PROCEEDINGS OF SCIENCE

Cyclostationarity for phased array radio telescope

Rodolphe Weber[*ab], **Rym Feliachi**[b] **and Albert-Jan Boonstra**[c]

[a] *Observatoire de Paris, Station de radioastronomie, F-18330 Nançay, France*

[b] *Laboratoire PRISME, Université d'Orléans, Site Galilée, 12 rue de Blois, F-45067 Orléans cedex 2, France*

[c] *ASTRON, Oude Hoogeveensedijk 4,7991 PD Dwingeloo, P.O. Box 2, 7990 AA Dwingeloo, The Netherlands*

E-mail: rodolphe.weber@univ-orleans.fr, rym.feliachi@univ-orleans.fr, boonstra@astron.nl

Most telecommunication signals contain hidden periodicities due to the periodic characteristics involved in the signals construction (carrier frequency, baud rate, coding scheme...). These periodicities are usually scrambled and hidden by the randomness of the transmitted message. However, by using a cyclostationary approach, this hidden periodicity can be recovered, making the identification of telecommunication signals possible. In this paper, it is shown how cyclostationary spatial processing techniques can limit the impact of the incoming radio frequency interference (RFI) for phased array radio telescopes. Cyclostationarity can be exploited for RFI detection purposes or for filtering purposes. These two RFI mitigation techniques are illustrated through simulations on data acquired with the Westerbork Telescope and the Low Frequency Array Radio telescope, LOFAR, in the Netherlands.

RFI mitigation workshop - RFI2010,
March 29-31, 2010
Groningen, the Netherlands

[*] Speaker.

PoS(RFI2010)033

1. What is the cyclostationarity ?

Mathematically, a cyclostationary process has the property that its statistics are periodic with time. For example, let us consider the second order statistics given by the correlation of a given process $r(t)$:

$$R_r(t,\tau) = \left\langle r(t-\frac{\tau}{2})r(t+\frac{\tau}{2})\right\rangle_\infty \tag{1.1}$$

where $\langle . \rangle_\infty$ represents the ensemble average operator or the time average operator.

If the process is modeled as stationary then $R_r(t,\tau) = R_r(\tau)$. If the process is modeled as cyclostationary then it exists T such as $R_r(t+T,\tau) = R_r(t,\tau)$. T is called the cyclic period. Review papers on cyclostationarity can be found in Gardner [2] or Serpedin[1].

To illustrate these concepts, let us consider the following simple baseband signal, $r(t) = \sum_{k\in Z} a_k g(t-kT)$, where a_k is a random digital message with power σ_a^2, $g(t)$, its pulse shape and T, its baud rate. The correlation of $r(t)$ becomes :

$$R_r(t,\tau) = \sigma_a^2 \sum_{k\in Z} g(t-kT+\frac{\tau}{2})g(t-kT-\frac{\tau}{2}) \tag{1.2}$$

One can easily verify that $R_r(t,\tau)$ is T-periodic. So, it can be decomposed in Fourier series and the corresponding Fourier coefficients are ($k \in Z$):

$$R_r^{\alpha=\frac{k}{T}}(\tau) = \frac{\sigma_a^2}{T} \underbrace{\int_{-\infty}^{+\infty} g(t+\frac{\tau}{2})g(t-\frac{\tau}{2})\exp(-j2\pi\alpha t)dt}_{r_g^\alpha(\tau)} \tag{1.3}$$

In practice, a direct approach is preferred and $R_r^\alpha(\tau)$, also called the cyclic correlation, is given by:

$$R_r^\alpha(\tau) = \left\langle r(t+\frac{\tau}{2})r(t-\frac{\tau}{2})\exp(-j2\pi\alpha t)\right\rangle_\infty \tag{1.4}$$

If $R_r^\alpha(\tau)$ is non-zero for some cyclic frequencies, α, with $\alpha \neq 0$, then $r(t)$ can be modeled as cyclostationary. Note that for $\alpha = 0$, we retrieve the expression of the classical correlation. Figure 1 gives the plots of these previous expressions for the considered example with a rectangular pulse function.

In the next section, these definitions and properties will be extended to the multi-dimensional case. For simplicity, we will only consider the case $\tau = 0$.

