

Mean multiplicity of light and heavy quark initiated jets

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After inserting the heavy quark mass dependence into QCD partonic evolution equations, we determine the mean charged hadron multiplicity of jets produced in high energy collisions. We thereby extend the so-called dead cone effect to the phenomenology of multiparticle production in QCD jets and find that the average multiplicity of heavy-quark initiated jets decreases significantly as compared to the massless case, even taking into account the weak decay products of the leading primary quark. We emphasize the relevance of our study as a complementary check of b -tagging techniques at hadron colliders like the Tevatron and the LHC.

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1. Introduction

High- p_t jets can be initiated either in a short-distance interaction among partons in high energy collisions such as pp , $p\bar{p}$, in the DIS $e^\pm p$, the e^+e^- annihilation and via electroweak (or new physics) processes. One well-known example is given by the decay chain of the top quark $t \rightarrow H^+ b$, where the b quark should start a jet. Thus the ability to identify jets from the fragmentation and hadronization of b quarks becomes very important for such Higgs boson searches. Needless to say, the relevance of b -tagging extends over many other channels in the quest for new physics at hadron colliders. The experimental identification of b -jets relies upon several of their properties in order to reject background, e.g. jets initiated by lighter quarks or gluons. First, the fragmentation is hard and the leading b -hadron retains a large part of the original b quark momentum. In addition, the weak decay products may have a large transverse momentum with respect to the jet axis therefore allowing separation from the rest of the cascade particles. Lastly, the relatively long lifetime of b -hadrons leading to displaced vertices which can be identified by using well-known impact parameter techniques [1]. Still, a fraction of light jets could be mis-identified as b -jets, especially at large transverse momentum of the jet. Now, let us point out that an essential difference between heavy and light quark jets results from kinematics constraints: the gluon radiation off a quark of mass m and energy $E \gg m$ is suppressed inside a forward cone with an opening angle $\Theta_m = m/E$, the so-called *dead cone* phenomenon [2,3].

In this work, we compute the average (charged) multiplicity of a jet initiated by a heavy quark. For this purpose, we extend the modified leading logarithmic approximation (MLLA) evolution equations [4] to the case where the jet is initiated by a heavy (charm, bottom) quark. The average multiplicity of light quarks jets produced in high energy collisions can be written as $N(Y) \propto \exp\left\{\int^Y \gamma(y)dy\right\}$ (Y and y are defined in section 2), where $\gamma \simeq \gamma_0 + \Delta\gamma$ is the anomalous dimension that accounts for soft and collinear gluons in the double logarithmic approximation (DLA) $\gamma_0 \simeq \sqrt{\alpha_s}$, in addition to hard collinear gluons $\Delta\gamma \simeq \alpha_s$ or single logarithms (SLs), which better account for energy conservation and the running of the coupling constant α_s [4].

In the present work we evaluate the mean multiplicity as a function of the mass of the heavy quark. Actually, the mass depends on the scale Q of the hard process in which the heavy quark participates. In the \overline{MS} renormalization scheme, for example, the running mass becomes a function of the strong coupling constant $\alpha_s(Q)$, and the pole mass defined through the renormalized heavy quark propagator (for a review see [5] and references therein). In order to assess the scheme-dependence of our calculations beyond leading-order we have considered a broad range of possible values for the charm and bottom masses: $m_c = 1 - 1.5$ GeV and $m_b = 3 - 5$ GeV, respectively [5]. Lastly, we will see that under the assumption of local parton hadron duality (LPHD) as hadronization model [6,7], light- and heavy-quark initiated jets show significant differences regarding particle multiplicities as a consequence of soft gluon suppression inside the dead cone. Such differences could be exploited by using auxiliary criteria complementing b -tagging procedures to be applied to jets with very large transverse momentum, as advocated in this study.

2. Kinematics and variables

As known from jet calculus for light quarks, the evolution time parameter determining the structure of the parton branching of the primary gluon is given by (for a review see [4] and references therein)

$$y = \ln \left(\frac{k_{\perp}}{Q_0} \right), \quad k_{\perp} = zQ \geq Q_0, \quad Q = E\Theta \geq Q_0, \quad (2.1)$$

