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Vacuum expectation value of A^2 from LQCD.

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We argue from LQCD that there is a non vanishing v.e.v of $A_a^\mu A_\mu^a$ in QCD in the Landau gauge. We use operator product expansion to provide a clear definition of $A_a^\mu A_\mu^a$ and extract a number both in the quenched and unquenched case.

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1. Introduction

"Natura abhorret vaccum" was an antique saying. Modern science has strangely supported this intuition when discovering that vacuum is very far from being empty. There are quantum fluctuations, virtual e^+e^- and $q\bar{q}$ pairs and vacuum expectation values (v.e.v). Our target here is the v.e.v of $A^2 \equiv A_a^\mu A_\mu^a$ in Landau gauge.

Is it legitimate to speak about a gauge dependent property of a gauge invariant vacuum ? Yes, of course, once we have understood that there are fields in the vacuum. These fields may be scrutinized in different gauges, the appearance will differ, the physics we learn will vary, and nevertheless the vacuum is gauge invariant. It is absolutely similar to the well known fact that a physical process, say some reaction, is decomposed differently into Feynman diagrams according to the chosen gauge, its appearance will depend on the gauge although the process is gauge invariant and the cross section will not depend on the gauge.

A^2 is certainly not a gauge invariant quantity, but in Landau gauge it is invariant for infinitesimal gauge tranformations, and consequently, for BRST transformations.

Why do we care about $\langle A^2 \rangle$? It is useful

- in order to understand the infrared properties of QCD. It is enough to quote an incomplete list of studies of QCD in the infrared which use $\langle A^2 \rangle$ [1]-[15], many of the authors being in the audience;
- in order to identify correctly non-perturbative corrections to renormalisation constants;
- in order to test and calibrate on a rather extensively computable case Operator Product Expansion : we find larger non-perturbative corrections than usually expected.

2. How to define $\langle A^2 \rangle$?

A naive estimate of $\langle A^2 \rangle$ produces an ultraviolet divergent quantity $\propto a^{-2}$ (a being the lattice spacing). Indeed we are interested in infrared modes and we must define a scale μ which, grossly speaking, separates the high and low modes. A way to perform rigorously the distinction between UV and IR modes is to use **Operator Product Expansion (OPE)**. One must then define a renormalisation scheme and a renormalisation scale.

3. Wilson Operator expansion

A momentum dependent quantity $Q(p^2)$ with vacuum quantum numbers can be expanded in inverse powers of μ^2

$$Q(p^2) = Q_{\rm pert}(p^2,\mu^2) + C^Q_{\rm wilson}(p^2,\mu^2)\langle A^2(\mu^2)\rangle + ... \tag{3.1}$$

where $Q_{\rm pert}(p^2,\mu^2)$ and $C^Q_{\rm wilson}(p^2,\mu^2) \sim 1/\mu^2$ are series in $\alpha(\mu^2)$ computable in perturbative QCD. Q can be the strong coupling constant, the quark field renormalisation constant, other renormalisation constants, etc. The coefficients $C^Q_{\rm wilson}$ are often called Wilson coefficients.

OPE has been extensively used in phenomenology since the pioneering work by SVZ [16][1].

[1] And we have the pleasure to have "Z" in the audience.

3.1 Criteria to check that we really measure $\langle A^2(\mu^2) \rangle$.

- The use of OPE is criticized under the argument that it is difficult to distinguish higher orders in the perturbative series (which are logarithmically suppressed $\propto 1/\log(p^2)$) from OPE contributions which are power suppressed ($\propto 1/p^2$) and that perturbative renormalons mimic a condensate, being also $\propto 1/p^2$. This issue has been carefully studied by Martinelli and Sachrajda (MS) in [17]. They show, using a specific model, that when subtracting with proper factors two quantities, say in our case Z_q and α_T, renormalon ambiguities cancel. One must still check that the renormalon-free neglected terms in the perturbative expansion are not larger than the condensate. The criterium proposed by MS is that we must check that the last perturbative contribution considered (say the order α^3 or α^4 depending on what is available) is significantly smaller than the power correction. This criterium has been checked in [14] concluding that the MS rule was satisfied on the full momentum range which is used.
- $\langle A^2(\mu^2) \rangle$ is not a lattice artefact. It is a quantity defined in the continuum limit. Therefore we must check that what we interpret as $\langle A^2(\mu^2) \rangle$ does not depend significanlty on the lattice spacing.
- We must also check that $\langle A^2(\mu^2) \rangle$ derived from different quantities are consistent provided we use the same renormalisation scheme and scale.
- $\langle A^2(\mu^2) \rangle$ depends on the vacuum and there is no reason for the values of $\langle A^2(\mu^2) \rangle$ extracted from $N_f = 0$ and $N_f = 2$ lattice data to agree.