2. Cyclostationary spatial processing

Consider a telescope array consisting of p antennas. The $p \times 1$ array output vector is noted $\mathbf{z}(t)$. It is assumed that the narrowband condition holds and that geometric delays for each antenna and each impinging source can be represented by phase shifts. In this case, the telescope correlation matrix $\mathbf{R_z}$ can be modeled as:

$$\begin{aligned}\mathbf{R_z} &= \left\langle \mathbf{z}(t)\mathbf{z}^H(t)\right\rangle_\infty \\ &= \mathbf{A_r}\mathbf{R}_r\mathbf{A_r}^H + \mathbf{A_s}\mathbf{R_s}\mathbf{A_s}^H + \mathbf{N}\end{aligned} \tag{2.1}$$

PoS(RFI2010)033

where $(.)^H$ is the conjugate transpose operator, $\mathbf{R_r}$ is the $K_r \times K_r$ correlation matrix due to the K_r α-cyclostationary sources (i.e. the RFI), $\mathbf{R_s}$ is the $K_s \times K_s$ correlation matrix due to the K_s others sources (i.e. stationary sources and non α-cyclostationary RFI if any) and $\mathbf{N}$ is the $p \times p$ correlation matrix due to the system noise. These matrices contain the signal information. Matrices $\mathbf{A_r}$ ($p \times K_r$) and $\mathbf{A_s}$ ($p \times K_s$) contain the spatial signatures of the impinging sources. At this stage, the phased array is not calibrated. Thus, all spatial signatures are modeled as random phase vectors.

If the cosmic sources are negligible and the system noise is calibrated (i.e. $\mathbf{R_z} \approx \mathbf{A_r}\mathbf{R}_r\mathbf{A_r}^H + \sigma^2\mathbf{I}$ where $\mathbf{I}$ is the identity matrix and σ^2 the system noise power), Boonstra [3] has demonstrated that a close estimate of the information contained in $\mathbf{A_r}$ can be derived from the eigenvalue decomposition (EVD) of $\mathbf{R_z}$.

Indeed, the subspace formed by the eigenvectors associated to the K_r largest eigenvalues (the signal subspace) will span the same dimensions as the RFI spatial signature matrix, $\mathbf{A_r}$, assuming that all other signal contributions in the matrix correlation model are negligible.

The same idea can be applied on the cyclic correlation matrix which is just a multidimensional extension of Equ. 1.4:

$$\mathbf{R}_{\mathbf{z}}^{\alpha} = \left\langle \mathbf{z}(t)\mathbf{z}^{\mathbf{H}}(t)\exp(-j2\pi\alpha t)\right\rangle_{\infty} \tag{2.2}$$

where α is one of the cyclic frequencies which characterize the RFI. Note that by replacing $(.)^H$ by a simple transpose operator $(.)^T$, another set of cyclic frequencies can be used, depending on the kind of modulation involved.

The great interest of the cyclic approach is that $\mathbf{R_z}^{\alpha}$ is asymptotically RFI-only dependent whatever will be the hypotheses on the cosmic sources or the system noise:

$$\mathbf{R_z}^{\alpha} = \mathbf{A_r}\mathbf{R_r}^{\alpha}\mathbf{A_r}^H \tag{2.3}$$

Thus, by using $\mathbf{R_z}^{\alpha}$ rather than $\mathbf{R_z}$, the RFI spatial signature estimation is more robust.

Remark: It is assumed that the K_r RFI sources have the same cyclostationary property. If not, the algorithm will be applied on each group of RFI.

In the following sections, two illustrations of this cyclostationary approach will be presented.

3. Example of cyclostationary spatial detection

Let us note $\mathbf{R_z}(t)$ the instantaneous correlation matrix of the array (i.e. $\mathbf{R_z}(t) = \mathbf{z}(t)\mathbf{z}^H(t)$). As illustrated in figure 2.a, the Fourier Transform of the data cube, formed by stacking all the instantaneous correlation matrices, provides another data cube where the k^{th} slice corresponds to the cyclic correlation matrix, $\mathbf{R_z}^{\alpha}$, at $\alpha = \frac{k}{L}$, L being the length of the Fourier Transform. Then, by computing the Frobenius norm, $\|.\|_F$, of each slice, we obtain a blind cyclic detection criterion which can be used even if the expected cyclic frequency is unknown.

Indeed, $\|\mathbf{R_z}^{\alpha}\|_F^2 = \sum_{k=1}^{K_r}\lambda_{r,k}^2 + \sum_{k=1}^{p-K_r}\lambda_{n,k}^2$ where $\lambda_{r,k}$ are the K_r dominant singular values and $\lambda_{n,k}$ are the $p-K_r$ other ones. Asymptotically and for the RFI cyclic frequency, the rank of $\mathbf{R_z}^{\alpha}$ is K_r. In particular, all the $\lambda_{n,k}$ are null. Thus, a non-zero value of $\|\mathbf{R_z}^{\alpha}\|_F^2$ will indicate the presence of the RFI.

