PROCEEDINGS OF SCIENCE

Threshold-improved predictions for charm production in deep-inelastic scattering

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PoS(DIS 2010)163

We have extended previous results on the threshold expansion of the gluon coefficient function for the charm contribution to the deep-inelastic structure function F_2 by deriving all threshold-enhanced contributions at the next-to-next-to-leading order. The size of these corrections is briefly illustrated, and a first step towards extending this improvement to more differential charm-production cross sections is presented.

XVIII International Workshop on Deep-Inelastic Scattering and Related Subjects
April 19-23, 2010, Convitto della Calza, Firenze, Italy

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1. Introduction

Deep-inelastic scattering (DIS), measured in fixed-target experiments and at HERA, provides core constraints on the parton distributions for the LHC. For some crucial processes, such as gauge-boson and Higgs production, these distributions are required at the next-to-next-to-leading order (NNLO) of perturbative QCD. Consequently coefficient functions at this accuracy are needed also for the extraction of the parton densities from (mainly) the structure function $F_2(x,Q^2)$ in DIS.

For the massless case, these quantities have been known for a long time [1]. However, a considerable part of F_2 at small Bjorken-x is due to the production of charm quarks which is dominated by the photon-gluon fusion process $\gamma^* g \to c\bar{c}X$. The NLO coefficient functions for F_2^c have been obtained in a semi-analytic manner [2]; the results are often used via the parametrizations of Ref. [3] (for minor corrections see Ref. [4]). The corresponding NNLO corrections are not known. Fully analytic NLO results have obtained in the asymptotic limit $m_c^2/Q^2 \to 0$ [5]. Recently these calculations have been extended to NNLO for the lowest even-integer Mellin moments [6].

It has been known for a long time, see, e.g., Refs. [7], that, at not too large values of Q^2, the convolution of the coefficient function for F_2^c and the gluon density is dominated by rather low partonic of-mass energies (CM). Hence the NNLO predictions of the threshold resummation [8, 9] can provide useful information on the dominant contribution to F_2^c. Previously the first two [10] and three [11] highest threshold logarithms have been determined at this and all higher orders.

In this contribution we employ recent developments concerning the structure of massive-particle amplitudes and the description of heavy-quark production in hadronic collisions [12, 13] to extend those results to four logarithms, i.e., we are now able to derive all threshold-enhanced terms at NNLO. We also include a brief update of the results of Ref. [10] for the transverse momentum distributions, calculated at NLO in Refs. [14], using a modern set of parton distributions [15].

2. Threshold resummation of the gluon coefficient function for F_2^c

The heavy-quark coefficient functions for F_2 are usually expressed in terms of the variables

$$\xi \;=\; \frac{Q^2}{m^2} \;, \quad \text{and} \quad \eta \;=\; \frac{1}{\rho} - 1 \quad \text{or} \quad \beta \;=\; \sqrt{1-\rho} \quad \text{with} \quad \rho \;=\; \frac{4m^2}{s} \;, \tag{2.1}$$

where s is the CM energy, m the mass of the heavy quark, and β the relative velocity of the heavy-quark pair. In terms of the threshold limit, ρ corresponds to the Bjorken variable x in massless DIS. Hence the dominant gluon coefficient function receives a double-logarithmic higher-order enhancement at $\beta \ll 1$. The resummation of these logarithms is performed in terms of the Mellin variable N conjugate to ρ. Up to terms suppressed by powers of $1/N$, the coefficient function reads

$$c_{2,g}\,(\alpha_s,N) \;=\; c_{2,g}^{(0)}(N) \cdot g_0(\alpha_s,N) \cdot \exp\left[\,G(\alpha_s,\ln N)\right] \;. \tag{2.2}$$

Here $c_{2,g}^{(0)}$ is the lowest-order coefficient function (see, e.g., Ref. [3]), and $g_0(\alpha_s,N)$ a matching coefficient. Its dependence on N, absent in the massless case, is due to Coulomb terms which are enhanced by a factor $1/\beta$ (see below). The resummation exponent G is of the standard form

$$G \;=\; \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \left[\int_{\mu^2}^{4m^2(1-z)^2} \frac{dq^2}{q^2}\, A_g(\alpha_s(q^2)) + D_{\gamma^* g\to c\bar{c}}(\alpha_s(4m^2[1-z]^2))\right] \;. \tag{2.3}$$

