Forward-backward and CP-violating asymmetries in rare semileptonic and radiative leptonic B-decays

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We study the forward-backward and the CP-violating asymmetries (both time-independent and time-dependent) in rare semileptonic and radiative leptonic $B_{d,s}$-decays and investigate the sensitivity of these asymmetries to the extensions of the Standard model.

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Introduction

Rare *B*-decays induced by flavor-changing neutral currents provide a valuable possibility of an indirect search of physics beyond the standard model (SM). *CP*-violation in beauty sector has been measured for the first time at *B*-factories BaBar and Belle in two-hadron *B*-meson decays [1]. Other interesting reactions, where *CP*-violating effects may be studied, are rare semileptonic and radiative leptonic decays induced by $b \to (d, s)(\gamma, \ell^+\ell^-)$ transitions (see, e.g., [2] and refs therein).

Obviously, an experimental study of *CP*-violating observables requires greater samples of beauty hadrons than those provided by the B-factories: since *CP*-violation effects in beauty sector are of order 10^{-3}, one needs *B*-meson data samples exceeding those from *B*-factories by at least two orders of magnitude. One expects such data samples of beauty particles at the LHC: e.g., the detector LHCb is expected to register about 10^{12} $b\bar{b}$-pairs per year, approximately by four orders of magnitude more than a yearly yield of $b\bar{b}$-pairs at the *B*-factories [3].

The time-independent *CP*-asymmetries in rare semileptonic decays were considered first for inclusive *B*-decays, i.e., for the process $b \to d\ell^+\ell^-$ [4]. An asymmetry of the order of a few percent in $b \to d\ell^+\ell^-$ has been predicted; for the $b \to s\ell^+\ell^-$-transitions the asymmetry is expected to be much smaller. Following [4], the time-independent *CP*-asymmetries in exclusive *B*-decays for several extentions of the SM have been analyzed [5, 6].

The time-dependent CP-asymmetries [7] were studied for the case of rare semileptonic $B_s \to \phi\ell^+\ell^-$–decays in Ref. [6].

This talk presents the results of our analysis [8] of the asymmetries (forward-backward, time-independent and time-dependent *CP*-violating asymmetries) in rare semileptonic and radiative leptonic $B_{s,d} \to (V, \gamma)\ell^+\ell^-$-decays. We make use of the technique of helicity amplitudes developed in [9, 10].

1. Theoretical overview

The effective Hamiltonian describing the $b \to s\ell^+\ell^-$ transition in the SM has the form [11][1]

$$
\begin{aligned}
H_{\text{eff}}^{\text{SM}}(b \to s\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{2\pi} V_{tb}V_{ts}^* \Bigg[&-2\frac{C_{7\gamma}(\mu)}{q^2} \cdot \bar{s}i\sigma_{\alpha\beta}q^\beta \left\{ m_b(1+\gamma^5) + m_s(1-\gamma^5) \right\} b \cdot \bar{\ell}\gamma^\alpha\ell \\
&+ C_{9V}^{\text{eff}(s)}(\mu, q^2) \cdot \bar{s}\gamma_\alpha \left(1-\gamma^5\right) b \cdot \bar{\ell}\gamma^\alpha\ell + C_{10A}(\mu) \cdot \bar{s}\gamma_\alpha \left(1-\gamma^5\right) b \cdot \bar{\ell}\gamma^\alpha\gamma^5\ell \Bigg],
\end{aligned}
\tag{1.1}
$$

where m_b (m_s) is the b (s)–quark mass, V_{ij} are the elements of the CKM matrix, μ is the renormalization scale, and q is the momentum of the $\ell^+\ell^-$ pair. The corresponding expression for the case of the $b \to d$ transition is self-evident.

The Wilson coefficient $C_{9V}^{\text{eff}}(\mu, q^2)$ contains the contributions of the virtual $\bar{u}u$ and $\bar{c}c$ pairs, which involve the integration over short and long distances. The long-distance effects are described by the neutral vector-meson resonances ρ, ω, and J/ψ, ψ'. We make use of the parameterization of $C_{9V}^{\text{eff}}(\mu, q^2)$ from [4] where the resonance contributions are modeled in a gauge-invariant way [12].