In practice, due the FFT finite length, $\mathbf{R_z}^{\alpha}$ is full rank. Thus, the $\lambda_{n,k}$ will be no more equal to zero making the detector less efficient. In that case, a better (but more computational heavy) approach could be to consider the maximum singular value only.

PoS(RFI2010)033

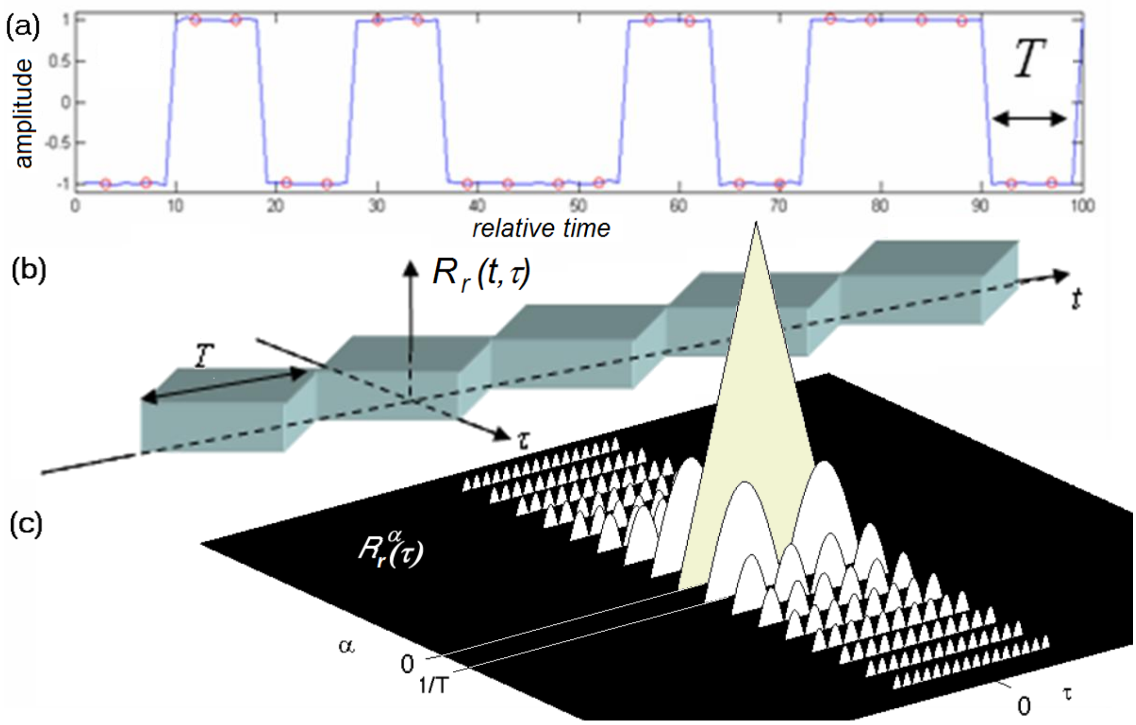

Figure 1: Random binary signal, $r(t) = \sum_{k\in Z} a_k g(t-kT)$ with $g(t)$ rectangular. **(a)** Temporal view. **(b)** correlation of $r(t)$. $R_r(t,\tau)$ is T-periodic. **(c)** the cyclic correlation $R_r^\alpha(\tau)$. $R_r^\alpha(\tau) \neq 0$ for $\alpha = \frac{k}{T}, k \in Z$. A stationary process will provide information only at $\alpha = 0$.

Figure 2.b gives an example with real data from the Westerbork radio telescope. More details can be found in [4].

4. Example of cyclostationary spatial filtering

If $\mathbf{A_r}$ can be estimated, we can filter out the RFI signals by applying a projector on the telescope output:

$$\mathbf{z}_{cleaned}(t) = \mathbf{P}.\mathbf{z}(t) \tag{4.1}$$

where $\mathbf{P}$ is the spatial projector defined by $\mathbf{P} = \mathbf{I} - \mathbf{A_r}(\mathbf{A_r}^H\mathbf{A_r})^{-1}\mathbf{A_r}^H$ and $\mathbf{I}$ is the pxp identity matrix. The projector is still valid, if $\mathbf{A_r}$ is replaced by another matrix,$\mathbf{U}_r$, which spans the same subspace. We have seen previously that such matrix can be derived from the EVD of the correlation matrix,$\mathbf{R_z}$, or from the SVD of the cyclic correlation matrix, $\mathbf{R_z}^\alpha$.

