

## On Confinement, Chiral Symmetry Breaking, and the $U_A(1)$ anomaly in Functional Approaches

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The so-called decoupling and scaling solutions of functional equations of Landau gauge Yang-Mills theory are briefly reviewed. In both types of solutions the positivity violation seen in the gluon propagator is taken as an indication of gluon confinement. In the scaling solution the resulting infrared singularities of the quark-gluon vertex are responsible for the linear potential between static quarks and are therefore signaling quark confinement. A corresponding description of the  $U_A(1)$  anomaly in functional approaches is only known for the scaling solution. Nevertheless, it seems puzzling at first sight that quark confinement is related to the dynamical and anomalous breaking of chiral symmetry in a self-consistent manner: One obtains either all these phenomena or none. For the scaling solution also fundamental scalar fields are confined. This provides evidence that within functional approaches static confinement is an universal property of the gauge sector even though it is formally represented in the functional equations of the matter sector.

*The many faces of QCD*

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## 1. Introduction: A note on the axial anomaly

This workshop is called *The many faces of QCD*, and actually most participants really experienced their work with QCD being multifaceted. First of all, most successes of QCD are related to processes with high-momentum transfer in which asymptotic freedom [1, 2] enables the use of perturbation theory. On the other hand, although QCD was invented 37 years ago [3] we only start to understand its infrared regime where we face all kind of strong-interaction phenomena, most prominently confinement, anomalous and dynamical chiral symmetry breaking, and the formation of relativistic bound states.

With respect to the anomaly I want to recall a seminal result [4] which may be loosely summarized as follows: the axial U(1) symmetry is always anomalously broken in vector-like gauge theories with vacuum angle  $\Theta = 0$ . One possibility to explain this anomaly rests on the existence of quark zero modes in topologically non-trivial fields [5, 6]: A random distribution of (not necessarily integer) winding number spots leads to a non-vanishing topological susceptibility in the thermodynamic limit. Via the index theorem one can then associate percolating quark zero modes, and they eventually cause the anomalous breaking of the axial U(1) symmetry.

As this explanation is so overwhelmingly successful the question arises whether it is the only existing one. And if another one is available, do these several explanations exclude each other? Here an historic example might be helpful: Everybody of us remembers from his graduate lectures how to derive Bloch waves in a periodic potential by employing the Schrödinger equation. A short look in Sidney Coleman's Erice Lectures "The uses of instantons" [7], however, tells us how to achieve the same by instanton calculus techniques. Of course, nobody of us would ever dare to believe that one would have to add instantons to the Schrödinger equation to obtain Bloch waves. The Schrödinger equation and instanton calculus are simply two different techniques to obtain the same physical result. On the other hand, one can hear quite often the opinion that one has to add to some non-perturbative techniques (as *e.g.* functional equations) the instantons (or other topologically non-trivial field configurations) by hand to obtain a non-vanishing topological susceptibility. The above comparison should, however, elucidate that adding to a consistent approach some other ingredients results in an incorrect treatment.

Accepting this, the following question arises: Where is the topological susceptibility encoded in an approach based on Green's functions? The decisive hint originates from the seventies [8] (see also [9]). Rephrasing this old result in modern language one may state that momentum-space Green's function can reflect the topological susceptibility only in their infrared behaviour because only these are related to the boundary conditions in (Euclidean) space-time.

Emphasizing with this introductory remark the special role of the axial anomaly in our understanding of QCD let me give a short outline of the following sections: After shortly reviewing the knowledge on the infrared structure of Landau gauge Yang-Mills theory I will focus on the positivity violation of the gluon propagator and potential implications for its analytic structure. For fundamental charges the corresponding gluon-matter vertex functions are analysed. Hereby it is demonstrated that the quark-gluon vertex may play a key role in the issue of quark confinement. The quark-gluon vertex is hereby twofold related in self-consistent manner to dynamical chiral symmetry breaking ( $D\chi$ SB): On the one hand, its strength triggers  $D\chi$ SB, on the other hand it is subject of  $D\chi$ SB and contains components which are only possible due to  $D\chi$ SB. A study of the

infrared properties of fundamentally charged scalars provides evidence that within functional approaches static confinement is an universal property of the gauge sector even though it is formally represented in the functional equations of the matter sector.<sup>1</sup> Last but not least, I will return to the question of the description of the axial anomaly within functional approaches.