The effective Hamiltonian for the $\bar{b} \to \bar{s}\ell^+\ell^-$ transition is obtained from (1.1) by interchanging b and s fields, i.e., by replacing $V_{qb}V_{qs}^* \to V_{qb}^*V_{qs}$, $\bar{s} \to \bar{b}$, $b \to s$, $m_b \leftrightarrow m_s$, $q \to -q$.

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[1]We use the following conventions: $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$, $\varepsilon^{0123} = -1$, $e = \sqrt{4\pi\alpha_{\text{em}}} > 0$.

The form factors for rare semileptonic transitions $\bar{B}(p_1, M_1) \to \bar{V}(p_2, M_2, \varepsilon)$ are defined in the standard way [13], where $q = p_1 - p_2$. The $B \to \gamma$ amplitudes are parametrized as follows [14], where $q = p - k$, $k^2 = 0$, $p^2 = M_1^2$.

2. Forward-backward and CP-violating asymmetries

The lepton forward-backward asymmetry $A_{FB}(\hat{s})$ is one of the differential distributions relatively stable with respect to QCD uncertainties and sensitive to the new physics effects [16]. Therefore it has been extensively studied both theoretically and experimentally [13]. We make use of the following definition of $A_{FB}(\hat{s})$ for $\bar{B} \to \bar{f}$ decays

$$A_{FB}(\hat{s}) = \frac{\int_0^1 d\cos\theta \, \frac{d^2\Gamma(\bar{B} \to \bar{f})}{d\hat{s}\,d\cos\theta} - \int_{-1}^0 d\cos\theta \, \frac{d^2\Gamma(\bar{B} \to \bar{f})}{d\hat{s}\,d\cos\theta}}{\frac{d\Gamma(\bar{B} \to \bar{f})}{d\hat{s}}}, \tag{2.1}$$

where $f = Vl^+l^-$ for rare semileptonic decay and $f = \gamma l^+l^-$ for rare radiative decay; $\hat{s} \equiv s/M_B^2$, $\sqrt{s}$ being the dilepton invariant mass. The asymmetry is calculated in the rest frame of the lepton pair, and the angle θ is defined as the angle between the $\bar{B}$-meson 3-momentum and the 3-momentum of the outgoing negative-charged lepton, l^-.

Equivalently, for $B \to f$ one defines

$$A_{FB}(\hat{s}) = \frac{\int_0^1 d\cos\theta_+ \, \frac{d^2\Gamma(B \to f)}{d\hat{s}\,d\cos\theta_+} - \int_{-1}^0 d\cos\theta_+ \, \frac{d^2\Gamma(B \to f)}{d\hat{s}\,d\cos\theta_+}}{\frac{d\Gamma(B \to f)}{d\hat{s}}} \tag{2.2}$$

where θ_+ is the angle between the B-meson 3-momentum and the 3-momentum of the outgoing positive-charged lepton, l^+. If CP-violating effects are neglected, both asymmetries (2.1) and (2.2) are equal to each other.

Time-dependent CP-violating asymmetry is defined in the B-meson rest frame as follows [17]

$$A_{CP}^{B_q \to f}(\tau) = \frac{\frac{d\Gamma(\bar{B}_q^0 \to f)}{d\tau} - \frac{d\Gamma(B_q^0 \to f)}{d\tau}}{\frac{d\Gamma(\bar{B}_q^0 \to f)}{d\tau} + \frac{d\Gamma(B_q^0 \to f)}{d\tau}}, \tag{2.3}$$

where f is the common final state for B^0 and $\bar{B}^0$ decays. In this case a pronounced CP violation is expected in interference between the oscillation and decay amplitudes [18]. Time-independent *CP*-asymmetry may be represented as follows:

$$A_{CP}^{B_q \to f}(\hat{s}) = \frac{\frac{d\Gamma(\bar{B}_q \to f)}{d\hat{s}} - \frac{d\Gamma(B_q \to f)}{d\hat{s}}}{\frac{d\Gamma(\bar{B}_q \to f)}{d\hat{s}} + \frac{d\Gamma(B_q \to f)}{d\hat{s}}}. \tag{2.4}$$