3.2 About Wilson coefficients

All Wilson coefficients $C^Q_{\text{wilson}}(p^2, \mu^2)$ are of the type

$$C^Q_{\text{wilson}}(p^2,\mu^2) = d^Q\text{tree}\; g^2(\mu^2)\, \frac{1+\mathscr{O}(\alpha)}{p^2} \quad \text{where}$$
$$d^Q\text{tree} = \frac{1}{12} \;\text{ for }\; Q = Z_q, \qquad d^Q\text{tree} = \frac{9}{32} \;\text{ for }\; Q = \alpha_T. \tag{3.2}$$

From eq. (3.1) and eq. (3.2) we see that there is always the same factor $\langle g^2(\mu^2)A^2(\mu^2)\rangle_{\overline{\text{MS}}}$ and we will therefore give the fitted values of the condensate as $\langle g^2(\mu^2)A^2(\mu^2)\rangle_{\overline{\text{MS}}}$. Indeed, in the following, we choose to renormalise the Wilson coefficient such that the local operator $A^2(\mu^2)$ is in the $\overline{\text{MS}}$ scheme, whichever prescription we take for the perturbative part of Q.

In practice one can show that the best and most general fitting formula is:

$$Q(p^2) = Q_{\text{pert}}(p^2,\mu^2)\left(1 + \frac{C^Q_{\text{wilson}}(p^2,\mu^2)}{Q_{\text{pert}}(p^2,\mu^2)}\;\langle A^2(\mu^2)\rangle_{\overline{\text{MS}}}\right) \tag{3.3}$$

At leading logarithm for the non-perturbative correction,

$$\frac{C^Q_{\text{wilson}}(p^2,\mu^2)}{Q_{\text{pert}}(p^2,\mu^2)}\;\langle A^2(\mu^2)\rangle_{\overline{\text{MS}}} = d^Q\text{tree}\;\langle g^2(\mu^2)A^2(\mu^2)\rangle_{\overline{\text{MS}}}\left(\frac{\alpha(p^2)}{\alpha(\mu^2)}\right)^e$$
$$\text{where} \qquad e = \frac{9}{44-\frac{8N_f}{3}}. \tag{3.4}$$

Notice that e is small. Therefore the corrective factor in eq. (3.4) is almost scale invariant. The exponent is the same for all quantities. In eq. (3.4) the only term which depends on the measured quantity is d^Qtree.

3.3 How to compute $\langle A^2(\mu^2)\rangle_{\overline{\mathrm{MS}}}$ from LQCD ?

- Compute Q_{latt} as measured from LQCD.
- Correct for hypercubic lattice artefacts i.e. artefacts related to the hypercubic geometry.
- Fit according to

$$Q(p^2) = Q_{\mathrm{pert}}(p^2,\mu^2)\left(1+\frac{C^Q_{\mathrm{wilson}}(p^2,\mu^2)}{Q_{\mathrm{pert}}(p^2,\mu^2)}\ \langle A^2(\mu^2)\rangle_{\overline{\mathrm{MS}}}\right)+c_{a2p2}\,a^2p^2$$

  where $C_{a2p2}\,a^2p^2$ is a simple model for the non-hypercubic remaining lattice artefacts, which turns out to give a good result, as we shall illustrate below.

Let us now apply this strategy.