## 2. Infrared Structure of Landau gauge Yang-Mills theory

The indefinite metric state space of a Yang-Mills theory can be classified according to the properties of the states under BRST transformation, see *e.g.* [11, 12, 13]. The BRST cohomology contains the physical states, the unphysical states form quartets. Such quartets do exist either as perturbative or non-perturbative ones [12, 14, 15]. One important ingredient in the construction of a BRST quartet generated by transverse gluons is the fact that a “mass” gap in transverse gluon correlations needs to be generated, *i.e.*, the massless transverse gluon states of perturbation theory have to disappear even though they should belong to quartets due to color antiscreening and superconvergence in QCD [16, 17]. Within this formulation one can provide a clear distinction between the confinement and the Higgs phase: In the former the colour charge is well-defined in the whole state space, in the latter it is not. A condition which leads to such a well-defined charge can be shown in Landau gauge by standard arguments employing functional equations and Slavnov-Taylor identities to be equivalent to an infrared enhanced ghost propagator [18, 17] which in turn then implies an infrared vanishing gluon propagator [19, 20, 21, 22, 23, 24, 25].<sup>2</sup>

The implications of a broken colour charge are quite straightforward [12]: In each channel in which the gauge potential contains an asymptotic massive vector field the global gauge symmetry generated by the colour charges is spontaneously broken. While this massive vector state results to be a BRST-singlet, the massless Goldstone boson states, which usually occur in some components of the Higgs field, replace the third component of the vector field in the elementary quartet and are thus unphysical. Since the broken charges are BRST-exact, this hidden symmetry breaking is not observable in the Hilbert space of physical states. Thus, if the gauge boson is massive it possesses three degenerate polarization states. Everything else would have been a surprise because with respect to the representations of the Poincaré group there are only two choices:

- massive and three polarization states, or
- massless and two polarization states.

With this remarks in mind let us now analyse the situation in QCD (*i.e.*, in the confinement phase) and assume hereby either of the two types of solutions found in functional equations, namely the scaling one with an infrared vanishing gluon propagator or the decoupling ones with an infrared finite gluon propagator, for a description of the latter solutions see refs. [27, 28] and references therein.

- An infrared vanishing gluon propagator has a vanishing screening length, the corresponding screening “mass” is thus infinite. Nevertheless one would not attribute an infinite gluon mass.

<sup>1</sup>However, one has to note that the corresponding lattice results reported by Axel Maas in this workshop [10] do not corroborate this evidence.

<sup>2</sup>This so-called scaling solution of functional equations has been debated quite intensively recently. One should note, however, that the violation of positivity for gluons is generally accepted, see *e.g.* [26].

- An infrared finite gluon propagator has a finite screening length, the corresponding screening “mass” is therefore finite. However, accepting the positivity violation of transverse gluons as a fact the question arises in which sense this relates to a mass.<sup>3</sup>
- Longitudinal and transverse gluons do not belong to the same BRST representation as there is no doubt that in the confinement phase the longitudinal gluon belongs to the perturbative elementary quartet. This implies that the longitudinal gluon stays a massless (unphysical) state. Putting now the transverse gluons into the same BRST representation<sup>4</sup> clearly contradicts the necessity of generating a “mass” gap for the transverse gluons.
- This is corroborated by the fact that glueballs (which are would-be physical states in pure Yang-Mills theory) do not contain any contribution of longitudinal gluons [29].

The only possible conclusion from this is that longitudinal and transverse gluons are not in the same representation of the Poincaré group. A Poincaré representation for a vector with two polarization states is certainly not the representation for a massive vector.

To summarize this argument: Without the longitudinal polarization as part of the Poincaré representation of the transverse gluons my choice is to refrain from statements like “The gluon is massive.” or phrases like “the gluon mass”, and this independent of what the value of the auto-correlation function of excitations of transverse gluons at vanishing virtuality  $k^2 = 0$  is. To my opinion, calling a gluon “massive” is confusing the issue of gluon confinement.