B-meson parameters	B_d^0	B_s^0
B-meson mass M_1 (GeV)	5.28	5.37
Width Γ (ps^{-1})	0.65	0.67
Mass difference Δm (ps^{-1})	0.507	17.77
Width difference $\Delta\Gamma$ (ps^{-1})	0.005	0.1

Table 1: Parameters of $B_{d,s}^0$-oscillations [2, 19, 20]

3. Numerical results

We are going to apply now the formulas derived above and to provide numerical results for the asymmetries. We use the following numerical parameters:

(i) Table 1 summarizes the parameters of the $B_{d,s}^0$-oscillations which we use for our numerical estimates.

(ii) The Wilson coefficients for the SM are evaluated at $\mu = 5$ GeV [11] for $C_2(M_W) = -1$: $C_1(\mu) = 0.241, C_2(\mu) = -1.1, a_1(\mu) = -0.126, C_{7\gamma}(\mu) = 0.312, C_{9V}(\mu) = -4.21$ and $C_{10A}(\mu) = 4.64$. Respectively, we use the running quark masses in the $\overline{\text{MS}}$ scheme at the same scale: $m_b = 4.2$ GeV, $m_s = 60 - 80$ MeV, and the d-quark mass is neglected. For the coefficient C_{eV}^{eff} we use the model proposed in [4] which takes into account resonances in a gauge-invariant way.

For the CKM matirx elements we use the values reported in the 2008 edition of PDG [19]: $A = 0.814, \lambda = 0.226, \bar{\rho} = 0.135, \bar{\eta} = 0.35$.

(iii) We make use of the form factor parameterizations for rare semileptonic decays from [21] and for rare radiative decays from [14]. The accuracy of these predictions for the form factors is expected to be at the level of 10-15%, which influences strongly the predictions for the decay rates. However, the form factor uncertainties cancel to a large extent in the asymmetries which therefore can be predicted with a few percent accuracy [15, 16]. For the decay constants of the B-mesons we use the values $f_B = 220 \pm 20$ MeV and $f_{B_s} = 240 \pm 20$ MeV.

(iv) A cut on the Bremsstrahlung photon spectrum at 20 MeV in the B-meson rest frame is applied. This corresponds to the expected level of the photon energy resolution of the LHCb detector.

3.1 Forward-backward asymmetry

The calculated forward-backward asymmetries are presented in Figs. 1–3.

The decay $\bar{B}_s \to \phi\mu^+\mu^-$ (Fig. 1) is of special interest: the detector LHCb may accumulate sufficient data sample for this decay already after the first few months of operation. Qualitatively, the asymmetry has the same structure as in the $B \to K^*\mu^+\mu^-$ decays: its behaviour at small $\hat{s}$ is sensitive to the invertion of the signs of $C_{7\gamma}$ and C_{10A} compared to the SM. Fig. 1 shows the influence of the sign invertion in one of the Wislon coefficients compared to the SM.