We have applied the classic (i.e. using $\mathbf{R}$) and cyclic (i.e. using $\mathbf{R}^\alpha$) spatial filtering to real observations acquired with the LOFAR radio telescope. The array configuration consisted of M=8 LOFAR antennas. The RFI to be filtered is a very strong transmitter (pager) at 170 MHz.

Figure 3.a shows the eigenvalues obtained from the classic and the cyclic correlation matrix. The cyclic frequency, α, of the pager has been first estimated from the data by using the cyclic blind detector of the previous section. The figure shows that the interferer signal subspace can be fairly well estimated using one dimension in the cyclic decomposition, whereas it needs two dimensions

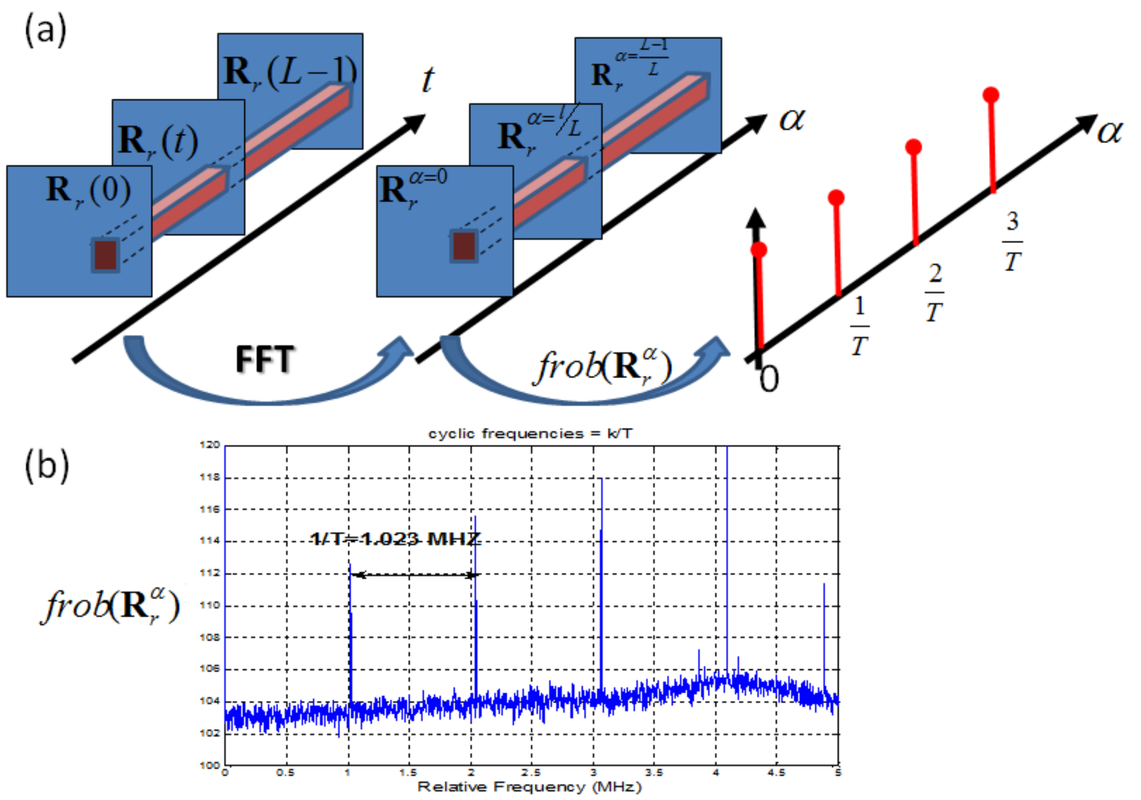

Figure 2: Cyclic spatial detection. (a) The different steps from the instantaneous correlation matrix, $\mathbf{R_z}(t)$, to the detection criterion, $\|\mathbf{R_z}^{\alpha}\|_F^2$. The spectral lines are the signature of cyclostationary RFI (b) Example with real data acquired with the Westerbork telescope. $p = 8$ antennas have been used. The RFI is a GPS satellite. We retrieve spectral lines at cyclic frequencies related to the GPS baudrate.

in the classic one. The more dimensions are used to remove the interferer, the more information about the cosmic sources is thrown away as well. We used therefore only the eigenvector corresponding to the strongest eigenvalue to build the projector for both methods. Figure 3.b shows the effect of the projector on the pager. Using the cyclic method, the pager is removed more effectively compared with the classic approach. More details can be found in [5].