## 2.1 Infrared Exponents for Gluons and Ghosts

As already stated the infrared behaviour of the one-parameter family of decoupling solutions is such that one obtains an infrared finite gluon propagator and otherwise infrared trivial Green’s functions [27, 28, 25]. The end-point of these solutions is the scaling solution. The infrared behaviour of all one-particle irreducible Green’s functions in the scaling solution is easily described in the simplified case with only one external scale  $p^2 \rightarrow 0$ : For a function with  $n$  external ghost and antighost as well as  $m$  gluon legs one obtains [30, 31]

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}. \quad (2.1)$$

This solution fulfills all functional equations and all Slavnov-Taylor identities. It verifies the hypothesis of infrared ghost dominance [32] and leads to infrared diverging 3- and 4-gluon vertices.

There is only one unique scaling solution with power laws for the Green’s functions [33, 34]. A detailed comparison of both type of solutions can be found *e.g.* in [25], an infrared analysis for both type of solutions is described *e.g.* in Ref. [35]. Although almost all lattice calculations of the gluon propagator favor the decoupling solution it is certainly worthwhile to study the scaling solution as a theoretical tool. And there is the possibility that the difference between these solutions depends on nothing else than a choice of gauge [36]. The latter interpretation is corroborated by the fact that lattice studies at strong coupling [37, 38, 39] reveal the existence of a regime where

<sup>3</sup>The clearest definition of mass in context of a relativistic quantum field theory is that mass is the square root of the first quadratic Casimir invariant of the Poincaré group,  $m := \sqrt{P_\mu P^\mu}$ .

<sup>4</sup>BRST multiplets are degenerate as the BRST charge commutes with the Hamiltonian.

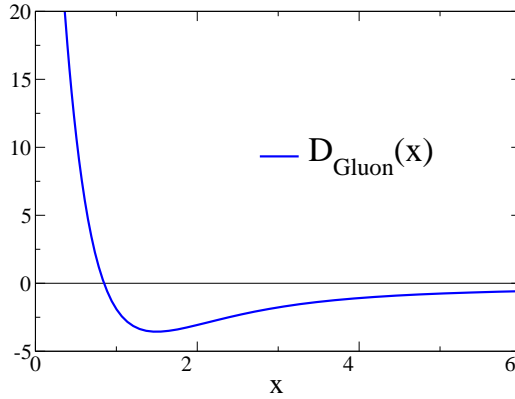
the scaling relation between the gluon and the ghost propagator is fulfilled, and the corresponding infrared exponent  $\kappa$  is very close to the value determined in a full class of truncated continuum studies with  $\kappa = \frac{93 - \sqrt{1201}}{98} \simeq 0.59535$ .

## 2.2 Positivity violation of the gluon propagator

The positivity violation of the (space-time) propagator of transverse gluons as predicted by the Oehme–Zimmermann superconvergence relation [16] and corresponding to the Kugo–Ojima [11] and Gribov–Zwanziger [32] scenarios has been a long-standing conjecture for which there is now compelling evidence, see *e.g.* Refs. [26, 40] and references therein. The basic features underlying these gluon properties, are the infrared suppression of correlations of transverse gluons and the infrared enhancement of ghost correlations as discussed above. A simple argument given by Zwanziger makes this at least for the scaling solution obvious: An infrared vanishing gluon propagator implies for the space-time gluon propagator being the Fourier transform of the momentum space gluon propagator:

$$0 = D_{gluon}(k^2 = 0) = \int d^4x \ D_{gluon}(x). \quad (2.2)$$

Therefore  $D_{gluon}(x)$  has to be negative for some values of  $x$ . Exactly this behaviour is seen in Fig. 1



**Figure 1:** The Fourier transform of the scaling solution for the gluon propagator.

in which the Fourier transform of the scaling solution for the gluon propagator is displayed.

In order to investigate the analytic structure of the gluon propagator we first parameterize the running coupling such that the numerical results for Euclidean scales are reproduced [41]:

$$\alpha_{\text{fit}}(p^2) = \frac{\alpha_S(0)}{1 + p^2/\Lambda_{\text{QCD}}^2} + \frac{4\pi}{\beta_0} \frac{p^2}{\Lambda_{\text{QCD}}^2 + p^2} \left( \frac{1}{\ln(p^2/\Lambda_{\text{QCD}}^2)} - \frac{1}{p^2/\Lambda_{\text{QCD}}^2 - 1} \right) \quad (2.3)$$

with  $\beta_0 = (11N_c - 2N_f)/3$ . In this expression the Landau pole has been subtracted, it is analytic in the complex  $p^2$  plane except the real timelike axis where the logarithm produces a cut for real  $p^2 < 0$ , and it obeys Cutkosky's rule.