Figs. 2 and 3 present A_{FB} for the radiative decays $\bar{B}_s \to \gamma\mu^+\mu^-$ and $\bar{B}_d \to \gamma\mu^+\mu^-$, respectively. Qualitatively, the asymmetry behaves at large and intermediate $\hat{s}$ similarly to the $B_s \to \phi\mu^+\mu^-$ decay, although has a larger magnitude. At small $\hat{s}$, however, the assymetry is influenced by the neutral light vector resonances ϕ, ω, and ρ^0 [14]: these resonances cause a strong distortion of the
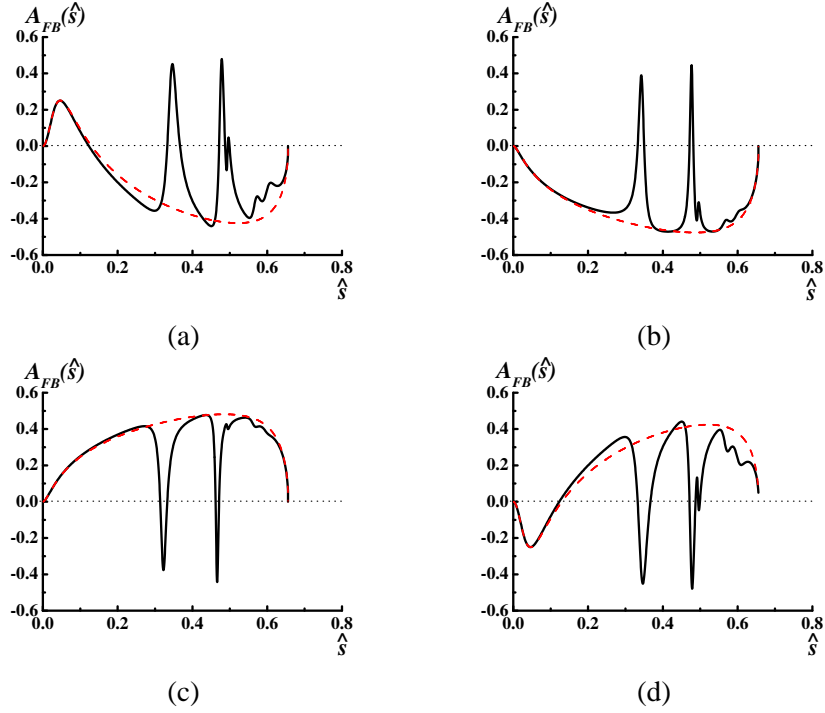
Figure 1: A_{FB} for rare semileptonic $\bar{B}_s \to \phi\mu^+\mu^-$ decays: (a) in the SM; (b) For $C_{7\gamma} = -C_{7\gamma}^{\mathrm{SM}}$, (c) For $C_{9V} = -C_{9V}^{\mathrm{SM}}$, (d) For $C_{10A} = -C_{10A}^{\mathrm{SM}}$. Solid line (black): the full asymmetry which takes into account the J/ψ, ψ', etc contributions. Dashed line (red): the non-resonant asymmetry.

full asymmetry compared to a nonresonance asymmetry. In particular, they leads to a visible shift of the "zero-point" compared to its location in the non-resonant asymmetry, which may be reliably calculated in the SM [22].

3.2 CP-violating asymmetries

We present the time-independent and the time-dependent CP-asymmetries in $B_d \to (\rho, \gamma)\mu^+\mu^-$. Concerning the $B_s \to (\phi, \gamma)\mu^+\mu^-$ decays we would like to mention the following: we have calculated these asymmetries and found that $A_{CP}(\hat{s})$, mainly due to flavor oscillations of the B_s mesons, is extremely small (smaller than 0.1%) and therefore cannot be studied experimentally; $A_{CP}(\tau)$ is not small but measuring this asummetry would require time resolution much smaller than the B_s lifetime.

3.2.1 Time-independent asymmetry

First, we would like to demonstrate the impact of flavor oscillations of the initial mesons on the resulting CP-violating asymmetries. Fig. 4 shows $A_{CP}(\hat{s})$ for $B_{d,s} \to \gamma\mu^+\mu^-$ decays. Obviously, flavour oscillations lead to a strong suppression of the the resulting CP-violating asymmetries in B_d decays and to a complete vanishing of A_{CP} in B_s decays.

