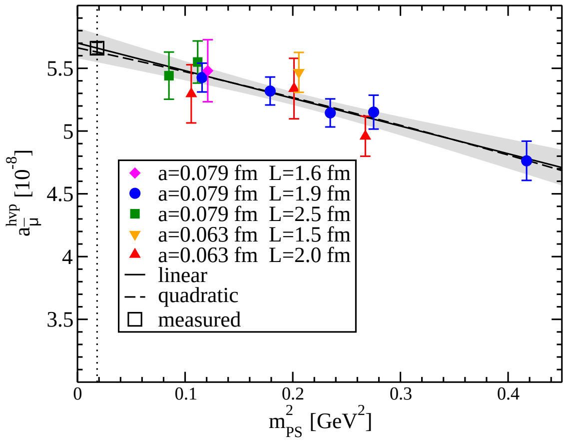

Figure 2: Leading-order hadronic correction for the muon, $a_\mu^{\rm hvp}$. This is our calculation of $a_\mu^{\rm hvp}$, using the modified method described in the text, compared to the two-flavor contribution to the experimental value.

in effective field theories of vector-mesons [7] and then add a polynomial expansion to provide a complete basis of functions. The resulting expression is

$$\Pi_{\rm low}(Q^2) = -\frac{5}{9}\sum_{i=1}^{M} g_{Vi}^2 \frac{m_{Vi}^2}{Q^2+m_{Vi}^2} + \sum_{n=0}^{N} a_n (Q^2)^n \,.$$

The sum over i gives the tree-level contribution for M vector mesons with masses m_{Vi} and couplings g_{Vi}. These masses and couplings have precise meanings of their own and are calculated in the same lattice computation. The form for $\Pi_{\rm low}$ is then matched to $\Pi_{\rm high}$ to provide a complete description that is suitable for numerical integration. An example of this interpolation and extrapolation is shown in Fig. 1. This method gives a fully non-perturbative determination of $a_l^{\rm hvp}$ that does not rely on QCD perturbation theory.

4. Modified method

We introduce a class of quantities that have the same physical limit as $a_l^{\rm hvp}$ but approach that limit more smoothly as a function of the pseudo-scalar meson mass m_{PS}. For any hadronic quantity H, we define

$$a_{\bar{l}}^{\rm hvp} = \alpha^2 \int_0^\infty dQ^2 \frac{1}{Q^2} w(Q^2/m_l^2 \cdot H_{\rm phys}^2/H^2)\, \Pi_R(Q^2)$$

where H is understood to be calculated at each value of m_{PS} and $H_{\rm phys} = H(m_{PS} \to m_\pi)$. By construction $a_{\bar{l}}^{\rm hvp} = a_l^{\rm hvp}$ as m_{PS} approaches m_π. Any choice of H leads to a valid definition but $H = m_V$, the lightest vector-meson mass, leads to a mild m_{PS} dependence and results in a well-controlled lattice calculation of $a_l^{\rm hvp}$ in the physical limit. The results for $a_{\bar{\mu}}^{\rm hvp}$ are shown in Fig. 2. The two-flavor contribution to the experimentally measured value of a_μ [2, 4] is also shown in the plot and we find good agreement between the linearly extrapolated lattice calculation and the measured value. As a check of the method, we perform the same calculation and comparison for the electron in Fig. 3 and for the tau in Fig. 4.

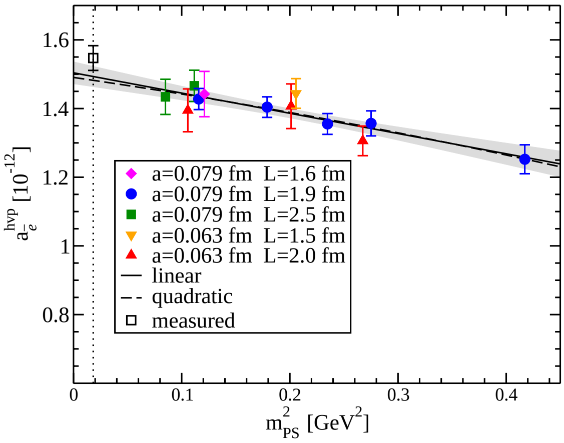

Figure 3: Leading-order hadronic correction for the electron, a_e^{hvp}. This is the same as in Fig. 2 but now for the electron.

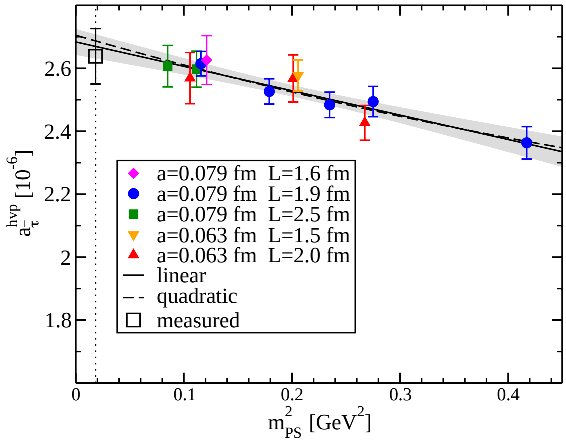

Figure 4: Leading-order hadronic correction for the tau, a_τ^{hvp}. This is the same as in Fig. 2 but now for the tau.

5. Conclusions

We have calculated the two-flavor contribution to the leading-order hadronic correction to the magnetic moments of the three charged leptons in the Standard Model: the electron, muon and tau. Combining an extensive study of all the systematics with a modification to the previously used method, we can reliably reproduce the two-flavor contribution to the experimentally measured values. The next systematic error that needs to be addressed, the omission of the strange and charm quark contributions, will be resolved by our ongoing four-flavor lattice calculation. This will open the possibility of a completely first-principles determination of the leading-order hadronic contribution to the Standard Model values for the lepton magnetic moments.

Acknowledgments

We are grateful to the John von Neumann Institute for Computing (NIC), the Jülich Supercomputing Center and the DESY Zeuthen Computing Center for their computing resources and support. This work has been supported in part by the DFG Sonderforschungsbereich/Transregio SFB/TR9-03, the DFG project Mu 757/13 and the U.S. DOE under Contract No. DE-AC05-06OR23177.

References

[1] Muon G-2 Collaboration, G. Bennett *et al.*, Phys. Rev. **D73**, 072003 (2006), hep-ex/0602035.

[2] F. Jegerlehner and A. Nyffeler, Phys. Rept. **477**, 1 (2009), arXiv:0902.3360.

[3] ETMC Collaboration, R. Baron *et al.*, JHEP **08**, 097 (2010), arXiv:0911.5061.

[4] X. Feng, K. Jansen, M. Petschlies, and D. B. Renner, in preparation.

[5] T. Blum, Phys. Rev. Lett. **91**, 052001 (2003), hep-lat/0212018.

[6] D. B. Renner, X. Feng, K. Jansen, and M. Petschlies, PoS **LAT2010**, 155 (2010), arXiv:1011.4231.

[7] C. Aubin and T. Blum, Phys. Rev. **D75**, 114502 (2007), hep-lat/0608011.