4. The strong coupling constant

There are many ways to define the strong coupling constant. We will use [18, 22] what we call the "Taylor coupling constant" which, thanks to Taylor's theorem [19], is only dependent on the gluon and ghost propagators.

We will use configurations with Wilson twisted quarks (Nf=2) from the ETM collaboration and compare them to quenched configurations (Nf=0).

4.1 Some definitions

The gluon propagator ($G^{(2)}$) and the ghost propagator ($F^{(2)}$) are defined as follows, $G(p^2,\Lambda)$ and $F(p^2,\Lambda)$ being named the "dressing functions":

$$\begin{aligned}
\left(G^{(2)}\right)^{ab}_{\mu\nu}(p^2,\Lambda) &= \frac{G(p^2,\Lambda)}{p^2}\,\delta_{ab}\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\ ,\\
\left(F^{(2)}\right)^{a,b}(p^2,\Lambda) &= -\delta_{ab}\,\frac{F(p^2,\Lambda)}{p^2}\ ;
\end{aligned}\tag{4.1}$$

Λ being some regularisation parameter: $\Lambda = a^{-1}(\beta)$ if, for instance, we specialise to lattice regularisation. The renormalised dressing functions, G_R and F_R are defined through :

$$\begin{aligned}
G_R(p^2,\mu^2) &= \lim_{\Lambda\to\infty} Z_3^{-1}(\mu^2,\Lambda)\,G(p^2,\Lambda)\\
F_R(p^2,\mu^2) &= \lim_{\Lambda\to\infty} \widetilde{Z}_3^{-1}(\mu^2,\Lambda)\,F(p^2,\Lambda)\ ,
\end{aligned}\tag{4.2}$$

with renormalisation condition

$$G_R(\mu^2,\mu^2) = F_R(\mu^2,\mu^2) = 1 \ . \tag{4.3}$$

Now, we will consider the ghost-gluon vertex which could be non-perturbatively obtained through a three-point Green function, defined by two ghost and one gluon fields, with amputated legs after dividing by two ghost and one gluon propagators. This vertex can be written quite generally as:

$$\widetilde{\Gamma}^{abc}_\nu(-q,k;q-k) = i g_0 f^{abc} \left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k) \right) \ , \tag{4.4}$$

The vertex renormalisation constant is defined as

$$\left(H_1^R(q,k) + H_2^R(q,k)\right)\big|_{q^2=\mu^2} = \lim_{\Lambda\to\infty} \widetilde{Z}_1(\mu^2,\Lambda) \left(H_1(q,k;\Lambda) + H_2(q,k;\Lambda)\right)\big|_{q^2=\mu^2} = 1, \tag{4.5}$$

The renormalised coupling constant is defined as

$$\begin{aligned} g_R(\mu^2) \ &= \ \lim_{\Lambda\to\infty} \widetilde{Z}_3(\mu^2,\Lambda) Z_3^{1/2}(\mu^2,\Lambda) g_0(\Lambda^2) \left(H_1(q,k;\Lambda) + H_2(q,k;\Lambda) \right)\bigg|_{q^2\equiv\mu^2} \\ &= \ \lim_{\Lambda\to\infty} g_0(\Lambda^2) \, \frac{Z_3^{1/2}(\mu^2,\Lambda^2)\widetilde{Z}_3(\mu^2,\Lambda^2)}{\widetilde{Z}_1(\mu^2,\Lambda^2)} \ . \end{aligned} \tag{4.6}$$

Now we choose a special kinematics where the incoming ghost momentum vanishes. Taylor's theorem states that $H_1(q,0;\Lambda) + H_2(q,0;\Lambda)$ is equal to 1 in full QCD for any value of q. Therefore, the renormalisation condition eq. (4.5) implies $\widetilde{Z}_1(\mu^2) = 1$ and then

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda^2)}{4\pi} G(\mu^2,\Lambda^2) F^2(\mu^2,\Lambda^2) \ ; \tag{4.7}$$

which only depends on the propagators.