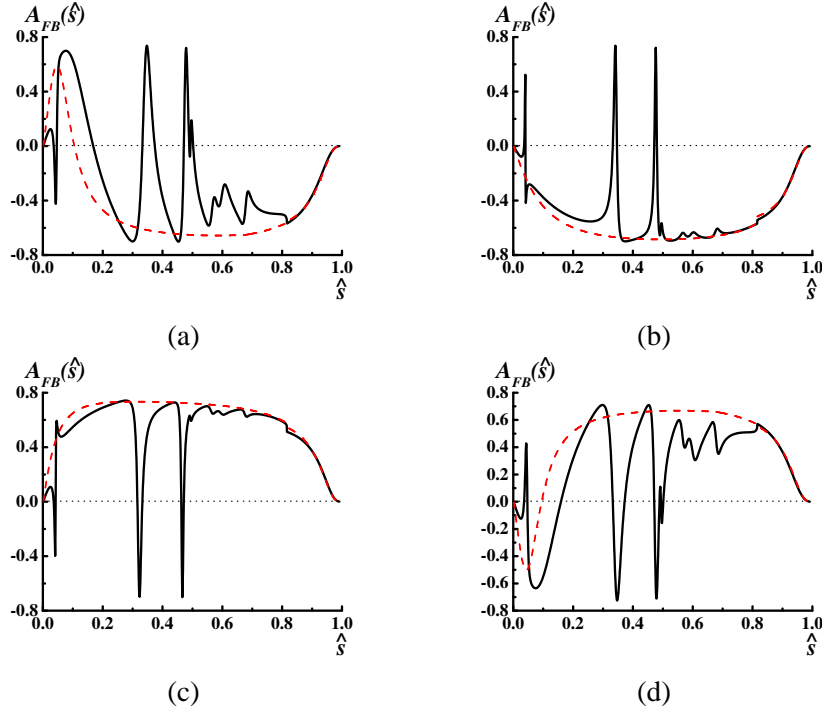Figs. 5 and 6 display $A_{CP}(s)$ for $B_d \to \rho\mu^+\mu^-$ and $B_d \to \gamma\mu^+\mu^-$ decays, respectively.

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Figure 2: $A_{FB}(\bar{s})$ for $\bar{B}_s \to \gamma\mu^+\mu^-$ decays: (a) In the SM. (b) For $C_{7\gamma} = -C_{7\gamma}^{SM}$. (c) For $C_{9V} = -C_{9V}^{SM}$. (d) For $C_{10A} = -C_{10A}^{SM}$. Solid line (black): the asymmetry calculated for the full amplitude of Ref. [14]. Dashed line (red): the asymmetry calculated for the amplitude without the contributions of light neutral vector mesons ϕ, the $c\bar{c}$ resonances (J/ψ, ψ', ...), Bremsstrahlung, and the weak annihilation.

For $B_d \to \rho\mu^+\mu^-$ decays, the asymmetry reaches a 30-40% level in the region of light vector resonances, and a level of 10% between the light and $c\bar{c}$ resonances. Notice that flavor oscillations enhance the asymmetry by a factor 2.

For $B_d \to \gamma\mu^+\mu^-$ decays the asymmetry is smaller and may be measured only in the region of light vector resonances.

Both for $B_d \to \rho\mu^+\mu^-$ and $B_d \to \gamma\mu^+\mu^-$ decays the asymmetry is sensitive to the signs of the Wilson coefficients C_7 and C_9. The asymmetry is however not sensitive to the inversion of the sign of C_{10}.

3.2.2 Time-dependent asymmetry

Fig. 7 plots the time-dependent asymmetry $A_{CP}(\tau)$ for $B_d \to \rho\mu^+\mu^-$ (a) $B_d \to \gamma\mu^+\mu^-$ (b) and $B_s \to \gamma\mu^+\mu^-$ (c) decays. The region around the J/ψ and ψ' resonances $0.33 \le \hat{s} \le 0.55$ was excluded from the integration while calculating the time-dependent asymmetries. This procedure corresponds to the analysis of the experimental data.

