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Beyond the heavy top limit in $gg \to H$ at LHC

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QCD corrections to Higgs boson production at the LHC are evaluated to NNLO. Taking advantage of the optical theorem and exact master integrals, we perform asymptotic expansion of the QCD cross-sections near the limit of infinitely heavy top quark and present a few first terms of the series. Convergence of the series is briefly discussed.

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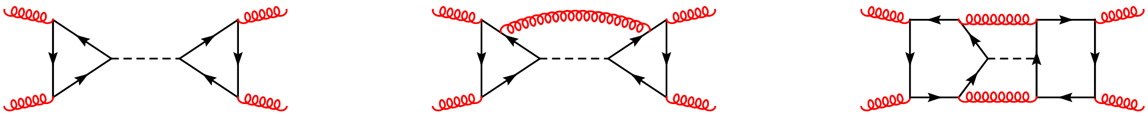

Figure 1: Sample forward scattering diagrams whose cuts correspond to the LO, NLO and NNLO corrections to $gg \to H$. Dashed, curly and solid lines represent Higgs bosons, gluons and top quarks, respectively.

1. Introduction

The results from the Large Hadron Collider (LHC) at CERN are expected to shed light on the mechanism of electroweak symmetry breaking, possibly by discovering the elusive Higgs boson. In the Standard Model, the dominant process of the Higgs boson production at the LHC is the gluon fusion, $gg \to H$, mediated by a top quark loop. During the last 20 years enormous efforts have been made to evaluate higher order corrections to this process.

The leading order (LO) result has been presented in Refs. [1] and already almost 15 years ago also the next-to-leading order (NLO) QCD corrections became available [2, 3, 4]. More recently also the next-to-next-to-leading order (NNLO) corrections have been evaluated [5, 6, 7, 8]. While the NLO results are exact in the top quark and Higgs boson masses, the NNLO results rely on the effective theory built in the limit of the large top quark mass (see, e.g., Refs. [9, 10] for the three-loop corrections to the effective ggH coupling). This approximation works surprisingly well at NLO, leading to $< 2\%$ deviations from the exact result for $M_H < 2M_t$ [11]. Recent numerical predictions of Higgs boson production in gluon fusion both at the Tevatron and the LHC are summarized in Ref. [12], including some results beyond the fixed-order perturbation theory.

NNLO effects of the finite top quark mass have been first addressed in Ref. [13], where the gluon-gluon channel has been considered in the limit of large center-of-mass energy $\sqrt{\hat{s}}$. Recently, in Ref. [14] the Higgs production cross-section was expanded in $\rho = M_H^2/M_t^2$ and calculated to $\mathcal{O}(\rho^6)$. Additional expansion around the soft limit (i.e. for $x = M_H^2/\hat{s} \to 1$) has been performed to a sufficiently high order to simplify the calculation.

In Ref. [15] we present the results of an independent calculation of these finite top mass effects, confirming the results of Ref. [14]. In this contribution the findings of Ref. [15] are summarized. We also asymptotically expand the QCD diagrams in $1/M_t$ and evaluate a few first terms in the series. However, our results are not expanded near the soft limit and the x-dependence of the cross section (valid below the top pair threshold, as discussed further) is retained.

2. Calculation of partonic cross-sections

The inclusive Higgs production in the collisions of protons originates from the corresponding QCD cross-sections of partons:

$$\hat{\sigma}_{ij\to H+X} = \hat{A}_{\rm LO}\left(\Delta_{ij}^{(0)} + \frac{\alpha_s}{\pi}\,\Delta_{ij}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta_{ij}^{(2)} + \ldots\right), \quad \hat{A}_{\rm LO} = \frac{G_F\,\alpha_s^2}{288\sqrt{2}\pi}\, f_0(\rho,0). \qquad (2.1)$$

Here ij denote one of the possible initial states: gg, qg, $q\bar{q}$, qq, or qq', and q and q' stand for (different) massless quark flavours. At the leading order, the only non-zero contribution is $\Delta_{gg}^{(0)} = \delta(1-x)$, and the function $f_0(\rho,0)$ given in Eq. (4) of Ref. [16] describes the mass dependence.