4.2 Perturbative running of α_T

The four-loops expression for the coupling constant in the Taylor scheme as a function of Λ_T ($\Lambda_{\rm QCD}$ in this scheme) is given by [20, 21, 22]:

$$\begin{aligned} \alpha_T(\mu^2) \ &= \ \frac{4\pi}{\beta_0 t} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\log(t)}{t} + \frac{\beta_1^2}{\beta_0^4} \frac{1}{t^2} \left(\left(\log(t) - \frac{1}{2} \right)^2 + \frac{\widetilde{\beta}_2 \beta_0}{\beta_1^2} - \frac{5}{4} \right) \right) \\ &+ \ \frac{1}{(\beta_0 t)^4} \left(\frac{\widetilde{\beta}_3}{2\beta_0} + \frac{1}{2} \left(\frac{\beta_1}{\beta_0} \right)^3 \left(-2\log^3(t) + 5\log^2(t) + \left(4 - 6\frac{\widetilde{\beta}_2\beta_0}{\beta_1^2} \right) \log(t) - 1 \right) \right) \end{aligned} \tag{4.8}$$

where $t = \ln \frac{\mu^2}{\Lambda_T^2}$ and coefficients are

$$\begin{aligned} \beta_0 \ &= \ 11 - \frac{2}{3} N_f \ , \ \ \beta_1 = \overline{\beta}_1 = 102 - \frac{38}{3} N_f \\ \widetilde{\beta}_2 \ &= \ 3040.48 \ - \ 625.387 \, N_f \ + \ 19.3833 \, N_f^2 \\ \widetilde{\beta}_3 \ &= \ 100541 \ - \ 24423.3 \, N_f \ + \ 1625.4 \, N_f^2 \ - \ 27.493 \, N_f^3 \ , \end{aligned} \tag{4.9}$$

Λ_T is converted into $\Lambda_{\overline{\rm MS}}$ by

$$\frac{\Lambda_{\overline{\rm MS}}}{\Lambda_T} = e^{-\dfrac{c_1}{2\beta_0}} = e^{-\dfrac{507-40N_f}{792-48N_f}} \ . \tag{4.10}$$

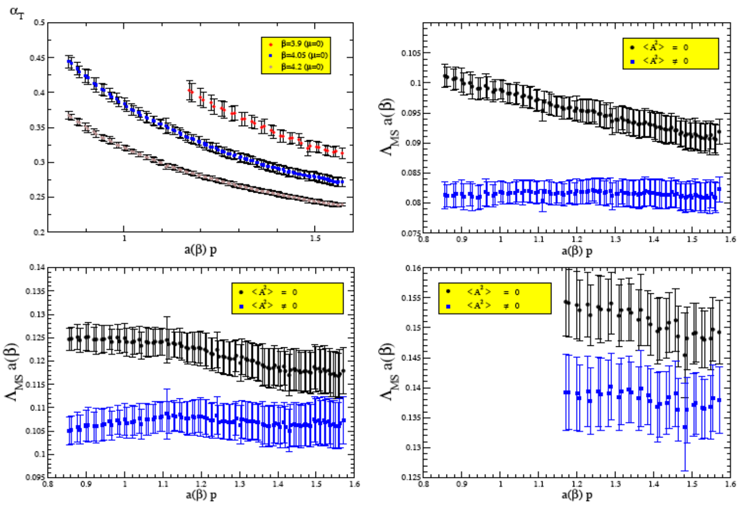
4.3 The non-perturbative contribution to α_T

It is easy to see that the dominant non-perturbative contribution to $\alpha_T(\mu^2)$ is the condensate $\langle A^2 \rangle$. To take it into account in our fits we will need the Wilson coefficient $C^{\alpha_T}_{\text{wilson}}$. This is possible up to three loops thanks to [8] where the $\langle A^2 \rangle$ correction to the ghost and gluon propagators have been computed. We will only write here the result at leading logarithm [22].

$$\alpha_T(\mu^2) \;=\; \alpha_T^{\text{pert}}(\mu^2) \left(1 + \frac{9}{\mu^2} \left(\frac{\ln \frac{\mu^2}{\Lambda^2_{QCD}}}{\ln \frac{\mu_0^2}{\Lambda^2_{QCD}}} \right)^{-9/(44-8N_f/3)} \frac{g_T^2(\mu_0^2)\langle A^2 \rangle_{R,\mu_0^2}}{4(N_C^2-1)} \right), \tag{4.11}$$