where k_{\perp} is the transverse momentum of the gluon emitted off the light quark, Q is the virtuality of the jet (or jet hardness), E the energy of the leading parton, $Q_0/E \leq \Theta \leq \Theta_0$ is the emission angle of the gluon ($\Theta \ll 1$), Θ_0 the total half opening angle of the jet being fixed by experimental requirements, and Q_0 is the collinear cut-off parameter. Let us define in this context the variable Y as $y = Y + \ln z$, $Y = \ln \left(\frac{Q}{Q_0} \right)$. The appearance of this scale is a consequence of angular ordering (AO) of successive parton branchings in QCD cascades [4, 7]. An important difference in the structure of light ($\ell \equiv q = u, d, s$) versus heavy quark ($h \equiv Q = c, b$) jets stems from the dynamical restriction on the phase space of primary gluon radiation in the heavy quark case, where the gluon radiation off an energetic quark Q with mass m and energy $E \gg m$ is suppressed inside the forward cone with an opening angle $\Theta_m = m/E$, the above-mentioned dead cone phenomenon [3].

The corresponding evolution time parameter for a jet initiated by a heavy quark with energy E and mass m appears in a natural way and reads $\tilde{y} = \ln \left(\frac{\kappa_{\perp}}{Q_0} \right)$, $\kappa_{\perp}^2 = k_{\perp}^2 + z^2 m^2$ [3], which for collinear emissions $\Theta \ll 1$ can also be rewritten in the form

$$\kappa_{\perp} = z\tilde{Q}, \quad \tilde{Q} = E (\Theta^2 + \Theta_m^2)^{\frac{1}{2}}, \quad (2.2)$$

with $\Theta \geq \Theta_m$ (see Fig. 1). An additional comment is in order concerning the AO for gluons emitted off the heavy quark. In (2.2), Θ is the emission angle of the primary gluon g being emitted off the heavy quark. Now let Θ' be the emission angle of a second gluon g' relative to the primary gluon with energy $\omega' \ll \omega$ and Θ'' the emission angle relative to the heavy quark; in this case the *incoherence* condition $\Theta'^2 \leq (\Theta^2 + \Theta_m^2)$ together with $\Theta'' > \Theta_m$ (the emission angle of the second gluon should still be larger than the dead cone) naturally leads (2.2) to become the proper evolution parameter for the gluon subject (for more details see [3]). For $\Theta_m = 0$, the standard AO ($\Theta' \leq \Theta$) is recovered. Therefore, for a massless quark, the virtuality of the jet simply reduces to $Q = E\Theta$ as given above. The same quantity κ_{\perp} determines the scale of the running coupling α_s in the gluon emission off the heavy quark. It can be related to the anomalous dimension of the process by

$$\gamma_0^2(\kappa_{\perp}) = 2N_c \frac{\alpha_s(\kappa_{\perp})}{\pi} = \frac{1}{\beta_0(\tilde{y} + \lambda)}, \quad \beta_0(n_f) = \frac{1}{4N_c} \left(\frac{11}{3}N_c - \frac{2}{3}n_f \right), \quad \lambda = \ln \frac{Q_0}{\Lambda_{QCD}}, \quad (2.3)$$

where n_f is the number of active flavours and N_c the number of colours. The variation of the effective coupling α_s as $n_f \rightarrow n_f + 1$ over the heavy quarks threshold has been suggested by next-to-leading (NLO) calculations in the \overline{MS} scheme [6] and is sub-leading in this frame. In this context $\beta_0(n_f)$ will be evaluated at the total number of quarks we consider in our application. The four scales of the process are related as follows,

$$\tilde{Q} \gg m \gg Q_0 \sim \Lambda_{QCD},$$

where $Q_0 \sim \Lambda_{QCD}$ corresponds to the limiting spectrum approximation [4]. Finally, the dead cone phenomenon imposes the following bounds of integration to the perturbative regime

$$\frac{m}{\tilde{Q}} \leq z \leq 1 - \frac{m}{\tilde{Q}}, \quad m^2 \leq \tilde{Q}^2 \leq E^2(\Theta_0^2 + \Theta_m^2), \quad (2.4)$$

which now account for the phase-space of the heavy quark jet. The last inequality states that the minimal transverse momentum of the jet $\tilde{Q} = E\Theta_m = m$ is given by the mass of the heavy quark, which enters the game as the natural cut-off parameter of the perturbative approach.

3. Definitions and notation

The multiplicity distribution is defined by the formula

$$P_n = \frac{\sigma_n}{\sum_{n=0}^{\infty} \sigma_n} = \frac{\sigma_n}{\sigma_{inel}}, \quad \sum_{n=0}^{\infty} P_n = 1 \quad (3.1)$$

where σ_n denotes the cross section of an n -particle yield process, σ_{inel} is the inelastic cross-section, and the sum runs over all possible values of n .