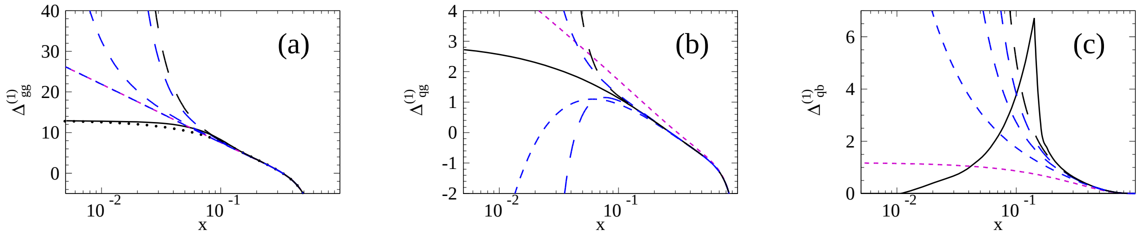

Figure 2: Partonic NLO cross sections for the (a) gg, (b) qg and (c) $q\bar{q}$ channel as functions of x for $M_H = 130$ GeV. The expansion in $\rho \to 0$ (dashed lines) is compared with the exact result (solid lines). Lines with longer dashes include higher order terms in ρ. The interpolation (see text) is shown as a dotted line.

The following discussion will focus on the x- and ρ-dependence of $\Delta^{(1)}_{ij}$ and $\Delta^{(2)}_{ij}$. In what follows, the "infinite top quark mass approximation" implies that $\Delta^{(k)}_{ij}$ are evaluated for $M_t \to \infty$, but $\hat{A}_{\rm LO}$ remains exact in M_t.

The calculation workflow can be summarized as follows. First, the diagrams are generated. To uniformly account for the real and virtual corrections we employ the optical theorem and compute the imaginary parts of the four-point forward-scattering amplitudes such as in Fig. 1. After taking traces we apply asymptotic expansion in the limit $M_t^2 \gg \hat{s}, M_H^2$. For cross-checks, we use two independent programs. The results are expressed in terms of factorized integrals of several kinds. The most non-trivial of them are two-loop four-point functions dependent on both $\hat{s}$ and M_H. After Laporta [17] reduction we end up with about 30 master integrals. The latter were studied in Ref. [8] (Appendix B), however, due to unfortunate misprints in that reference we re-computed the integrals with soft expansion and differential equation methods. Finally, we add renormalization and mass factorization contributions and obtain a few first terms in the expansion of $\Delta^{(k)}_{ij}$ in powers of ρ, where coefficients are functions of x.

3. NLO and NNLO results

The exact integral representations of the NLO functions $\Delta^{(1)}_{gg}$, $\Delta^{(1)}_{qg}$ and $\Delta^{(1)}_{q\bar{q}}$ can be found in Refs. [2, 3, 4]. In Fig. 2 we compare the x-dependence of the exact answers (evaluated for $M_H = 130$ GeV and $M_t = 173.1$ GeV) to the $\mathcal{O}(\rho^n)$ approximations for successive n.

In agreement with previous observations, the leading term in ρ is smooth and demonstrates a reasonably good agreement with the exact curve. However, the higher order terms in ρ introduce divergences at $x \to 0$ which are the most obvious for the $q\bar{q}$ channel. This signifies the breakdown of the assumption that $M_t^2 \gg \hat{s}$ for large $\hat{s}$. Note, however, the decent convergence above the threshold for the top quark pair production, i.e., for $x > x_{\rm th} = M_H^2/(4M_t^2)$ (in Fig. 2, $x_{\rm th} \approx 0.14$). In order to improve the $x = 0$ behaviour of the most important gg channel, we use its $\hat{s} \to \infty$ asymptotics found in Ref. [13]. Since this channel does not demonstrate any threshold effects, we construct some smooth interpolation between the $\mathcal{O}(\rho^n)$ result and the value at $x = 0$ in the region $0 < x < x_m$ with some $x_m < x_{\rm th}$. The result (dots in Fig. 2) agrees well with the exact curve.