The infrared exponent  $\kappa$  is an irrational number, and thus the gluon propagator possesses a cut on the negative real  $p^2$  axis. It is possible to fit the solution for the gluon propagator quite accurately without introducing further singularities in the complex  $p^2$  plane. The fit to the gluon renormalization function [40]

$$Z_{\text{fit}}(p^2) = w \left( \frac{p^2}{\Lambda_{\text{QCD}}^2 + p^2} \right)^{2\kappa} (\alpha_{\text{fit}}(p^2))^{-\gamma} \quad (2.4)$$

works quite precisely. Hereby  $w$  is a normalization parameter, and  $\gamma = (-13N_c + 4N_f)/(22N_c - 4N_f)$  is the one-loop value for the anomalous dimension of the gluon propagator. The discontinuity of (2.4) along the cut vanishes for  $p^2 \rightarrow 0^-$ , diverges to  $+\infty$  at  $p^2 = -\Lambda_{\text{QCD}}^2$  and goes to zero for  $p^2 \rightarrow \infty$ . The function (2.4) contains only four parameters: the overall magnitude which due to renormalization properties is arbitrary (it is determined via the choice of the renormalization scale), the scale  $\Lambda_{\text{QCD}}$ , the infrared exponent  $\kappa$  and the anomalous dimension of the gluon  $\gamma$ . The latter two are not free parameters:  $\kappa$  is determined from the infrared properties of the DSEs and for  $\gamma$  its one-loop value is used. Thus we have found a parameterization of the gluon propagator which has effectively only one parameter, the scale  $\Lambda_{\text{QCD}}$ . It is important to note that the gluon propagator possesses a form such that *Wick rotation is possible!*

### 3. Quarks/Matter: Confinement vs. $D\chi$ SB & $U_A(1)$ anomaly

Due to the infrared suppression of the gluon propagator, present in the scaling **and** in the decoupling solutions, quark confinement (or, generally, confinement of fundamental charges) cannot be generated by any type of gluon exchange together with infrared-bounded vertex functions. Therefore it is mandatory to study the functional equations for the quark propagator together with the one for the quark-gluon vertex in a self-consistent way [42, 43]. An important difference of the quarks as compared to Yang-Mills fields arises: As the former possess a mass, and as  $D\chi$ SB does occur, the quark propagator will always approach a constant in the infrared.

#### 3.1 Dynamically induced scalar quark confinement

The fully dressed quark-gluon vertex consists of twelve linearly independent Dirac tensors. Half of the coefficient functions would vanish if chiral symmetry were realized in the Wigner-Weyl mode. From a solution of the Dyson-Schwinger equations we infer that these “scalar” structures are, in the chiral limit, generated non-perturbatively together with the dynamical quark mass function in a self-consistent fashion. This implies the important result that dynamical chiral symmetry breaking manifests itself not only in the propagator but also in the quark-gluon vertex.

From an infrared analysis one obtains an infrared divergent solution for the quark-gluon vertex such that Dirac vector and “scalar” components of this vertex are infrared divergent with exponent  $-\kappa - \frac{1}{2}$  if either all momenta or the gluon momentum vanish [43]. A numerical solution of a truncated set of Dyson-Schwinger equations confirms this infrared behaviour. The essential components to obtain this solution are the “scalar” Dirac amplitudes of the quark-gluon vertex and the scalar part of the quark propagator. Both are only present when chiral symmetry is broken, either explicitly or dynamically.

In order to determine how this self-consistent quark propagator and quark-gluon vertex solution relates to quark confinement, the anomalous infrared exponent of the four-quark function is calculated. The static quark potential can be obtained from this four-quark one-particle irreducible Green function. In the scaling solution it behaves like  $(p^2)^{-2}$  for  $p^2 \rightarrow 0$  due to the infrared enhancement of the quark-gluon vertex for vanishing gluon momentum. Using a well-known relation one obtains for the static quark-antiquark potential  $V(\mathbf{r})$ :

$$V(\mathbf{r}) \sim \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^4} \Big|_{p^0=0} e^{i\mathbf{p}\mathbf{r}} \sim |\mathbf{r}| \quad (3.1)$$

Therefore the infrared divergence of the quark-gluon vertex, as found in the scaling solution of the coupled system of Dyson-Schwinger equations, the vertex overcompensates the infrared suppression of the gluon propagator such that one obtains a linearly rising potential.

