PROCEEDINGS OF SCIENCE

A Gas of Bags

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A system with an exponential mass spectrum (Hagedorn-like or bag-like) leads to a thermodynamics which is identical with that of a two phase coexistence at a fixed temperature. A gas of bags is thermodynamically unstable.

XLVIII International Winter Meeting on Nuclear Physics in Memoriam of Ileana Iori
25-29 January 2010
Bormio, Italy

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PoS(BORMIO2010)058

1. Introduction

Hadron properties and their mass spectrum have been derived in the bootstrap model (Hagedorn model) [1, 2] and the bag model [3]. This spectrum is rather poorly known experimentally, but it has suggested an exponential form which the theoretical spectra from both models reproduce, exactly in the bag model, in leading order in the bootstrap model. Given such an exponential spectrum, one should easily derive the associated thermodynamics.

Such exponential systems are thermostats and are well known in standard thermodynamics. Such are, for instance, any two phase systems in coexistence at fixed pressure, like a mixture of liquid water and ice, as seen in Fig. 1, or liquid water and its saturated vapor. *Erroneously*, the partition function calculated with this spectrum seems to lead to the expectation that such a system can exist over a range of temperatures, with an upper bound called the limiting temperature T_H:

$$Z(T) = \int dE\, \rho(E) \exp\left(-\frac{E}{T}\right) = \frac{T\, T_H}{T_H - T} \qquad (1.1)$$

$$\text{when } \rho(E) = \exp\left(+\frac{E}{T_H}\right).$$

In contrast to the above conclusion, we have shown [4, 5] that the correct result implies that such exponential systems are characterized by one and only one temperature T_H, and confer the same temperature T_H to any other "system" coupled to it. Thus the Hagedorn thermostat cannot exist at any temperature different than T_H any more than a water-ice system at atmospheric pressure can exist at any temperature other than 0°C.

It is immediately shown [4, 5] that such a system, when allowed to equilibrate in temperature and in particle number with a gas of particles of mass m, leads to a particle concentration which is volume independent:

$$\frac{N(m)}{V} = g(m) \left(\frac{mT_H}{2\pi}\right)^{\frac{3}{2}} \exp\left(-\frac{m}{T_H}\right), \qquad (1.2)$$

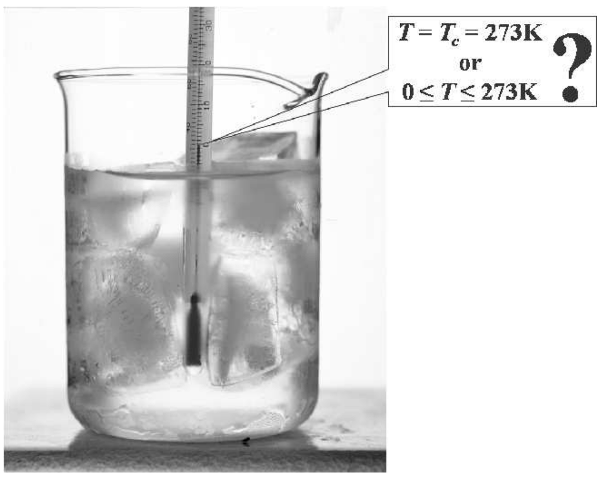


Figure 1: Liquid water with ice in equilibrium at atmospheric pressure is an example of a thermostat with an exponential spectrum and will always have the same temperature no matter how much energy is put into it.

where $g(m)$ is the degeneracy of the particle. In other words, the gas is a saturated vapor and the thermostat is not only an infinite heat/energy reservoir, but also an infinite particle reservoir.

2. The bag model

The bag model, in its simple form (no conserved charges), can be easily visualized. There are two vacua: 1) a lower energy vacuum, the hadronic vacuum; 2) a higher energy vacuum, or partonic vacuum. A bubble, or bag, of partonic vacuum of volume V can be opened at a cost:

$$E = pV = BV\,, \tag{2.1}$$

where B is called the bag constant and is the constant pressure exerted by the hadronic vacuum on the bag.

The bag pressure, B, can be counteracted by the pressure of the partonic black body:

$$p = \frac{g\pi^2}{90} T_H^4 = B \tag{2.2}$$

This can occur only at one temperature $T_H = \left(B\frac{90}{g\pi^2}\right)^{\frac{1}{4}}$.