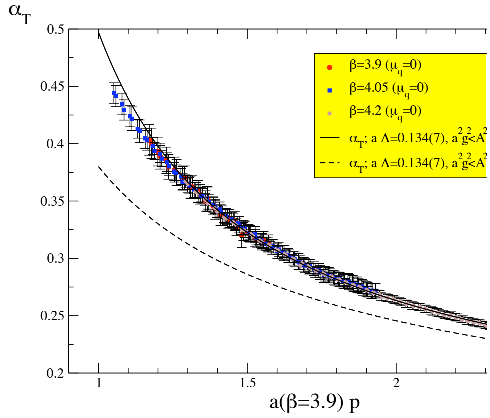
where μ_0 is our reference renormalisation scale which we take to be 10 GeV.

4.4 Results of the lattice simulation



We perform the lattice calculation of $\alpha_T(\mu^2)$ from eq. (4.7), using $N_f = 2$ with twisted dynamical quarks from the ETMC collaboration. We eliminate the hypercubic lattice artefacts. The results depend on the dynamical quark mass. For three values of the lattice spacing (labelled by β) we extrapolate to zero mass leading to the three sets of data in the upper-left plot of the figure aside.

We invert eq. (4.8) to extract Λ_T as a function of μ^2 and then $\Lambda_{\overline{\text{MS}}}$ from eq. (4.10). Since Λ_T and $\Lambda_{\overline{\text{MS}}}$ are constants independent on the scale, the result should be a nice "plateau" if we were in the full perturbative regime. This is obviously not the case as shown in black for the three lattice spacings in the upper-right and two down plots. We then use eq. (4.11), fitting $g^2(\mu^2)\langle A^2 \rangle_{R,\mu_0^2}$ to have $\Lambda_{\overline{\text{MS}}}$ as a nice plateau. The result is shown in blue. It looks quite convincing. **From the comparison of the black and blue curves it is evident that, although the fit has been performed in the rather high-energy range 2.6-6.4, a sizeable non perturbative correction is needed**.



In the plot on the left we have merged the three curves of $\alpha_T(\mu^2)$ by fitting the lattice spacing ratios to have the best matching. The compatibility of the three curves is quite impressive and the needed lattice spacing ratios agree very well with what is extracted from the interquark potential [22]. The fit has been performed both using the leading logarithm [22] (LL) and the three loop [8] ($\mathscr{O}(\alpha^4)$) formula for the Wilson coefficient. The result is:

$$\Lambda_{\overline{\rm MS}} = 330 \pm 32^{+0}_{-33}\ \text{MeV} \tag{4.12}$$
$$g^2(\mu^2)\langle A^2\rangle_{R,\mu} = 4.4 \pm 1.5 \pm 0.7\ \text{GeV}^2(\text{LL}) \qquad g^2(\mu^2)\langle A^2\rangle_{R,\mu} = 2.7 \pm 1.0 \pm 0.7\text{GeV}^2(\mathscr{O}(\alpha^4))$$

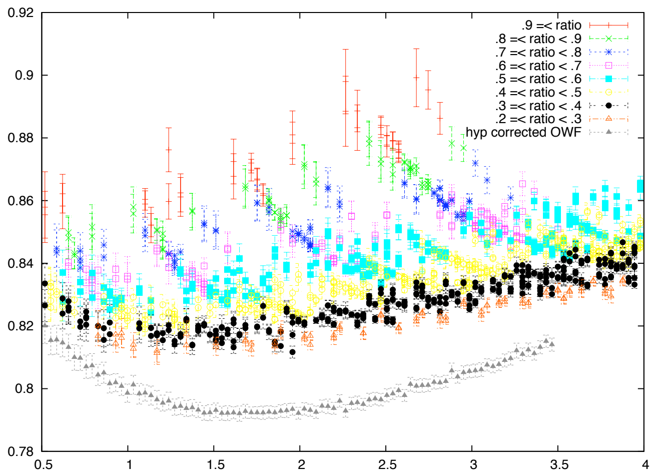
5. The quark field renormalisation constant Z_q