The asymmetry in B_d decays reaches a level of 10% at the time-scale of a few B-meson lifetimes ($\tau_{B_d}=1.53$ ps) and may be studied experimentally. It also exhibits a sensitivity the the extentions of the SM.
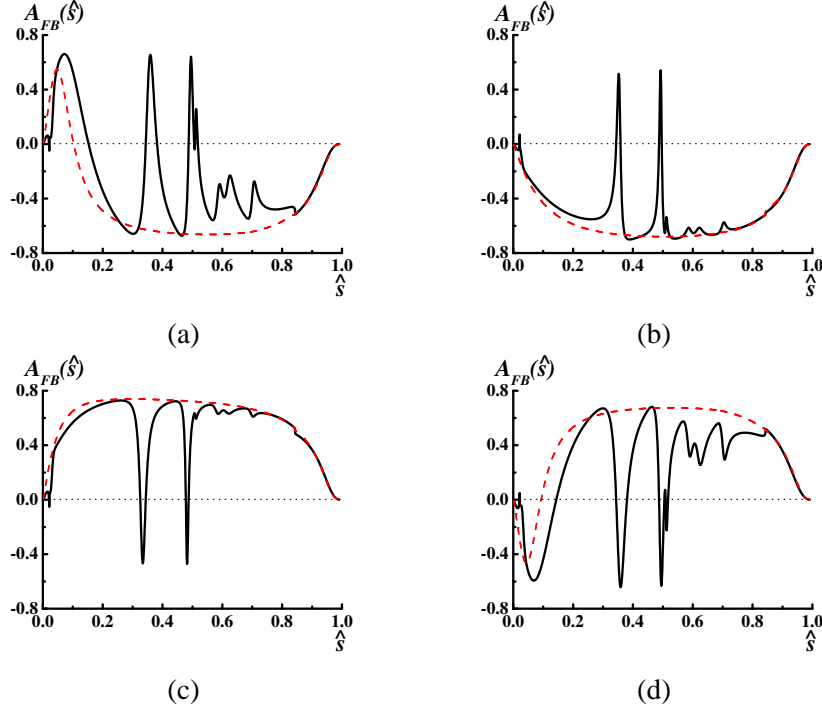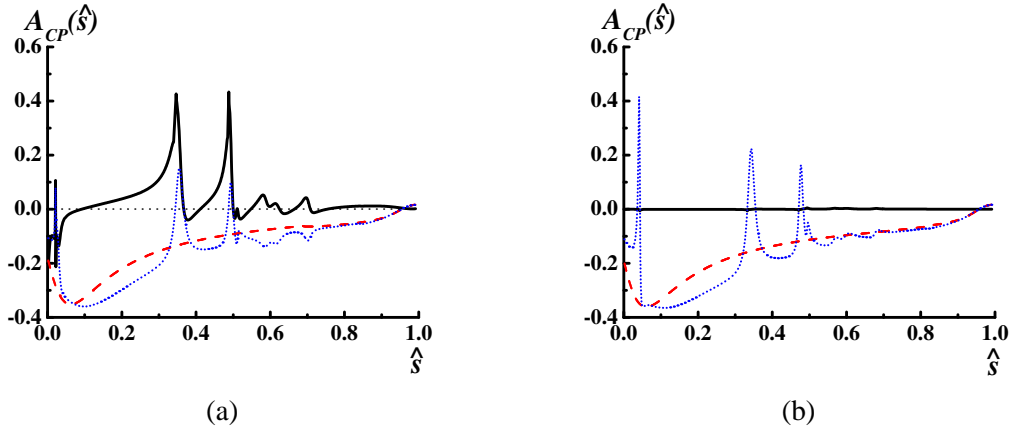
Figure 3: $A_{FB}(\bar{s})$ for $\bar{B}_d \to \gamma \mu^+ \mu^-$ decays: (a) In the SM. (b) For $C_{7\gamma} = -C_{7\gamma}^{SM}$. (c) For $C_{9V} = -C_{9V}^{SM}$. (d) For $C_{10A} = -C_{10A}^{SM}$. Solid line (black): the asymmetry calculated for the full amplitude of Ref. [14]. Dashed line (red): the asymmetry calculated for the amplitude without the contributions of light neutral vector mesons ω, ρ^0, the $c\bar{c}$ resonances (J/ψ, ψ', ...) Bremsstrahlung, and the weak annihilation.