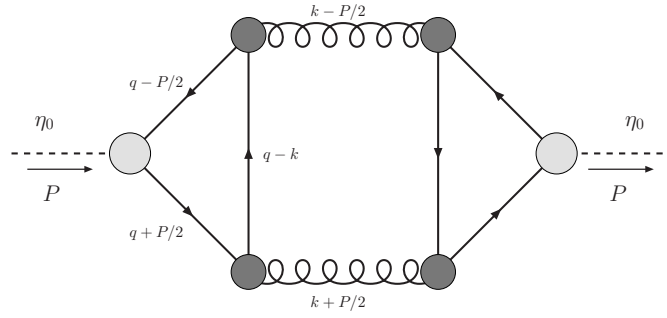
### 3.2 Fundamentally charged scalar field

Given the complications with the many tensor structures for quarks, and given the cost for fermions on the lattice, it seems natural to use fundamentally charged scalars as a laboratory to study confinement. In this context the scalar propagator and the scalar-gluon vertex were investigated on the lattice [44, 10] and analytically [45, 46, 47]. Different than the quark Green's functions the tensor structure of the scalar ones is strongly simplified. Compared to two components in the fermionic propagator, the scalar propagator features only a single structure. Similarly the vertex depending on two independent momenta can be decomposed into two tensors (instead of twelve).

A scalar possesses self-interactions and therefore the number of terms in the Dyson-Schwinger and Functional Renormalization Group equations is significantly increased. For the derivation of the Dyson-Schwinger equations one may employ the MATHEMATICA package DoDSE [48]. (A package for Functional Renormalization Group equations will be published soon [49].) In the uniform scaling limit, applying the constraints on the infrared exponents arising from the comparison of the inequivalent towers of Dyson-Schwinger and Functional Renormalization Group equations [34], the system of equations for the anomalous exponents simplifies. One obtains the scaling and the decoupling solutions with an unaltered Yang-Mills sector. In the case of the scaling solution for a massive scalar, the scalar-gluon vertex can show two distinct behaviours [45, 46]. In the one to be discussed further it exhibits the same infrared exponent as the quark-gluon vertex.

The uniform scaling uncovers only a small part of the potential infrared enhancements. Vertex functions may also become divergent when only a subset of the external momenta vanish. Such kinematic divergences provide a mechanism for the long-range interaction of quarks as described in the section above. It is gratifying to realize that the kinematic divergences of the scalar-gluon vertex are identical to those of the quark-gluon vertex. These singularities induce a confining interaction in the four-scalar vertex function as they did in the case of the four-quark vertex function in the case of scalar QCD. Their Fourier transform leads to a linearly rising static potential.

This result provides the possibility that within functional approaches static confinement is an universal property of the gauge sector even though it is formally represented in the functional equations of the matter sector. Unfortunately, these results are not corroborated by the lattice results, see Refs. [44, 10] for more details.



**Figure 2:** Contribution to the  $\eta'$  mass due to the infrared divergence of the product of quark-gluon vertices and gluon propagator,  $\Gamma_\mu D^{\mu\nu} \Gamma_\nu \propto 1/(k \pm P/2)^4$ .

### 3.3 $U_A(1)$ : $\eta'$ mass from infrared divergent Green functions

Based on purely dimensional arguments [8] one can conclude that the diamond diagram depicted in Fig. 2 supplies a non-vanishing contribution to the mass of the pseudo-scalar flavor-singlet meson in the chiral limit if the effective one-gluon-exchange diverges with the gluon-momentum like  $1/k^4$ . As discussed above this is exactly the behaviour found in the scaling solution for the product of two quark-gluon vertices and the gluon propagator when the exchanged gluons become soft,  $\Gamma_\mu(p, q; k) D^{\mu\nu}(k) \Gamma_\nu(r, s; k) \propto 1/k^4$ . An explicit calculation [50] verifies the corresponding generation of a flavor-singlet mass. However, it has to be noted that the exchange of more than two gluons also generate contributions to the  $\eta'$  mass. As a matter of fact infinitely many diagrams contribute. Under this aspect it is reassuring that the diagram of Fig. 2 provides the leading contribution. Expressing the result in terms of the topological susceptibility  $\chi^2$  one obtains  $\chi^2 = (160\text{MeV})^4$  as compared to the phenomenological value  $\chi^2 = (180\text{MeV})^4$  [50].