The first term resums the collinear gluons emitted off the initial gluon, the corresponding 'cusp anomalous dimension' A_g is known to order α_s^3 [16]. The second term collects soft and final-state emissions. Following the methods of Refs. [12, 13] we find

$$D_{\gamma^* g\to c\bar{c}} \;=\; 1/2\, D_{gg\to Higgs} + D_{Q\bar{Q}} \;, \tag{2.4}$$

where the latter heavy-quark coefficient is known to order α_s^2 [13] (obviously only the colour-octet result is required in the present case), and the former even to order α_s^3 [17].

The above information is sufficient to predict the highest four powers of $\ln N$ at all orders in α_s (cf., e.g., Ref. [18]), provided that the matching function g_0 is known at NLO. g_0 is of the form

$$g_0(\alpha_s, N) \;=\; g_0^h(\alpha_s)\cdot g_0^c(\alpha_s, N)\;. \tag{2.5}$$

The Coulomb contribution g_0^c can be determined by Mellin transforming the partonic cross section in non-relativistic QCD, calculated for the colour-singlet case to NNLO in Ref. [19]. The required octet results are obtained by the colour-factor replacement $C_F \to C_F - C_A/2$. The NLO contribution to the N-independent hard matching constant g_0^h had not been determined before this research. We have extracted this coefficient – which will be presented elsewhere [20] – analytically by integrating the intermediate results of Ref. [2] (distributed as a FORTRAN program), and checked our result numerically, for some relevant values of ξ, using the parametrization of Ref. [3].

3. Threshold approximation to the NNLO coefficient function

The above N-space results can be readily expanded in α_s and then Mellin inverted using, e.g., App. A of Ref. [21] and the fact that the leading-order coefficient function is linear in β near threshold (we normalize the coefficient functions as in Refs. [2, 3]),

$$c_{2,g}^{(0)}(\xi,\beta) \;=\; \pi T_f\, \beta\, (1+\xi/4)^{-1} + \mathcal{O}(\beta^3)\;. \tag{3.1}$$

At NLO one thus recovers the threshold expansion (with $T_f = 1/2$, $C_A = 3$ and $C_F = 4/3$ in QCD)

$$\begin{aligned} c_{2,g}^{(1)}(\xi,\beta) \;=\; \frac{c_{2,g}^{(0)}}{(4\pi)^2}\Big\{ & 4C_A\ln^2(8\beta^2) - 20C_A\ln(8\beta^2) + c_0(\xi) + (2C_F - C_A)\,\frac{\pi^2}{\beta} \\ & + \ln\frac{\mu^2}{m^2}\big[-4C_A\ln(4\beta^2) + \bar{c}_0(\xi)\big] + \mathcal{O}(\beta^2)\Big\}\;. \end{aligned} \tag{3.2}$$

The logarithmic and $1/\beta$ contributions have first been given in Ref. [3]. The scale term $\bar{c}_0(\xi)$ is fixed by renormalization-group constraints and reads

$$\bar{c}_0(\xi) \;=\; 4C_A\,(2 + \ln(1+\xi/4)) - 4/3\, T_f\;, \tag{3.3}$$

where the final term arises from the transformation of α_s to the standard $\overline{\mathrm{MS}}$ scheme [22] which was not performed in Ref. [3]. The corresponding scale-independent contribution $c_0(\xi)$ is not available in the literature yet, the full result will be presented in Ref. [20]. Here we can, for brevity, only provide its numerical values at the two scales used in our illustrations below,

$$c_0(1.956) \;=\; 88.28\;, \quad c_0(19.56) \;=\; 70.23\;. \tag{3.4}$$

For F_2^c, hence $n_f = 3$ light flavours, our corresponding new NNLO results are numerically given by