- We compute the Fourier transform of the lattice quark propagator for momentum p: it is a 12×12 matrix $S(p)$. We define the renormalised quark field

$$q_{\rm R} = Z_q\, q_{\rm bare} \qquad \text{whence} \qquad Z_q(\mu^2 = p^2) \equiv \frac{-i}{12p^2}\text{Tr}\left[\frac{S^{-1}_{\rm bare}(p)\, \not{p}}{p^2}\right] \tag{5.1}$$

- We suppress the hypercubic artefacts which in this case are particularly large.
- We perform the fit

$$Z_{\rm qlatt}(p^2) = Z_{\rm pert}(p^2,\mu^2)\left(1 + \frac{C^Z_{\rm wilson}(p^2,\mu^2)}{Z_{\rm pert}(p^2,\mu^2)}\langle A^2(\mu^2)\rangle\right) + c_{\rm a2p2}\, a^2 p^2 \tag{5.2}$$

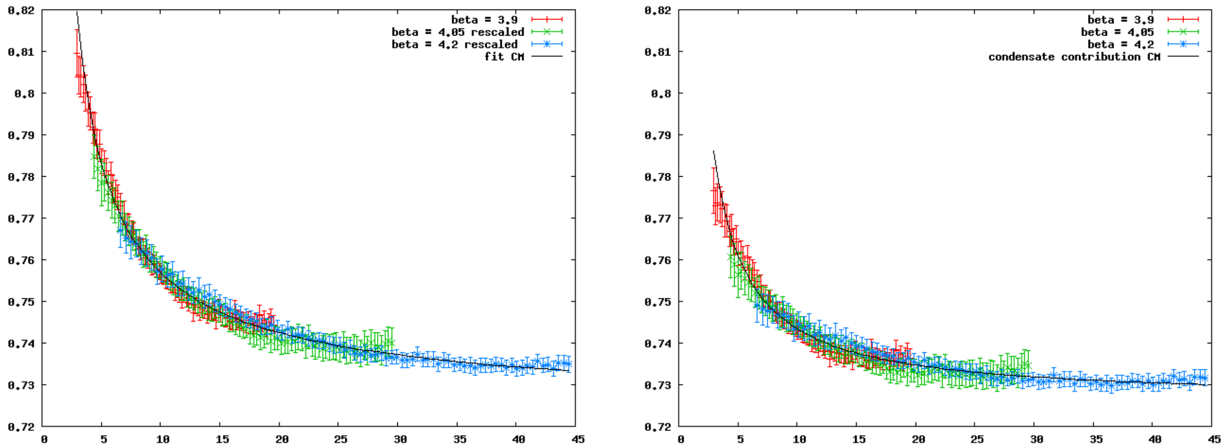


The plot on the left shows the result for $Z_{\rm qlatt}$ as a function of p^2 stemming directly from the lattice calculation. It is very far from the smooth dependence expected in the continuum. One sees a “half-fishbone" structure which is a dramatic expression of hypercubic artefacts. Let us define

$$p^{[4]} = \sum_{\mu=1}^{4} p_\mu^4 \qquad \text{ratio} \equiv \frac{p^{[4]}}{(p^2)^2}$$

In the above plot the color code corresponds to the value of the parameter “ratio" which is bounded $0.25 \leq \text{ratio} \leq 1$. It is visible that the hypercubic artefacts increase with this parameter. We reduce drastically these artefacts by an extrapolation down to ratio$= 0$ [12] :

$$Z_q^{\text{latt}}(a^2 p^2, a^4 p^{[4]}, a^6 p^{[6]}, ap_4, a^2\Lambda_{\text{QCD}}^2) = Z_q^{\text{hyp\_corrected}}(a^2 p^2, a^2\Lambda_{\text{QCD}}^2) + c_{a2p4}\, a^2 \frac{p^{[4]}}{p^2} + c_{a4p4} a^4\, p^{[4]} \tag{5.3}$$