In this picture the infrared divergence of the quark-gluon vertex plays an important role in a confinement-based explanation of the  $\eta'$  mass, the topological susceptibility and the  $U_A(1)$  anomaly. This provides evidence that the confining field configurations of QCD are topologically non-trivial. For example, when removing center vortices from a lattice ensemble, the string tension vanishes and the Landau gauge ghost propagator becomes infrared suppressed [51].<sup>5</sup>

The appearance of the correct infrared singularity in a single Feynman diagram is the case only for the scaling solution. One may therefore speculate that for a decoupling solution only a resummation of infinitely many diagrams will be able to describe the axial anomaly.

## 4. Summary

The unique scaling and the family of decoupling solutions of the functional equations of Yang-Mills theory have been presented. It has been conjectured that the appearance of several solutions is related to the choice of gauge [36]. This would especially imply that all these solutions give the same values for physical observables.

A relatively simple form for the analytic structure of the gluon propagator has been suggested [40]. It has the remarkable property that it allows a Wick rotation.

<sup>5</sup>For an introduction to confining field configurations see *e.g.* Refs. [52, 53].



The quark-gluon vertex plays a double role in dynamical chiral symmetry breaking: This vertex triggers and is subject of the symmetry breaking [43]. This results in a quite complicated Dirac structure of the static linearly rising quark potential. Analytical results for the scaling solution for a fundamentally charged scalar exist [45]. However, lattice results do not corroborate them [44].

The infrared singularities of the quark-gluon vertex for soft gluon momenta generate in the scaling scenario an  $\eta'$  mass and the axial anomaly [50]. To my best knowledge, it is yet unknown how the axial anomaly is encoded in the elementary Green's functions of the decoupling solution.

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## References

- [1] D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30** (1973) 1343.
- [2] H. D. Politzer, Phys. Rev. Lett. **30** (1973) 1346.
- [3] H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. B **47** (1973) 365.
- [4] C. Vafa and E. Witten, Nucl. Phys. B **234** (1984) 173; Phys. Rev. Lett. **53** (1984) 535.
- [5] G. 't Hooft, Phys. Rev. D **14** (1976) 3432 [Erratum-ibid. D **18** (1978) 2199].
- [6] H. Leutwyler and A. V. Smilga, Phys. Rev. D **46** (1992) 5607.
- [7] S. R. Coleman, Subnucl. Ser. **15** (1979) 805.
- [8] J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.
- [9] L. von Smekal, A. Mecke and R. Alkofer, arXiv:hep-ph/9707210.
- [10] A. Maas, PoS **FacesQCD** (2011) 033 [arXiv:1102.0901 [hep-lat]].
- [11] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. **66**, 1 (1979).
- [12] N. Nakanishi and I. Ojima, World Sci. Lect. Notes Phys. **27** (1990) 1.
- [13] L. von Smekal and R. Alkofer, arXiv:hep-ph/0009219.
- [14] N. Alkofer and R. Alkofer, PoS **FacesQCD** (2011) 043.
- [15] N. Alkofer and R. Alkofer, arXiv:1102.2753 [hep-th].
- [16] R. Oehme and W. Zimmermann, Phys. Rev. D **21**, 471 (1980).
- [17] R. Alkofer and L. von Smekal, Phys. Rept. **353** (2001) 281 [arXiv:hep-ph/0007355].
- [18] T. Kugo, arXiv:hep-th/9511033.
- [19] L. von Smekal, R. Alkofer and A. Hauck, Phys. Rev. Lett. **79** (1997) 3591 [arXiv:hep-ph/9705242].