For the quark channels, we use a simplified approach. (Corresponding $x = 0$ asymptotics became available in Ref. [18], but the hadronic results do not differ much from those presented

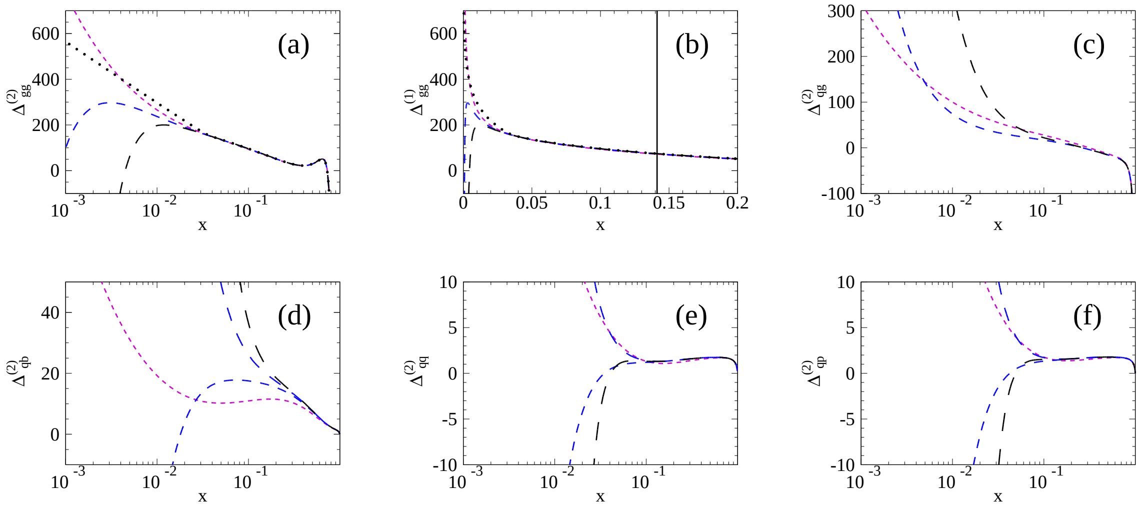

Figure 3: Partonic NNLO cross sections for the (a) *gg*, (c) *qg*, (d) $q\bar{q}$, (e) *qq*, (f) qq' channels functions of x for $M_H = 130$ GeV. Lines with longer dashes include higher order terms in ρ. In (b) we also show the *gg* channel in the linear scale. The dotted line in (a) and (b) corresponds to the matched result.

here.) For $x > x_{\rm th}$, we use the ρ-expansion including all known $\mathcal{O}(\rho^n)$ corrections, and for $x < x_{\rm th}$ – the $\mathcal{O}(\rho^0)$ approximation. The error in NLO hadronic predictions then is less than 50%, and if we assume that the same error estimate applies at NNLO, the overall effect is smaller than scale uncertainties.

The NNLO diagrams require considerably more effort and CPU time. Utilizing the virtual corrections published in Refs. [19, 16] we were able to evaluate $\mathcal{O}(\rho^n)$ terms in the quantities $\Delta_{ij}^{(2)}$ for expansion depth $n = 0,1,2$ for $gg \to H$, and $n = 0,1,2,3$ for the other channels; results are expressed in terms of harmonic polylogarithms of maximal weight 3.

The $\mathcal{O}(\rho^0)$ terms exactly reproduce the findings of Ref. [8]; expanding the higher $\mathcal{O}(\rho^n)$ corrections in $(1-x) \ll 1$ we find complete agreement with Ref. [14]. In Fig. 3 we present the x-dependence of the functions $\Delta_{ij}^{(2)}$ for $ij = gg, qg, q\bar{q}, qq, qq'$. As with the NLO, the higher order terms in ρ develop singularities near $x \to 0$, and below $x_{\rm th}$ the results converge. The dotted curves in Figs. 3(a) and (b) demonstrate extrapolations analogous to the NLO approach described above. The further numerical analysis is based on the these extrapolations, and the "simplified recipe" applied to quark channels.

4. Hadronic results

The hadronic cross sections are given by the convolution of the QCD cross sections $\hat{\sigma}_{ij\to H+X}$ with the corresponding parton distribution functions (PDFs). To discuss the numerical effect of our calculation we decompose the prediction of the total cross section into its LO, NLO and NNLO contributions: $\sigma_{pp'\to H+X}(s) = \sigma^{\rm LO} + \delta\sigma^{\rm NLO} + \delta\sigma^{\rm NNLO}$, and denote the heavy top quark approximation with an additional subscript ∞.

