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Technicolor and Lattice Gauge Theory*

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Technicolor and other theories of dynamical electroweak symmetry breaking invoke chiral symmetry breaking triggered by strong gauge-dynamics, analogous to that found in *QCD*, to explain the observed W, Z, and fermion masses. In this talk we describe why a realistic theory of dynamical electroweak symmetry breaking must, relative to *QCD*, produce an enhanced fermion condensate. We quantify the degree to which the technicolor condensate must be enhanced in order to yield the observed quark masses, and still be consistent with phenomenological constraints on flavor-changing neutral-currents. Lattice studies of technicolor and related theories provide the only way to demonstrate that such enhancements are possible and, hopefully, to discover viable candidate models. We comment briefly on the current status of non-perturbative investigations of dynamical electroweak symmetry breaking, and provide a "wish-list" of phenomenologically-relevant properties that are important to calculate in these theories.

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*Portions of this manuscript have previously appeared in [1], and a more detailed discussion can be found there.
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1. Technicolor and Extended Technicolor

The earliest models [2 – 4] of dynamical electroweak symmetry breaking include a new asymptotically free non-abelian gauge theory (“technicolor”) and additional massless fermions (“technifermions” transforming under a vectorial representation of the gauge group) which feel this new force. The global chiral symmetry of the fermions is spontaneously broken by the formation of a technifermion condensate, just as the approximate chiral $\mathrm{SU}(2) \times SU(2)$ symmetry in QCD is broken down to SU(2) isospin by the formation of a quark condensate. If the quantum numbers of the technifermions are chosen correctly (*e.g.* by choosing technifermions in the fundamental representation of an $\mathrm{SU}(N)$ technicolor gauge group, with the left-handed technifermions being weak doublets and the right-handed ones weak singlets), this condensate can break the electroweak interactions down to electromagnetism.

While technicolor chiral symmetry breaking can give mass to the W and Z particles, additional interactions must be introduced to produce the masses of the standard model fermions. The most thoroughly studied mechanism for this invokes “extended technicolor” (ETC) gauge interactions [5, 6]. In ETC, technicolor and flavor are embedded into a larger gauge group, which is broken at a sequence of mass scales down to the residual, exact technicolor gauge symmetry. The massive gauge bosons associated with this breaking mediate transitions between quarks/leptons and technifermions, giving rise to the couplings necessary to produce fermion masses.

As noted by Eichten and Lane [6], however, the additional interactions introduced to generate ordinary fermion masses cannot be flavor-universal, and would therefore also generically give rise to flavor-changing neutral-current (FCNC) processes. In particular they showed that, absent any “GIM-like" mechanism [7 – 9] for suppressing flavor-changing neutral currents, the ETC scale associated with strange-quark mass generation must be larger than of order 10^3 TeV in order to avoid unacceptably large (CP-conserving) contributions to neutral K-meson mixing. To obtain quark masses that are large enough therefore requires an enhancement of the technifermion condensate over that expected naively by scaling from QCD. Such an enhancement can occur in “walking" technicolor theories [10 – 15] in which the gauge coupling runs very slowly,[1] or in “strong-ETC" theories [18 – 21] in which the ETC interactions themselves are strong enough to help drive technifermion chiral symmetry breaking.[2]

2. Constraints on Λ_{ETC} from neutral meson mixing

At low energies, the flavor-changing four-fermion interactions induced by ETC boson exchange alter the predicted rate of neutral meson mixing. Ref. [24] has derived constraints on general $\Delta F = 2$ four-fermion operators that affect neutral Kaon, D-meson, and B-meson mixing, including the effects of running from the new physics scale down to the meson scale and interpolating between quark and meson degrees of freedom. Their limits on the coefficients (C^1_j) of the

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[1] For some examples of proposed models of walking technicolor, see [16] and [17] and references therein.

[2] It is also notable that walking technicolor and strong-ETC theories are quite different from QCD, and may be far less constrained by precision electroweak measurements [22, 23, 4].

FCNC operators involving LH current-current interactions:

$$C^1_K (\bar{s}_L \gamma^\mu d_L)(\bar{s}_L \gamma_\mu d_L) \tag{2.1}$$

$$C^1_D (\bar{c}_L \gamma^\mu u_L)(\bar{c}_L \gamma_\mu u_L) \tag{2.2}$$

$$C^1_{B_d} (\bar{b}_L \gamma^\mu d_L)(\bar{b}_L \gamma_\mu d_L) \tag{2.3}$$

$$C^1_{B_s} (\bar{b}_L \gamma^\mu s_L)(\bar{b}_L \gamma_\mu s_L)\,, \tag{2.4}$$

are listed in the left column of Table 1. In the case of an ETC model with arbitrary flavor structure and no assumed ETC contribution to CP-violation, one has[3] $C^1_i = \Lambda^{-2}_{ETC}$ and the limits on the Λ_{ETC} from [24] are as shown in the right-hand column of Table 1. The lower bound on Λ_{ETC} from *D*-meson mixing is now the strongest, with that from Kaon mixing a close second and those from *B*-meson mixing far weaker. Since the charm quark is so much heavier than the strange quark, requiring an ETC model to produce m_c from interactions at a scale of over 1000 TeV is a significantly stronger constraint on model-building than the requirement of producing m_s at that scale.[4]