- [20] P. Watson and R. Alkofer, Phys. Rev. Lett. **86** (2001) 5239 [arXiv:hep-ph/0102332].
- [21] D. Zwanziger, Phys. Rev. D **65** (2002) 094039 [arXiv:hep-th/0109224].
- [22] C. Lerche and L. von Smekal, Phys. Rev. D **65** (2002) 125006 [arXiv:hep-ph/0202194].
- [23] C. S. Fischer and R. Alkofer, Phys. Lett. B **536** (2002) 177 [arXiv:hep-ph/0202202].
- [24] J. M. Pawłowski *et al.*, Phys. Rev. Lett. **93** (2004) 152002 [arXiv: hep-th/0312324].
- [25] C.S. Fischer, A. Maas and J.M. Pawłowski, Ann. Phys. **324** (2009) 2408 [arXiv:0810.1987 [hep-ph]].
- [26] P. O. Bowman *et al.*, Phys. Rev. D **76** (2007) 094505 [arXiv:hep-lat/0703022].
- [27] A.C. Aguilar, D. Binosi, J. Papavassiliou, Phys. Rev. D **78** (2008) 025010 [arXiv:0802.1870 [hep-ph]].
- [28] P. Boucaud *et al.*, JHEP **0806** (2008) 099 [arXiv: 0803.2161 [hep-ph]].
- [29] V. Mathieu, PoS **QCD-TNT09** (2009) 024 [arXiv:0910.4855 [hep-ph]]; talk given at this workshop.
- [30] R. Alkofer, C. S. Fischer and F. J. Llanes-Estrada, Phys. Lett. B **611** (2005) 279 [arXiv:hep-th/0412330].
- [31] M. Q. Huber *et al.*, Phys. Lett. B **659** (2008) 434 [arXiv:0705.3809 [hep-ph]].
- [32] D. Zwanziger, Phys. Rev. D **69** (2004) 016002 [arXiv: hep-ph/0303028].
- [33] C. S. Fischer and J. M. Pawłowski, Phys. Rev. D **75** (2007) 025012 [arXiv:hep-th/0609009].
- [34] C. S. Fischer and J. M. Pawłowski, Phys. Rev. D **80** (2009) 025023 [arXiv:0903.2193 [hep-th]].
- [35] R. Alkofer, M.Q. Huber, K. Schwenzer, Phys. Rev. D **81** (2010) 105010 [arXiv:0801.2762 [hep-th]].
- [36] A. Maas, Phys. Lett. B **689** (2010) 107 [arXiv:0907. 5185 [hep-lat]];
- [37] A. Sternbeck and L. von Smekal, Eur. Phys. J. C **68** (2010) 487 [arXiv:0811.4300 [hep-lat]].
- [38] A. Maas *et al.*, Eur. Phys. J. C **68** (2010) 183 [arXiv: 0912.4203 [hep-lat]].
- [39] A. Cucchieri and T. Mendes, Phys. Rev. D **81** (2010) 016005 [arXiv:0904.4033 [hep-lat]].
- [40] R. Alkofer *et al.*, Phys. Rev. D **70** (2004) 014014; [arXiv:hep-ph/0309077];  
Nucl. Phys. Proc. Suppl. **141** (2005) 122 [arXiv:hep-ph/0309078].
- [41] C. S. Fischer and R. Alkofer, Phys. Rev. D **67**, 094020 (2003) [arXiv:hep-ph/0301094].
- [42] R. Alkofer, C.S. Fischer, F.J. Llanes-Estrada, Mod.Phys Lett. **A23** (2008) 1105 [hep-ph/0607293].
- [43] R. Alkofer *et al.*, Annals Phys. **324** (2009) 106 [arXiv: 0804.3042 [hep-ph]].
- [44] A. Maas, Eur. Phys. J. C **71** (2011) 1548 [arXiv:1007.0729 [hep-lat]].
- [45] L. Fister, R. Alkofer and K. Schwenzer, Phys. Lett. B **688** (2010) 237 [arXiv:1003.1668 [hep-th]].
- [46] L. Fister, Diploma thesis, University Graz, 2009 [arXiv:1002.1649 [hep-th]].
- [47] R. Alkofer, L. Fister, A. Maas and V. Macher, arXiv:1011.5831 [hep-ph].
- [48] R. Alkofer, M. Q. Huber and K. Schwenzer, Comput. Phys. Commun. **180** (2009) 965 [arXiv:0808.2939 [hep-th]].
- [49] M. Q. Huber and J. Braun, private communication.
- [50] R. Alkofer, C. S. Fischer and R. Williams, Eur. Phys. J. A **38** (2008) 53 [arXiv:0804.3478 [hep-ph]].
- [51] J. Gattnar, K. Langfeld and H. Reinhardt, Phys. Rev. Lett. **93** (2004) 061601 [arXiv:hep-lat/0403011].
- [52] R. Alkofer and J. Greensite, J. Phys. G **34**, S3 (2007) [arXiv:hep-ph/0610365].
- [53] J. Greensite, Lect. Notes Phys. **821** (2011) 1.