$$
\begin{aligned}
c_{2,g}^{(2)}(\xi,\beta) \;\simeq\; \frac{c_{2,g}^{(0)}}{(4\pi)^4}\Big\{ & \ln^4\beta\,1152 - \ln^3\beta\,\big(1545. + 1152\,L\big) \\
& + \ln^2\beta\,\Big(-3570. + 48\,c_0(\xi) + \big(118.0 + 48\,\bar{c}_0(\xi)\big)L + 288L^2 - 16\,\pi^2\beta^{-1}\Big) \\
& + \ln\beta\,\Big(2403. - 20.19\,c_0(\xi) + \big(2223. - 20.19\,\bar{c}_0(\xi) - 24\,c_0(\xi)\big)L \\
& \quad + \big(291.3 - 24\,\bar{c}_0(\xi)\big)L^2 + \pi^2\beta^{-1}[2.910 + 8L]\Big) \;+\; O(\beta^{-2})\Big\}
\end{aligned} \tag{3.5}
$$

with $L \equiv \ln(\mu^2/m^2)$, where the coefficients with a decimal point are approximate. In addition to the terms given here, also the non-logarithmic $1/\beta$ Coulomb contributions are now known.

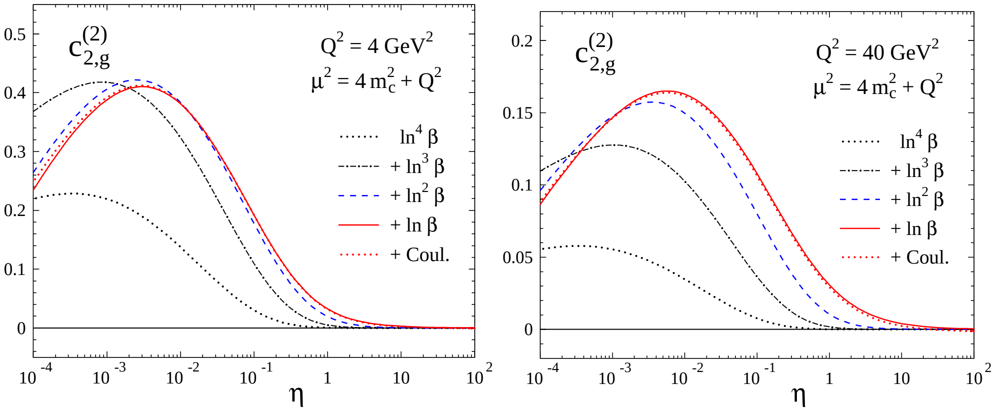

Figure 1: Successive approximations to the NNLO gluon coefficient function for F_2^c in terms of threshold logarithms and $1/\beta$ Coulomb contributions at two typical scales Q^2 for a charm pole-mass $m = 1.43$ GeV.

The threshold expansion (3.5) of the NNLO coefficient function is shown in Fig. 1 for a standard choice of the renormalization/factorization scale μ. Keeping only the highest two logarithms is obviously insufficient. The new $\ln\beta$ contribution is rather small at the lower, but definitely relevant at the higher scale, while the non-logarithmic NNLO Coulomb terms are small in both cases. The resulting estimates for the NNLO corrections to F_2^c are illustrated in Fig. 2. In the region $10^{-4} \lesssim x \lesssim 10^{-2}$ these amount to no more than about $5-10\%$ at $Q^2 = 40\,\text{GeV}^2$, but reach $15-30\%$ at $Q^2 = 4\,\text{GeV}^2$, which the largest effects occurring at the upper end of the above x-range.

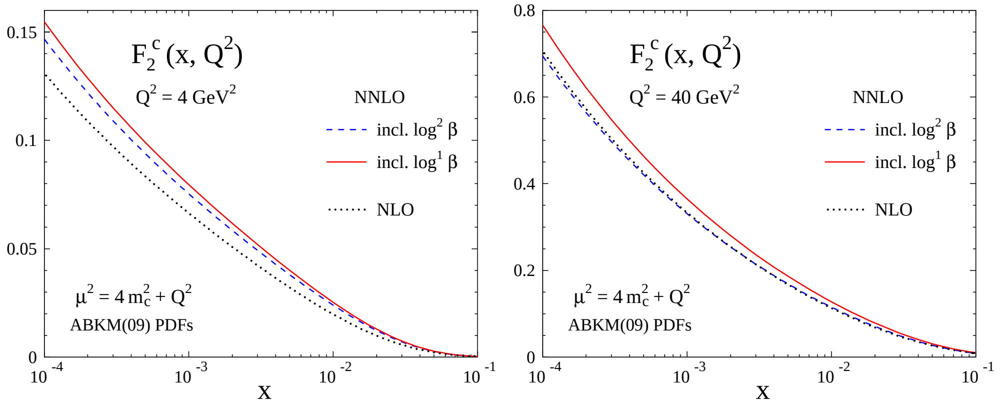

Figure 2: NLO and threshold-estimated NNLO results for the charm contribution to the structure function F_2 using the respective parton distributions and strong coupling constants of Ref. [15] with $m_c = 1.43$ GeV.