The enthalpy density of the bag is:

$$H = \frac{g\pi^2}{30} T_H^4 + B\,, \tag{2.3}$$

and the entropy is:

$$S = \frac{H}{T_H} \equiv \frac{m}{T_H} \text{ or } \rho(E) = \exp S = \exp\left(\frac{m}{T_H}\right). \tag{2.4}$$

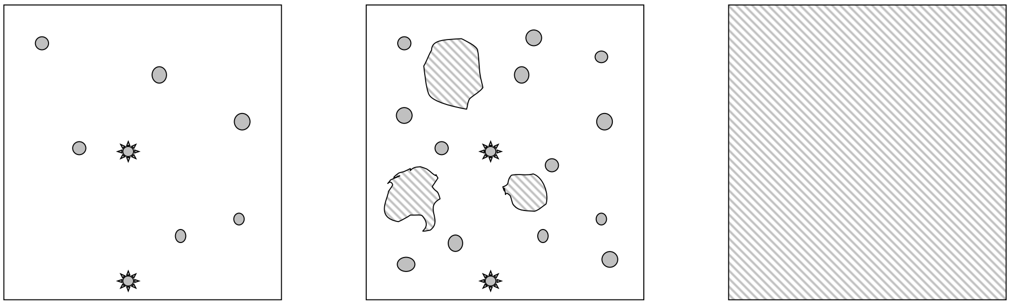
This shows that the bag is characterized by an exponential spectrum.

The analogy with a bubble of vapor forming in a liquid at a hydrostatic pressure $P = B$ is complete and compelling. There is a temperature, T_B, at which the saturated vapor pressure is $p(T_B) = B$. At any temperature $T < T_B$, no bubble can exist in the liquid. At $T = T_B$, a bubble can finally form. This bubble can be of an arbitrary size, as the liquid evaporates *isothermally* into the bubble. In fact, the system exists between two extreme regimes: 1) all liquid; 2) all saturated vapor. Accordingly, the "energy density" interpolates linearly between the two corresponding limits.

In conclusion, we have described the bubble formation (boiling) in a liquid at fixed pressure. We can translate this picture into the case of a bag according to a simple dictionary:

liquid	$\rightarrow$	hadronic vacuum
interior of vapor bubble	$\rightarrow$	partonic vacuum
saturated vapor	$\rightarrow$	partonic blackbody radiation

In order to complete the picture, we can add that a liquid cannot - ever - generate a bubble at $T < T_B$. However, a black body phonon spectrum can exist at any temperature $T < T_B$. Similarly, the hadronic vacuum cannot support a bag at temperature $T < T_H$, but can have a black-body spectrum of its own, like a saturated gas of pions and any other particle with a non-exponential spectrum. See Fig. 2.



(a) $T < T_H$: Non saturated gas of π etc.

(b) $T = T_H$: Gas of bags and saturated gas of π etc.

(c) $T = T_H$: One big bag

Figure 2: Schematics of a Hagedorn system at different temperatures. Notice how below T_H there are no bags, whereas any system with bag(s) present is at $T = T_H$.

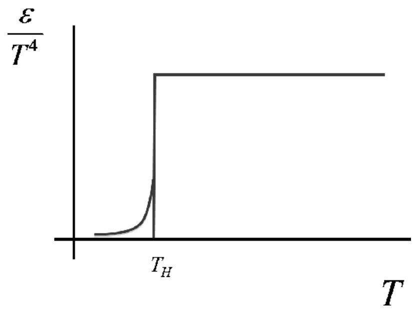


Figure 3: As a function of energy, the temperature of the Hagedorn system will first increase and then reach the limiting temperature of T_H.

What happens in the bag model when we heat the hadronic vacuum at various temperatures? At $T < T_H$, the hadronic vacuum is permeated with a black-body radiation made up of all existing particles/antiparticles, such as pions, with a non-exponential intrinsic spectrum. The energy density is:

$$E(T) = \frac{1}{V}\sum N(m_i)\left(m_i + \frac{3}{2}T\right) = \sum\left(\frac{m_i T}{2\pi}\right)^{\frac{3}{2}}\left(m_i + \frac{3}{2}T\right)\exp\left(-\frac{m_i}{T}\right). \tag{2.5}$$

As T increases, the energy density increases until the temperature hits T_H. At this time the bag(s) appears, intermingled with the saturated vapor. The energy density at this temperature can go from that of the saturated vapor to that of the bag. The latter corresponds to all the accessible space taken up by the partonic vacuum and the partonic blackbody. No temperature higher than T_H is possible unless an external pressure is added to the bag pressure, either dynamically or through a formal constraint. See Figs. 2 and 3.