5. Conclusions

In this paper, we have described two RFI mitigation approaches based on the cyclostationary properties of the RFI. The first method is a blind cyclostationary detector , the second one is a cyclic spatial filtering. Both are based on the subspace decomposition of the cyclic correlation matrix. These methods seem to be an attractive alternative to the classic method based on (cross)-power statistics. Feliachi ([6]) has described in her PhD manuscript other applications of cyclostationarity for phased array radio telescopes.

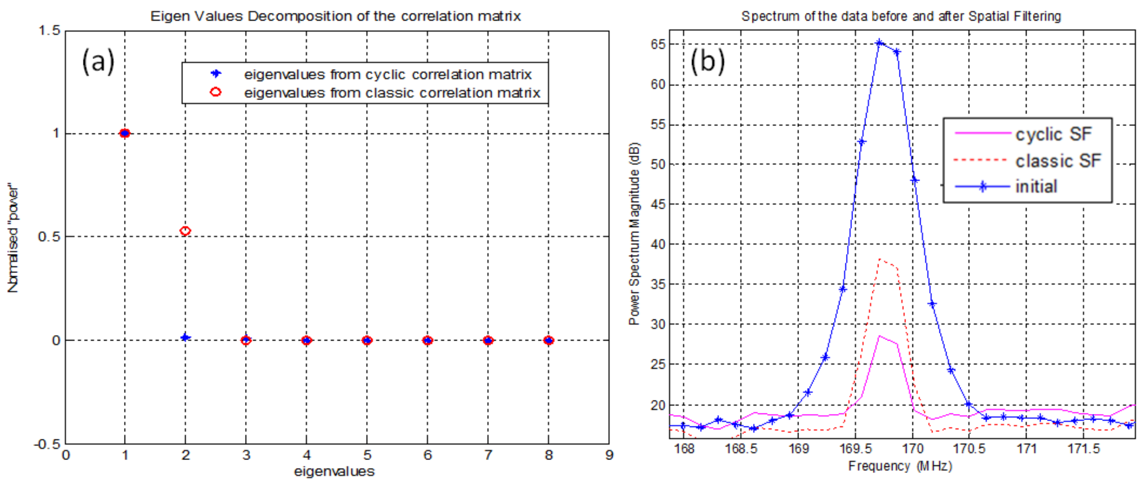

Figure 3: Spatial filtering (SF) of real data acquired with the LOFAR telescope. $p = 8$ antennas have been used. (a) The eigenvalue decomposition of the classic and the cyclic correlation matrices. The correlation matrices have been estimated over $L = 65536$ samples. (b) Spectrum of one antenna output after applying cyclic and classic spatial filtering. A strong transmitter is present in the dataset at 170 MHz.

Acknowledgments

The authors would like to thank the European Commission Framework Program 7 (Project PrepSKA (contract no 212243)) and The French funding agency ANR (contract ANR-09-BLAN-0225-04) for funding part of this work.

References

[1] Serpedin, E., Panduru, F., Sari, I. & Giannakis, G. B. 2005, Bibliography on cyclostationarity (Signal Processing),12, 85

[2] Gardner, W.A, Napolitano, A.& Paura, L. 2006, Cyclostationarity: Half a century of research (Signal Processing), 4 ,86

[3] Boonstra, A. J. 2005, Radio Frequency Interference Mitigation in Radio Astronomy, Thesis, Delft university, The Netherlands.

[4] Millot, A., Feliachi, R., Weber, R., Leger, C. 2010, Wireless water quality monitoring in the ISM frequency band: a software and cognitive radio approach (Eusipco'2010), Aalborg, Denmark.

[5] Feliachi, R., Weber, R., Boonstra, A. J. 2009, Cyclic spatial filtering in radio astronomy : application to LOFAR data (Eusipco'2009), Glasgow, Scotland.

[6] Feliachi, R. 2010, Spatial processing of cyclostationary interferers for phased arrays radio telescopes, Thesis, University of Orléans, France http://www.skads-eu.org/PDF/RymFeliachi-thesis2010.pdf