The merged plot results with $\beta = 4.05$ and $\beta = 4.2$ rescaled to the $\beta = 3.9$. The l.h.s shows the data corrected for all lattice artefacts. The r.h.s shows the same data furthermore corrected by the perturbative running factor up to 10 GeV. The horizontal axix is p^2 in GeV ^{2}. The black line on the l.h.s corresponds to the global fit with perturbative running and the three-loops (Chetyrkin-Maier [8]) Wilson coefficient for the $1/p^2$ term. The black line on the r.h.s corresponds only to the $1/p^2$ times the three loops wilson coefficient added to $Z_q^{\text{pert}}((10\,\text{GeV})^2, 6/3.9) = 0.726$

Once this non-perturbative hypercubic correction is performed, we merge the results for three lattice spacings. The merged result is shown in the figure above l.h.s. In the r.h.s these data are corrected by the perturbative running factor up to 10 GeV. If non-perturbative corrections were absent the curve should be flat. This is obviously not the case. The results are shown in table 1.

β	a^2 fm^2	Z_q^{pert}	c_{a2p2}	$g^2\langle A^2\rangle_{tree}$	$g^2\langle A^2\rangle_{CM}$
3.9	0.00689	0.726(5)	0.0201(13)	3.20(38)	2.62(31)
4.05	0.00456	0.742(5)	0.0200(15)	3.09(65)	2.57(54)
4.2	0.00303	0.760(3)	0.0194(8)	3.23(55)	2.74(47)
average			0.0201(3)	3.18(28)	2.64(23)

Table 1: Results for Z_q^{pert} (10GeV) and c_{a2p2} eq. (5.2) and the estimated $g^2\langle A^2\rangle$ v.e.v from the tree level $1/p^2$ term and from the Chetyrkin-Maier [8] (CM) Wilson coefficient.

6. conclusion

It is striking that the fitted value of $g^2\langle A^2\rangle$ are independent of the lattice spacings. It is even more striking that the values extracted from the quark renormalisation constant and the coupling constant are perfectly compatible.

This is very encouraging in the sense that all our criteria to identify really a $g^2\langle A^2\rangle$ condensate are fulfilled.

N_f	order $g^2\langle A^2\rangle_{tree}$	Z_q	α_T	3 gluons
0	LL	9.4(3)	5.2(1.1)	10(3)
0	$O(\alpha^4)$	9.0(3)	3.7(8)	
2	LL	2.7(4)	4.4(1.6)	
2	$O(\alpha^4)$	2.55(36)	2.7(1.0)	

Table 2: Comparison of estimates of $g^2\langle A^2\rangle$ from different quantities at $N_f = 0$ and $N_f = 2$. All are taken at the scale $\mu = 10$ GeV. LL means leading logarithm for the Wilson coefficient. $O(\alpha^4)$ refers to Chetyrkin-Maier's [8] computation.

In table 2 we perform a global comparison of the estimates of $g^2(\mu^2)\langle A^2\rangle_\mu$ both in the quenched and unquenched case. The agreement between $g^2(\mu^2)\langle A^2\rangle_\mu$ estimated from different observables is not very good in the quenched case. This does not induce in our mind any doubt about the existence of a $g^2(\mu^2)\langle A^2\rangle_\mu$ ($\mu = 10$ GeV) condensate in Landau gauge for the following reason: we have performed a large number of fits with different inputs, and we have never found $g^2(\mu^2)\langle A^2\rangle_\mu$ compatible with zero at better than four sigmas. On the other hand the fitting procedure is certainly delicate due to many correlations between $g^2(\mu^2)\langle A^2\rangle_\mu$ and lattice artefacts. Our final conclusion is **that the accurate estimate of its value needs some improvements in our fitting method due to several correlations difficult to disentangle. There are however strong evidences in favor of the existence of a positive condensate $g^2(\mu^2)\langle A^2\rangle_\mu$ in the range 2-10 GeV^2 in the $\overline{MS}$ scheme at $\mu = 10$ GeV**.

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