Figure 4: The influence of B-meson flavor oscillations upon CP-violating asymmetries: (a) $B_d \to \gamma \mu^+ \mu^-$, (b) $B_s \to \gamma \mu^+ \mu^-$. Dashed line (red): A_{CP} without resonances and without flavor oscillations; Dotted line (blue): A_{CP} with resonances but without flavor oscillations; Solid line (black): A_{CP} after flavor oscillations have been taken into account.

Conclusions

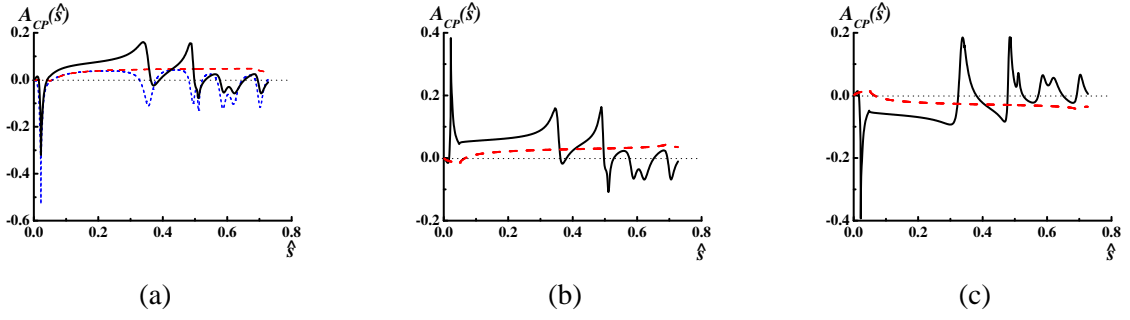We presented the analysis of the forward-backward and the CP-violating asymmetries in rare

Figure 5: Time-independent CP-asymmetry $A_{CP}(\hat{s})$ in $B_d \to \rho\mu^+\mu^-$ decays. (a) SM (b) $C_{7\gamma} = -C_{7\gamma}^{SM}$ (c) $C_{9V} = -C_{9V}^{SM}$. Flavor oscillations have been taken into account. Solid line (black) line: full asymmetry. Dashed (red) line: nonresonant asymmetry. Dotted (blue) line shows the asymmetry if flavor oscillations are not taken into account.

semileptonic and radiative leptonic *B*-decays. Our results may be summarized as follows:

1. We obtained the analytic results for the time-dependent and time-independent *CP*-asymmetries in rare radiative leptonic *B*-decays $B_{d,s} \to \gamma\ell^+\ell^-$.

2. We presented numerical results for the forward-backward asymmetry in $B_s \to \phi\mu^+\mu^-$ decays which may be measured in the near future at the LHCb. This asymmetry, as could be expected, has a very similar shape to the asymmetry in $B_d \to K^*\mu^+\mu^-$ decays and thus may be used for "measuring" the signs of the Wilson coefficients $C_{7\gamma}$, C_{9V}, and C_{10A}.

3. We studied the forward-backward asymmetry in $B_{d,s} \to \gamma\ell^+\ell^-$ decays taking into account the vector resonance contributions, the Bremsstrahlung, and the weak annihilation effects. We noticed that the light neutral vector resonances strongly distort the shape of the asymmetry at small values of the dilepton invariant mass. In particular, in the SM these resonances lead to a sizeable shift of the zero point of the full asymmetry compared to the zero-point of the non-resonant asymmetry. The A_{FB} in this reaction reaches 60% and thus may be studied experimentally at the LHC and the future Super-B factory.
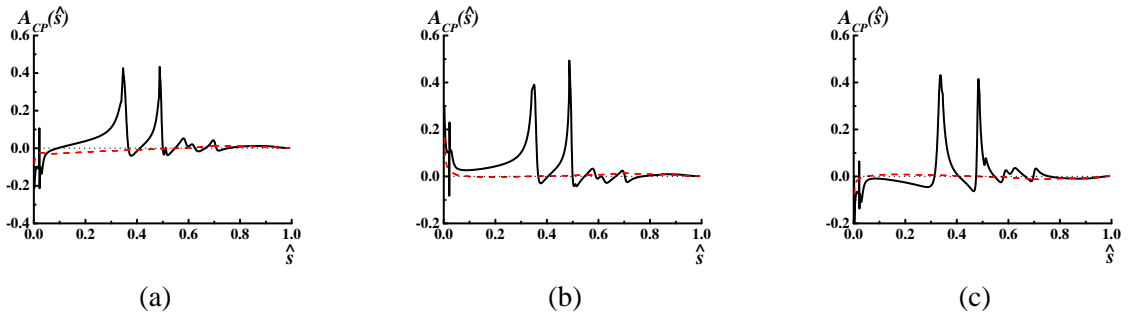