It is often more convenient to represent multiplicity distributions by their moments. All such sets can be obtained from the generating functional $Z(y, u)$ [4] defined by

$$Z(y, u) = \sum_{n=0}^{\infty} P_n(y) (1 + u)^n$$

at the energy scale y . For fixed y , we can drop this variable from the *azimuthally* averaged generating functional $Z(u)$. The average multiplicity is defined by the formula,

$$\langle n \rangle \equiv N = \sum_{n=0}^{\infty} P_n n, \quad P_n = \frac{1}{n!} \left. \frac{d^n Z(u)}{du^n} \right|_{u=-1}. \quad (3.2)$$

We will compute the average multiplicity (3.2) of partons in jets to be denoted hereafter as N_A , with $A = Q, q, g$, corresponding to a heavy, light quark or gluon initiated jet respectively.

Once arrived at his point, let us make an important distinction between two different particle sources populating heavy-quark initiated jets. On the one hand, parton cascade from gluon emission yields the QCD component of the total jet multiplicity (the main object of our present study), *excluding weak decay products of the leading primary quark* at the final stage of hadronization. On the other hand, the latter products coming from the leading flavoured hadron should be taken into account in the measured multiplicities of jets. We shall denote the average charged hadron multiplicity from the latter source as N_A^{dc} . Hence the total charged average multiplicity, N_A^{total} , reads

$$N_A^{total} = N_A^{ch} + N_A^{dc}; \quad A = q, Q. \quad (3.3)$$

As a consequence of the LPHD, $N_A^{ch} = \mathcal{K}^{ch} \times N_A$ [6, 7], where the free parameter \mathcal{K}^{ch} normalizes the average multiplicity of partons to the average multiplicity of charged hadrons. For charm and bottom quarks, we will respectively set the values $N_c^{dc} = 2.60 \pm 0.15$ and $N_b^{dc} = 5.55 \pm 0.09$ [3, 8], while in light quark jets one expects $N_q^{dc} = 1.2 \pm 0.1$ [9]. In this work, we advocate the use of such a difference between average jet multiplicities as a signature to distinguish *a posteriori* heavy from light quark jets, particularly in *b*-tagging techniques applied to the analysis of many interesting decay channels.

4. QCD evolution equations

The splitting functions

$$P(z, \alpha_s) = \alpha_s P^{(0)}(z) + \alpha_s^2 P^{(1)}(z) + \dots \quad (4.1)$$

where $P^{(0)}(z)$ and $P^{(1)}(z)$ are respectively the LO and NLO splitting functions, can be associated to each vertex of the process in the partonic shower. $P(z, \alpha_s)$ determines the decay probability of a parent parton (quark, anti-quark, gluon) into two offspring partons of energy fractions z and $1 - z$. In this work, we are rather concerned with calculations which only involve the LO $P^{(0)}(z)$ splitting functions in the evolution equations [4].

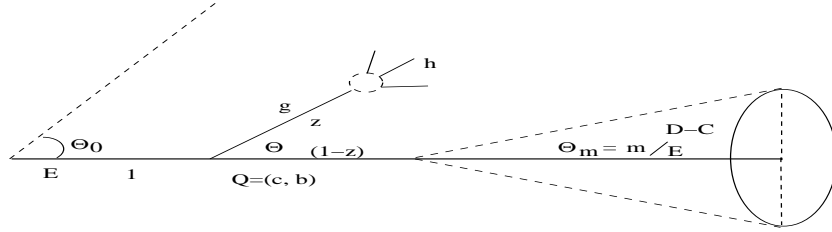


FIG. 1: Parton splitting in the process $Q \rightarrow Qg$: a dead cone with opening angle Θ_m is schematically shown.

Let us start by considering the the splitting process, $Q \rightarrow Qg$, Q being a heavy quark and g the emitted gluon which is displayed in Fig.1; the corresponding LO splitting function reads [10, 11]

$$P_{Qg}^{(0)}(z) = \frac{C_F}{N_c} \left[\frac{1}{z} - 1 + \frac{z}{2} - \frac{z(1-z)m^2}{k_\perp^2 + z^2 m^2} \right], \quad P_{QQ}^{(0)}(z) = P_{Qg}^{(0)}(1-z) \quad (4.2)$$

where $k_\perp \approx \min(zE\Theta, (1-z)E\Theta)$ is the transverse momentum of the soft gluon being emitted off the heavy quark. The previous formula (4.2) has the following physical interpretation, for $k_\perp \ll z^2 m^2$, the corresponding limit reads $P_{Qg}(z) \rightarrow \frac{C_F}{2N_c} z$ and that is why, at leading logarithmic approximation, the forward emission of soft and collinear gluons off the heavy quark becomes suppressed once $\Theta \ll \Theta_m$, while the emission of hard and collinear gluons dominates in this region.