Table 1: Limits from the UTFit Collaboration [24] on coefficients of left-handed four-fermion operators contributing to neutral meson mixing (left column) and the implied lower bound on the ETC scale (right column). The bounds in the first four rows apply when one assumes ETC does not contribute to CP violation; the bound in the last row applies if one assumes that ETC does contribute to CP violation in the Kaon system.

Bound on operator coefficient (GeV^{-2})	Implied lower limit on ETC scale (10^3 TeV)
$-9.6 \times 10^{-13} < \Re(C^1_K) < 9.6 \times 10^{-13}$	1.0
$\lvert C^1_D \rvert < 7.2 \times 10^{-13}$	1.5
$\lvert C^1_{B_d} \rvert < 2.3 \times 10^{-11}$	0.21
$\lvert C^1_{B_s} \rvert < 1.1 \times 10^{-9}$	0.03
$-4.4 \times 10^{-15} < \Im(C^1_K) < 2.8 \times 10^{-15}$	10

3. Condensate Enhancement and γ_m

In studying how ETC theories produce quark masses, the primary operator of interest has the form[5]

$$\frac{(\bar{Q}^a_L \gamma^\mu q^j_L)(u^i_R \gamma_\mu U^a_R)}{\Lambda^2_{ETC}}\,, \tag{3.1}$$

where the Q^a_L and U^a_R are technifermions (a is a technicolor index), and the q^j_L and u^i_R are left-handed quark doublet and right-handed up-quark gauge-eigenstate fields (i and j are family indices). This

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[3] Here we assume there is no flavor symmetry suppressing tree-level flavor-changing neutral currents [1, 24].

[4] Note that if one, instead, assumes that ETC contributes to CP-violation in the Kaon system, then the relevant bound on Λ_{ETC} comes from the imaginary part of C^1_K and is a factor of ten more severe (see last row of Table 1).

[5] In an ETC gauge theory, we would expect $1/\Lambda^2_{ETC} \equiv g^2_{ETC}/M^2_{ETC}$ where g_{ETC} and M_{ETC} are the appropriate extended technicolor coupling and gauge-boson mass, respectively. At energies below M_{ETC}, these parameters always appear (to leading order in the ETC interactions) in this ratio – and therefore, we use Λ_{ETC} for simplicity.

operator will give rise, after technifermion chiral symmetry breaking at the weak scale, to a fermion mass term of order

$$\mathscr{M}_{ij} = \frac{\langle \bar{U}_L U_R \rangle_{\Lambda_{ETC}}}{\Lambda^2_{ETC}} \ . \tag{3.2}$$

Here it is important to note that the technifermion condensate, $\langle \bar{U}_L U_R \rangle_{\Lambda_{ETC}}$ is renormalized at the ETC scale [10 – 15]. It is related to the condensate at the technicolor (electroweak symmetry breaking) scale by

$$\langle \bar{U}_L U_R \rangle_{\Lambda_{ETC}} = \exp\left(\int_{\Lambda_{TC}}^{\Lambda_{ETC}} \gamma_m(\alpha_{TC}(\mu)) \frac{d\mu}{\mu} \right) \langle \bar{U}_L U_R \rangle_{\Lambda_{TC}} \ , \tag{3.3}$$

where $\gamma_m(\alpha_{TC}(\mu))$ is the anomalous dimension of the technifermion mass operator.[6] Using an estimate of the technifermion condensate, and a calculation of the anomalous dimension of the mass operator, we may estimate the size of quark mass which can arise in a technicolor theory for a given ETC scale.

In a theory of walking technicolor [10 – 15], the gauge coupling runs very slowly just above the technicolor scale Λ_{TC}. The largest enhancement occurs in the limit of “extreme walking" in which the technicolor coupling, and hence the anomalous dimension γ_m, remains approximately constant from the technicolor scale, Λ_{TC}, all the way to the ETC scale, Λ_{ETC}. In the limit of extreme walking, one obtains

$$\langle \bar{U}_L U_R \rangle_{\Lambda_{ETC}} = \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{\gamma_m} \langle \bar{U}_L U_R \rangle_{\Lambda_{TC}} \ . \tag{3.4}$$

We may now use (3.4) to quantify the enhancement of the technicolor condensate required to produce the observed quark masses in a walking model. Specifically, we will investigate the size of the quark mass which can be achieved in the limit of extreme walking for various γ_m, and an ETC scale of 10^3 TeV (which, as shown above, should suffice to meet the CP-conserving FCNC constraints in the K - and D-meson systems). The calculation requires an estimate of the technicolor scale Λ_{TC} and the technicolor condensate renormalized at the electroweak scale, $\langle \bar{U}_L U_R \rangle_{\Lambda_{TC}}$.