In Fig. 4 we show the M_H-dependence of the NNLO contribution to the hadronic cross section from $\Delta_{qg}^{(2)}$, $\Delta_{q\bar{q}}^{(2)}$, and $\Delta_{qq}^{(2)}$ normalized to the infinte top quark mass result. In all cases the power-

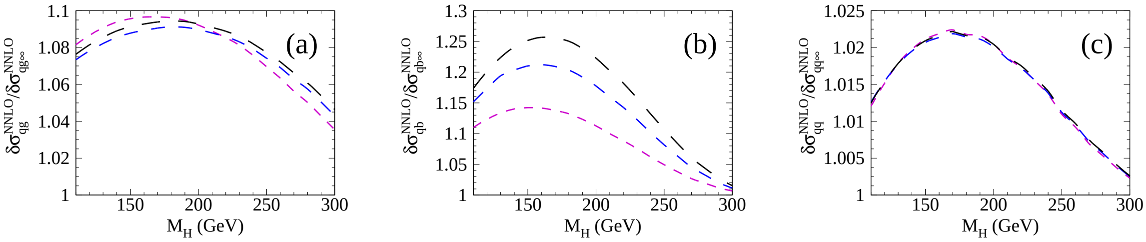

Figure 4: Ratio of the NNLO hadronic cross section including successive higher orders in $1/M_t$ (from short to long dashes) normalized to the infinite top quark mass result, (a) qg, (b) $q\bar{q}$, (c) qq. The qq' result is almost identical to that of qq.

suppressed terms lead to an increase of the cross section between 4% and 10% for the quark-gluon and up to 25% for the quark-anti-quark channel in our range of Higgs boson masses. The very rapid convergence is observed for the qq and qq' channels where the contribution beyond the $1/M_t^2$ term is practically zero.

The NNLO corrections to the most important channel, gg, are shown in Fig. 5(a), also normalized to the infinite top quark mass result. Finally, in Fig. 5(b) we compute the total gluon-induced cross-section taking into account the exact LO and NLO contributions. Different curves correspond to the effect of successive ρ terms in the NNLO piece. This plot can be directly compared to the left panel of Fig. 7 in Ref. [14], and the (minor) differences can be traced back to the different matching procedures. As can be seen, the effects of matching near $x = 0$ and M_t-suppressed corrections nearly cancel and the final deviation from the heavy top mass result is below 1%.

5. Conclusion

We present the NNLO production cross section of the Standard Model Higgs boson including the finite top quark mass effects. We observe rapid convergence of the series below the threshold for the production of real top quarks, i.e. for $\hat{s} \leq 4M_t^2$. To rectify the spurious $1/x^n$ behaviour for the dominant gluon-gluon channel we match our results to the large $\hat{s}$ limit.

The numerical impact of the top quark mass suppressed terms is below approximately 1% and thus about a factor ten smaller than the uncertainty from scale variation. Let us, however, stress that this result was not obvious a priori. Our calculation justifies the use of the heavy top quark mass approximation when evaluating the NNLO cross section.

In addition, we independently confirm the results of Ref. [8] for the infinite top quark mass and the M_t-suppressed terms calculated in Ref. [14].

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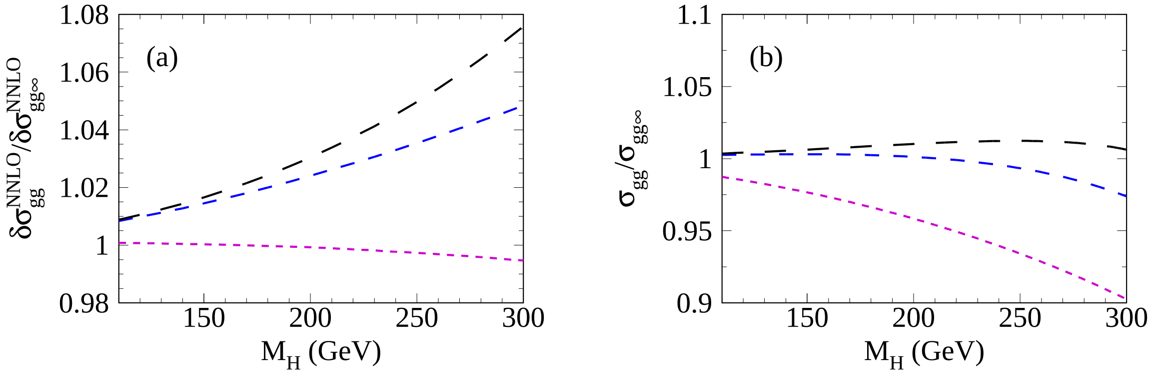

Figure 5: (a) Ratio of the NNLO hadronic cross section (*gg* contribution) including successive higher orders in $1/M_t$ normalized to the infinite top quark mass result. (b) Prediction for the gluon-induced inclusive Higgs production cross section up to NNLO normalized to the heavy top limit.

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