For the massless process $g \rightarrow gg$, we adopt the standard three gluon vertex kernel [4, 13]

$$P_{gg}^{(0)}(z) = \frac{1}{z} - (1-z)[2 - z(1-z)], \quad (4.3)$$

and finally for $g \rightarrow Q\bar{Q}$, we take [10, 11]

$$P_{gQ}^{(0)}(z) = \frac{1}{4N_c} \left[1 - 2z(1-z) + \frac{2z(1-z)m^2}{k_\perp^2 + m^2} \right], \quad (4.4)$$

which needs to be resummed together with the three gluon vertex contribution. However, as a first approach to this problem, we neglect the production of heavy quark pairs inside gluon and quark jets, making use of [4, 13]

$$P_{gq}^{(0)}(z) = P_{gQ}^{(0)}(z)|_{m=0} = \frac{1}{4N_c} [1 - 2z(1-z)]. \quad (4.5)$$

Including mass effects in the evolution equations also requires the replacement of the massless quark propagator $1/k_{\perp}^2$ by the massive quark propagator $1/(k_{\perp}^2 + z^2 m^2)$ [11], such that the phase space for soft and collinear gluon emissions off the heavy quark can be written in the form [11]

$$d^2\sigma_{Qg} \simeq \gamma_0^2 \frac{dk_{\perp}^2}{k_{\perp}^2 + z^2 m^2} dz P_{Qg}^{(0)}(z), \quad (4.6)$$

where $P_{Qg}^{(0)}(z)$ is given by (4.2). Working out the structure of (4.2) and setting $k_{\perp} \approx zE\Theta$ one has

$$P_{Qg}^{(0)}(z) = \frac{C_F}{N_c} \left[\frac{1}{z} \frac{\Theta^2}{\Theta^2 + \Theta_m^2} - 1 + \frac{z}{2} + \frac{\Theta_m^2}{\Theta^2 + \Theta_m^2} \right]. \quad (4.7)$$

The system of QCD evolution equations for the heavy quark initiated jet is found to read [12]

$$\frac{N_c}{C_F} \frac{dN_Q}{d\tilde{Y}} = \varepsilon_1(\tilde{Y}, L_m) \int_{\tilde{Y}_m}^{\tilde{Y}_{ev}} d\tilde{y} \gamma_0^2(\tilde{y}) N_g(\tilde{y}) - \varepsilon_2(\tilde{Y}, L_m) \gamma_0^2(\tilde{Y}) N_g(\tilde{Y}), \quad (4.8)$$

$$\frac{dN_g}{d\tilde{Y}} = \int_{\tilde{Y}_m}^{\tilde{Y}_{ev}} d\tilde{y} \gamma_0^2(\tilde{y}) N_g(\tilde{y}) - A(\tilde{Y}, L_m) \gamma_0^2(\tilde{Y}) N_g(\tilde{Y}), \quad (4.9)$$

where

$$A(\tilde{Y}, L_m) = a(n_f) - \left[2 + \frac{n_f}{2N_c} \left(1 - 2\frac{C_F}{N_c} \right) \right] e^{-\tilde{Y} + L_m} + \frac{1}{2} \left[1 + \frac{n_f}{N_c} \left(1 - 2\frac{C_F}{N_c} \right) \right] e^{-2\tilde{Y} + 2L_m}, \quad (4.10)$$

with

$$a(n_f) = \frac{1}{4N_c} \left[\frac{11}{3} N_c + \frac{2}{3} n_f \left(1 - 2\frac{C_F}{N_c} \right) \right], \quad \tilde{Y}_m \equiv L_m = \ln \frac{m}{Q_0}, \quad (4.11)$$

and

$$\varepsilon_1(\tilde{Y}, L_m) = 1 - e^{-2\tilde{Y} + 2L_m}, \quad \varepsilon_2(\tilde{Y}, L_m) = \frac{3}{4} - \frac{3}{2} e^{-\tilde{Y} + L_m} - e^{-2\tilde{Y} + 2L_m}. \quad (4.12)$$