3. Resonance gas, or a gas of bags

No bag can form at $T < T_H$. At $T = T_H$, however, bags can form. Two related questions arise:

1) What is the mass distribution of the bags?

2) Do these bags contribute in any way to the pressure (equation of state) of the system?

3.1 Mass distribution of bags

The exponential spectrum tells us that bags are thermodynamically indifferent to coalescence or fragmentation [4, 5]. However, if one incorporates the translational degrees of freedom of the bags, one obtains the concentration of bags of mass m as:

$$\frac{N(m)}{V} = g(m)\left(\frac{mT_H}{2\pi}\right)^{\frac{3}{2}} \exp\left(-\frac{m}{T_H}\right) = \left(\frac{mT_H}{2\pi}\right)^{\frac{3}{2}} \quad \text{when } g(m) = \exp\left(\frac{m}{T_H}\right). \tag{3.1}$$

The most probable bag is the bag with the largest mass possible, which is $m \to \infty$. Given the finite size of the bag proportional to m, the most probable configuration is a single infinite bag representing the liquid. Notice that bags, whatever their mass, will carry an average kinetic energy of $\frac{3}{2}T_H$. For more details, see Ref. [4] and Ref. [5].

While, in principle, the "little" bags of the distribution peaking at $m \to \infty$ can be considered part of the saturated vapor, the above description, like in any liquid-vapor system, implies the essential separation of vapor and liquid. The vapor is always at $T = T_H$ and the pressure is *independent of volume*. These considerations drastically limit the role of a "resonance" or "bag" gas in the description of the thermodynamic equilibrium of the system [4, 5].

4. The shape of the bag and its surface energy

In the standard bag model, only volume terms are present, and there is no surface energy. Therefore, a new question arises: "What is the shape of a bag?" Or better, "How many shapes can a bag of finite volume assume?"

In lattice models, it is possible to enumerate such number of ways. The most probable way is typically highly dendritic spaghetti-like. The example of clusters in a lattice gas is illuminating. The presence of a surface energy for $T < T_c$ forces the most probable shape to be compact. Above T_c, the surface energy disappears and the shape is dendritic. Fig. 4 shows how the probability of finding a cluster with a given surface changes when the temperature is altered. This is vividly manifested in the dimensionality change as one crosses T_c, as shown in Fig. 5. In the absence of surface energy, the fluctuations of the bag shape are already those of a supercritical system at the unique stable temperature T_H. In other words, there is no criticality.

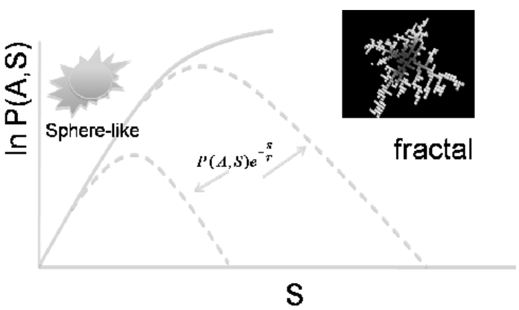


Figure 4: The shape of the clusters changes as a function of temperature, controlled by the Boltzmann factor for the surface energy.

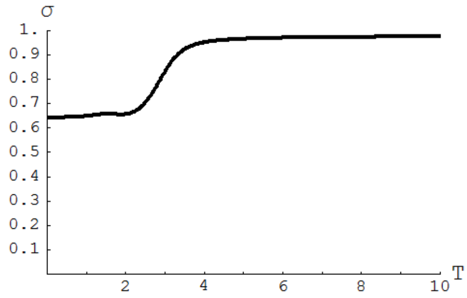


Figure 5: Surface dimensionality ($S \propto A^{\sigma}$) of 3-dimensional lattice clusters with surface as a function of temperature.