Two estimates of the scales associated with technicolor chiral symmetry breaking are commonly used in the literature: Naive Dimensional Analysis (NDA) [25 – 27] and simple dimensional analysis (DA) as applied in [6]. In Naive Dimensional Analysis, one associates Λ_{TC} with the "chiral symmetry breaking scale" for the technicolor theory, $\Lambda_{TC} = \Lambda_{\chi SB} \approx 4\pi v$ and $\langle \bar{U}_L U_R \rangle_{\Lambda_{TC}} \approx 4\pi v^3 \approx (580\,\text{GeV})^3$, (where $v \approx 250$ GeV is the analog of f_π in QCD). In the simple dimensional estimates one simply assumes that all technicolor scales are given by $\Lambda_{TC} \approx 1$ TeV, and hence $\langle \bar{U}_L U_R \rangle_{\Lambda_{TC}} \approx (1\,\text{TeV})^3$.

In Table 2 we estimate the size of quark mass corresponding to various (constant) values of γ_m and an ETC scale of 10^3 TeV. We show these values in the range $0 \leq \gamma_m \leq 2.0$ since $\gamma_m \simeq 0$ in a "running" technicolor theory, and conformal group representation unitarity implies that $\gamma_m \leq 2.0$ [28]. The usual Schwinger-Dyson analysis used to analyze technicolor theories would imply that $\gamma_m \leq 1.0$ in walking technicolor theories [10 – 15], while the values $1.0 \leq \gamma_m \leq 2.0$ could occur in strong-ETC theories [18 – 21].

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[6] For a discussion of the potential scheme-dependence of γ_m, see [1].

Table 2: Size of the quark mass m_q generated by technicolor dynamics assuming an ETC scale $\Lambda_{ETC} = 1000$ TeV and various values for the anomalous dimension γ_m of the mass operator. In the row labeled NDA [DA], the value of the techniquark condensate at the technicolor scale is taken to be $\langle \bar{T}T \rangle \approx (580\,\mathrm{GeV})^3$ $[(1000\,\mathrm{GeV})^3]$. Values of γ_m of 1.0 or less correspond to walking theories [10 – 15]; values greater than 1.0 correspond to strong-ETC theories [18 – 21].

γ_m	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0
m_q^{NDA}	0.2 MeV	0.8 MeV	3.5 MeV	15 MeV	63 MeV	260 MeV	1.1 GeV	4.7 GeV	20 GeV
m_q^{DA}	1 MeV	5.6 MeV	32 MeV	180 MeV	1 GeV	5.6 GeV	32 GeV	180 GeV	1 TeV

4. Discussion

Examining Table 2, we see that generating the charm quark mass from ETC dynamics at a scale of order 10^3 TeV requires an anomalous dimension γ_m close to or exceeding one, even in the case of the more generous DA estimate of the technifermion condensate. It is therefore important for nonpertubative studies of strong technicolor dynamics to determine how large γ_m can be in specific candidate theories of walking technicolor. Lattice Monte Carlo studies to date [29 – 39] prefer values of $\gamma_m \lesssim 1.0$ in the theories studied so far. Values of γ_m substantially less than one would require a lower ETC scale, which would necessitate the construction of ETC theories with approximate flavor symmetries [7 – 9] and corresponding GIM-like partial cancellations of flavor-changing contributions.

If a "walking" theory with $\gamma_m \gtrsim 1$ is found, then a number of interesting questions should also be investigated, including:

- What is the complete phase diagram for theories of this sort, as a function of the number of "colors" and "flavors" [40]?
- Can γ_m be larger than one?
- What is the value of the electroweak S [22, 23] parameter[7]?
- Is there a (pseudo-)dilaton with Higgs-like couplings[8]?
- What are the properties of the lightest vector-mesons which would appear in WW scattering?
- Are there other marginal or relevant operators, and can they be useful in generating quark masses á la strong-ETC [18 – 21]?

Results presented at this conference [39] are intriguing, and we look forward to a thorough exploration of the properties of candidate theories of dynamical electroweak symmetry breaking.

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[7] See [41] for a recent conjecture on this topic.

[8] For recent discussions in this regard, see [42 – 45].

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