There are the following kinds of power suppressed corrections to the heavy quark multiplicity: the leading integral term of (4.8) is $\mathcal{O}(\frac{m^2}{Q^2})$ suppressed, while subleading MLLA corrections appear in the standard form $\mathcal{O}(\sqrt{\alpha_s})$ like in the massless case, finally $\mathcal{O}(\frac{m}{Q}\sqrt{\alpha_s})$ and $\mathcal{O}(\frac{m^2}{Q^2}\sqrt{\alpha_s})$, which are new in this context. Massless results can be recovered after setting $m/\tilde{Q} \rightarrow Q_0/Q$ in (4.10) and (4.12). For massive particles however, these terms are somewhat larger and can not be neglected in our approach unless they are evaluated for much higher energies than at present colliders. On top of that, the corresponding massless equations in the high energy limit are obtained from (4.8) and (4.9) simply by setting $\tilde{y} \rightarrow y$, $\tilde{Y} \rightarrow Y$, $Y_{ev} \rightarrow Y$, $Y_m \rightarrow 0$, $\varepsilon_1 \rightarrow 1$,

$$\varepsilon_2 \rightarrow \tilde{\varepsilon}_2 = \frac{3}{4} - \frac{3}{2} e^{-Y} + \mathcal{O}(e^{-2Y}), \quad A \rightarrow \tilde{A} = a(n_f) - \left[2 + \frac{n_f}{2N_c} \left(1 - 2\frac{C_F}{N_c} \right) \right] e^{-Y} + \mathcal{O}(e^{-2Y}),$$

and are written in the standard form [13]

$$\frac{N_c}{C_F} \frac{dN_q}{dY} = \int_0^Y dy \gamma_0^2 N_g(y) - \tilde{\varepsilon}_2 \gamma_0^2 N_g(Y), \quad \frac{dN_g}{dY} = \int_0^Y dy \gamma_0^2 N_g(y) - \tilde{A} \gamma_0^2 N_g(Y), \quad (4.13)$$

with the initial condition $N_{g,q}(Y=0) = 1$ at threshold. Notice that (4.8) and (4.9) are valid only for $m \gg Q_0$ and therefore $m \rightarrow 0$ does not reproduce the correct limit, which has to be smooth as

given by the massless equations (4.13). Since heavy quarks are less sensitive to recoil effects, the subtraction terms $\propto e^{-\tilde{Y}+L_m}$ and $\propto e^{-2\tilde{Y}+2L_m}$ in $\epsilon_2(\tilde{Y}, L_m)$ diminish the role of energy conservation as compared to massless quark initiated jets. As a consistency check, upon integration over \tilde{Y} of the DLA term in Eq.(4.8), the phase space structure of the radiated quanta reads

$$N_Q(\ln \tilde{Q}) \approx 1 + \frac{C_F}{N_c} \int_0^{\Theta_0^2} \frac{\Theta^2 d\Theta^2}{(\Theta^2 + \Theta_m^2)^2} \int_{m/\tilde{Q}}^{1-m/\tilde{Q}} \frac{dz}{z} [\gamma_0^2 N_g](\ln z \tilde{Q}). \quad (4.14)$$

Notice that the lower bound over Θ^2 in (4.14) (\tilde{Y} in (4.8)) can be taken down to “0” ($Y_m = L_m$ in (4.8)) because the heavy quark mass plays the role of collinear cut-off parameter.

5. Phenomenological consequences

Working out the structure of (4.8) and (4.9), we obtain the rough difference between the light and heavy quark jet multiplicities, which yields,

$$N_q - N_Q \stackrel{E \rightarrow \infty}{\approx} \left[1 - \exp\left(-2\sqrt{\frac{L_m}{\beta_0}}\right) \right] N_q, \quad N_q \propto \exp 2\sqrt{\frac{\tilde{Y}}{\beta_0}}. \quad (5.1)$$

It can be seen that (5.1) is exponentially increasing because it is dominated by the leading DLA energy dependence of N_q . According to (5.1), the gap arising from the dead cone effect should be bigger for the b than for the c quark at the primary state bremsstrahlung radiation off the heavy quark jet. The approximated solution of the evolution equations leads to the rough behaviour of $N_q - N_Q$ in (5.1), which is not exact in its present form. In Fig.2 (left), we display the numerical solution of the evolution equations (4.13) for N_q and (4.8) for N_Q corresponding to the heavy quark mass intervals $m_c = 1 - 1.5$ GeV, $m_b = 3 - 5$ GeV. Let us remark that the gap arising between the light quark jet multiplicity and the heavy quark jet multiplicity follows the trends given by (5.1) asymptotically with $E \rightarrow \infty$. Finally, as expected for massless quarks $L_m = 0$, the difference $N_q - N_Q$ vanishes. In this study we advocate the role of mean multiplicities of jets as a potentially