5. Effects of surface energy

What are the consequences of a possible surface energy? First of all, it would make the situation of a gas of bags even more precarious, since there would be an ever greater tendency to maximize the drop size and to minimize the surface. Even more interesting is the effect of the surface on the bag temperature. The surface translates into an additional pressure on the bag. The enthalpy of the bag becomes:

$$EV = H = [f(T) + B]V + c_s V^{2/3}. \tag{5.1}$$

The pressure is:

$$p = \frac{1}{3} f(T) - \left(B + \frac{2}{3} c_s V^{-1/3} \right) = 0 \text{ at equilibrium,} \tag{5.2}$$

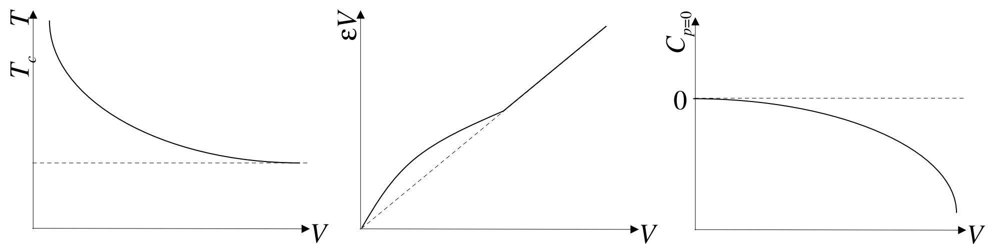
for which:

$$T = f^{-1} \left[3 \left(B + \frac{2}{3} c_s V^{-1/3} \right) \right]. \tag{5.3}$$

In Fig. 6 we show the dependence of the bag size on the temperature, energy, and heat capacity. Notice that the temperature of a bag increases and tends to infinity with decreasing bag size.

The introduction of a temperature dependent surface energy is also easily implemented. If:

$$c_s = c_s^0 \left(1 - \frac{T}{T_c} \right), \tag{5.4}$$



(a) Temperature (b) Energy (c) Heat Capacity

Figure 6: Effects of bags having a surface energy on various thermodynamic properties.

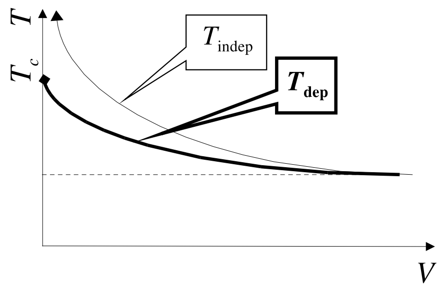
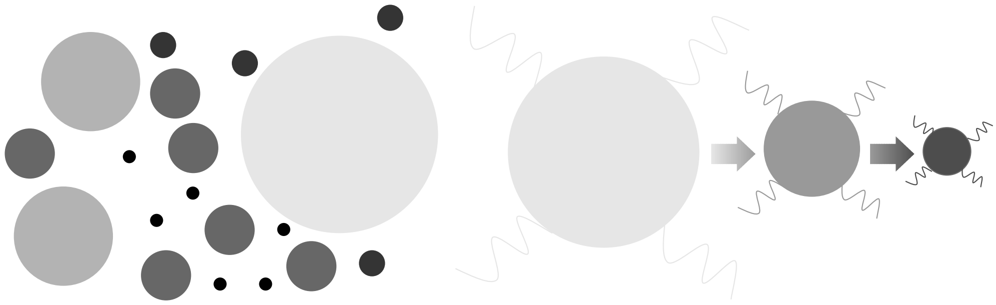


Figure 7: Effects of the surface energy varying with temperature. Notice how the temperature stays finite with this change when previously it diverges.



(a) Stability of a gas of bags (b) The decay of a bag with surface

Figure 8: Schematic picture of the effects of a bag having surface energy.

we have for the stability condition:

$$\sigma T^4 = 3\left[B + \frac{2}{3}c_s^0\left(1 - \frac{T}{T_c}\right)V^{-2/3}\right]. \tag{5.5}$$

Compared to the previous situation, the bag temperature goes to T_c rather than infinity as the bag becomes small, as seen in Fig. 7. It also preserves the trend that the large bag sizes go to a temperature of T_H.

This temperature dependence of the bag leads to two interesting conclusions:

1) A gas of bags of different sizes, and thus *of different temperatures*, is out of equilibrium. The system will tend to make one single bag of maximum size and minimum surface.

2) If a bag decays, its temperature, as manifested by the decay products such as pions, will progressively increase as the bag evaporates.

These conclusions are shown in Fig. 8.

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