4. The p_T-differential charm structure function

Experimentally the inclusive structure function F_2^c is determined via (theory-dependent) extrapolations of more differential cross sections. As an example we consider the p_T-unintegrated structure function dF_2/dp_T, calculated at NLO in Refs. [14]. First NNLO estimates based on the next-to-leading logarithmic (NLL) threshold resummation were derived in Ref. [10]. In Fig. 3 we present an update of these predictions, using an independent code and up-to-date parton densities [15].

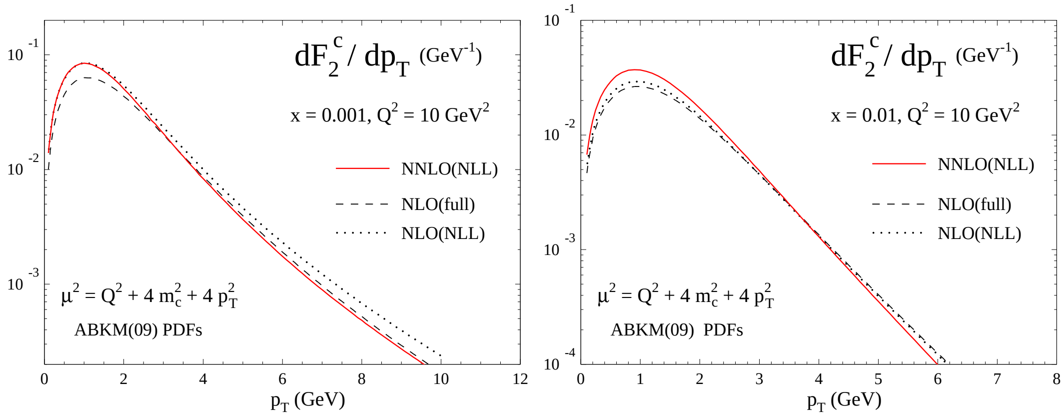

Figure 3: NNLO estimates for the p_T-unintegrated charm structure function F_2 for two typical values of x. At NLO the results for the NLL expanded coefficient function are compared compared to the exact values.

The NLO comparison of the complete and NLL expanded results indicates that the latter are reliable at $x \simeq 0.01$, but not at $x \simeq 0.001$. The estimated NNLO corrections are large and positive around the peak of the distribution, where they amount to as much as 40%. More work is needed to arrive at quantitatively reliable NNLO predictions for this and other differential cross sections. It is interesting to note, however, that a considerable excess over the NLO results has been observed in HERA measurements of charm production including very low values of p_T [23].

5. Summary and Outlook

We have determined the next-to-next-to-leading logarithmic (NNLL) resummation exponent and the one-loop matching function for the dominant gluon coefficient function for the heavy-quark structure functions F_2^h in deep-inelastic lepton-hadron scattering. The results have been used to obtain all threshold-enhanced NNLO contributions to this coefficient function, which we have illustrated for the especially important case of charm production.

At present, these results provide the only reliable estimate of the NNLO effects at small scales, $Q^2 \not\gtrsim 10\,m_c^2$. At larger scales, it may be useful to combine these threshold contributions, the Mellin moments (with respect to x) of the large-ξ limits [6] and the leading large-η (small-x) logarithms [24], in order to obtain an all-η approximate NNLO coefficient function.

As an example for less inclusive quantities, we have also presented NNLO threshold estimates for the p_T-differential structure function. Also here the accuracy reached in Ref. [10] needs to be improved for quantitatively reliable predictions. Present results indicate considerably larger NNLO corrections than for F_2^c close to the peak of the distribution at rather low values of p_T.

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