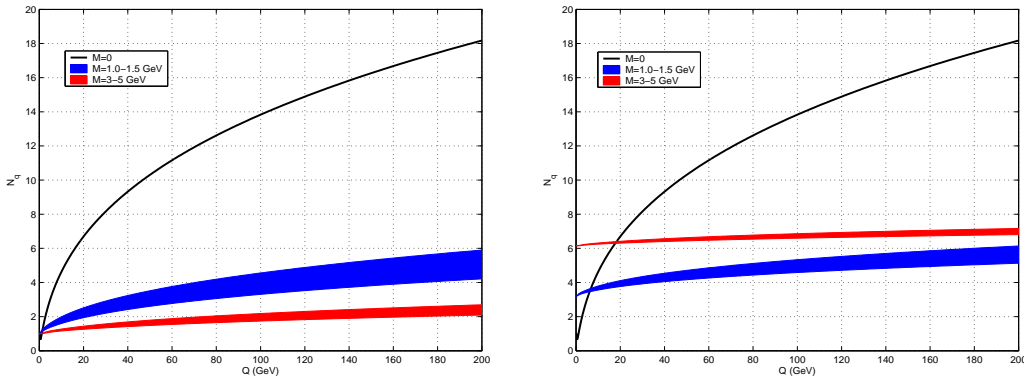


FIG. 2: Massless and massive quark jet average multiplicity N_Q for the primary state radiation (left) and N_Q^{total} including heavy quark flavour decays (right) as a function of the jet hardness Q . Bands indicate m_c and m_b in the $[1, 1.5]$ and $[3, 5]$ GeV intervals respectively.

useful signature for b -tagging and new associated physics [14] when combined with other selection criteria. In Fig. 2 (right), we plot as function of the jet hardness Q^1 , the total average jet multiplicity (3.3), which accounts for the primary state radiation off the heavy quark together with the decay products from the final-state flavoured hadrons, which were introduced in section 3. For these predictions, we set $\mathcal{K}^{ch} = 0.6$ in (3.3), and $Q_0 \sim \Lambda_{QCD} = 230$ MeV [4]. Moreover, the flavour decays constants $N_c^{dc} = 2.60 \pm 0.15$ and $N_b^{dc} = 5.55 \pm 0.09$ are independent of the hard process inside the cascade, such that N_A^{dc} can be added in the whole energy range. For instance, such values were obtained by the OPAL collaboration at the Z^0 peak of the e^+e^- annihilation. In this experiment, D^* mesons were properly reconstructed in order to provide samples of events with varying c and b purity, such that it became possible to measure light and heavy quark charged hadron multiplicities separately [8]. As compared to the average multiplicities of the primary state radiation displayed in Fig. 2 (left), after accounting for N_A^{dc} , the b quark jet multiplicity becomes slightly higher than the c quark jet multiplicity, although both remain suppressed because of the dead cone effect.

6. Conclusions

Thus, our present work focusing on the differences of the average charged hadron multiplicity between jets initiated by gluons, light or heavy quarks could indeed represent a helpful auxiliary criterion to tag such heavy flavours from background for jet hardness $Q \gtrsim 40$ GeV. Notice that we are suggesting as a potential signature the *a posteriori* comparison between average jet multiplicities corresponding to different samples of events where other criteria to discriminate heavy from light quark initiated jets were first applied. Fig.2 (right) plainly demonstrate that the separation between light quark jets and heavy quark jets is allowed above a few tens of GeV with the foreseen errors of the experimentally measured average multiplicities of jets. The difference between light quark jet multiplicities and heavy quark jet multiplicities $N_q - N_Q$ in one jet is exponentially increasing because of suppression of forward gluons in the angular region around the heavy quark direction. This result is not drastically affected after accounting for heavy flavour decays multiplicities, such that it can still be used as an important signature for b -tagging and the search of new physics in a jet together with other selection criteria. Notice that the measurement of such observables require the previous reconstruction of jets at hadron colliders [15].

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¹The energy range $100 \leq Q(\text{GeV}) \leq 200$ should be realistic for Tevatron and LHC phenomenology.

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