Figure 6: Time-independent CP-asymmetry $A_{CP}(\hat{s})$ in $B_d \to \gamma\mu^+\mu^-$ decays. (a) SM (b) $C_{7\gamma} = -C_{7\gamma}^{SM}$ (c) $C_{9V} = -C_{9V}^{SM}$. Solid (black) line: full asymmetry. Dashed (red) line: nonresonant asymmetry. Flavor oscillations have been taken into account.
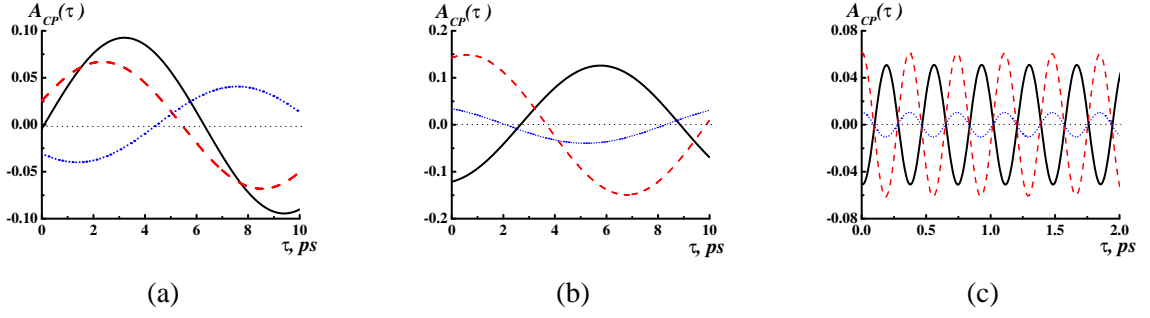
(a)	(b)	(c)

Figure 7: Time-dependent asymmetry $A_{CP}(\tau)$: (a) $B_d \to \rho \mu^+ \mu^-$; (b) $B_d \to \gamma \mu^+ \mu^-$; (c) $B_s \to \gamma \mu^+ \mu^-$. Solid line (black): SM. Dashed line (red): $C_{7\gamma} = -C_{7\gamma}^{\mathrm{SM}}$. Dotted line (blue): $C_{9V} = -C_{9V}^{\mathrm{SM}}$.

4. We analysed the CP-violating asymmetries (both time-dependent and time-independent) in $B_d \to \rho \mu^+ \mu^-$, $B_s \to \phi \mu^+ \mu^-$, and $B_{s,d} \to \gamma \mu^+ \mu^-$ decays.

   The asymmetries in B_s decays are found to be very small and therefore to be of no practical interest.

   The asymmetries in B_d decays reach measurable values and thus might provide additional tests of the SM and its extentions. These potentially interesting cases are: (i) The time-independent CP-violating asymmetry $A_{CP}(\hat{s})$ in $B_d \to \rho \mu^+ \mu^-$ decays in the region below $c\bar{c}$ resonances (10-30 % level) and $A_{CP}(\hat{s})$ in $B_d \to \gamma \mu^+ \mu^-$ in the region of light neutral vector resonances (5-10 % level). (ii) The time-dependent CP-violating asymmetry $A_{CP}(\tau)$ in $B_d \to (\rho, \gamma)\mu^+ \mu^-$ decays (10% level).

4. Acknowledgments

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